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Optimization in Allocating Goods to Shop Shelves Utilizing Genetic Algorithm under Expanded Shelf Position Case

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Abstract

How to allocate goods in shop shelves makes great influence to sales amount. Searching best fit allocation of goods to shelves is a kind of combinatorial problem. This becomes a problem of integer programming and utilizing genetic algorithm may be an effective method. Reviewing past researches, there are few researches made on this. Formerly, we have presented a papers concerning optimization in allocating goods to shop shelves utilizing genetic algorithm. In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examine the problem that allows goods to be allocated in multiple shelves and introduce the concept of sales profits and sales probabilities. Expansion of shelf position is executed. Optimization in allocating goods to shop shelves is investigated. Utilizing genetic algorithm, optimum solution is pursued and verified by a numerical example. Various patterns of problems must be examined hereafter.

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1 Introduction

Displaying method in the shop makes influence to sales amount, therefore various ideas are devised. What kind of items should be placed where in the shop, how to guide customers to what aisle in the shop are the big issues to be discussed. Searching best fit allocation of goods to shelves is also an important issue to be solved. In this paper, we seek how to optimize in allocating goods to shop shelves.

As for allocating good to shop shelves, following items are well known (Nagashima, 2005).

Shelf height is classified as follows.

- Shelf of 135cm height: Customers can see the whole space of the shop. Specialty stores often use this type.
- Shelf of 150cm height: Female customers may feel pressure to the shelf height. This height may be the upper limit to look over the shop.
- Shelf of 180cm height: It becomes hard to look over the shop. Therefore it should not be used for island display (display at the center or inside the shop).

Next, we show the following three functions of shelf for display.

1. Exhibition of goods function
2. Stock function
3. Display function

Effective range for exhibition is generally said to be 45cm-150cm. The range of 75cm-135cm is called golden zone especially. For the lower part under 45cm, goods are stocked as well as displaying.

Reviewing past papers, there are many papers concerning lay out problem. As for the problem of the distribution of equipment, we can see B. Korte *et al*. (2005), M. Gen *et al*. (1997) for the general research book. There are many researches made on this. Yamada *et al*. (2004) handles the lay out problem considering the aisle structure and intra-department material flow. Y. Wu *et al.* (2002) and Yamada *et al.* (2004) handle this problem considering aisle structure. Ito *et al.* (2006) considers multi-floor facility problem.

Although there are many researches on corresponding theme as stated above, we can hardly find researches on the problem of optimization in allocating goods to shop shelves.

Formerly, we have presented a paper concerning optimization in allocating goods to shop shelves utilizing genetic algorithm (Takeyasu *et al.*,2008). In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examine the problem that allows goods to be allocated in multiple shelves and introduce the concept of sales profits and sales probabilities. Expansion of shelf position is executed. Optimization in allocating goods to shop shelves is investigated. Utilizing genetic algorithm, optimum solution is pursued and verified by a numerical example.

The rest of the paper is organized as follows. Problem description is stated in section 2. Genetic Algorithm is developed in section 3. Numerical example is exhibited in section 4 which is followed by the remarks of section 5. Section 6 is a summary.

2 Problem description

Shelf model is constructed as Figure 1. There are five shelf positions. Shelf position 1 is mainly to put big and heavy goods including stock function. Shelf position 3, 4 at the height of the range 75cm to 135cm are the space of golden zone. Thus, we can use shelves properly by assuming these shelves. In numerical example, we examine using these five shelves. First of all, we make problem description in the case there is only one shelf (case 1). Then we expand to the case there are multiple shelves (case 2).

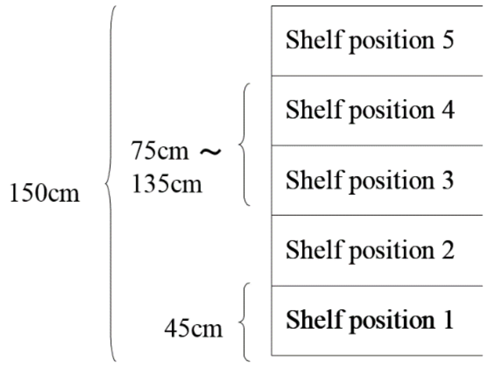

Figure 1: Shelf Model

(1) Case1: The case that there is only one shelf

Although there are few cases that there is only one shelf, it makes the foundation for multiple shelves case. Therefore we pick it up as a fundamental one. Suppose shelf position k is from 1 to L (Figure 2).

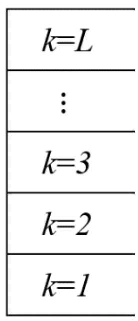

Figure 2: Shelf Position

Suppose there are N amount of goods $(i = 1, \cdots, N)$. Set sales profit of goods i as H^i. Table 1 shows the sales probabilities when each goods is placed at each shelf position. The values in this table are written for example.

Table 1: Sales probability for each goods

Day of the Week	Time Zone(t)	Shelf $j=1$			Shelf $j=2$			…	Shelf $j=m$		
		Shelf Position			Shelf Position			…	Shelf Position		
		$k=1$	…	$k=L_1$	$k=1$	…	$k=L_2$	…	$k=1$	…	$k=L_m$
(Mon.)	0-1(t=1)	0.01	…								
	1-2(t=2)	0.02									
	…										
	23-24(t=24)	0.03									
(Tue.)	0-1(t=25)	0.02									
	1-2(t=26)	0.02									
	…										
	23-24(t=48)	0.03									
…	…	…	…	…	…	…	…	…	…	…	…
(Sun.)	0-1(t=145)	0.02									
	1-2(t=146)	0.03									
	…										
	23-24(t=168)	0.04									

Suppose goods are sold in the period from t_1 to t_n. In addition, a new goods i is replenished when goods i is sold out. Set the accumulated sales probability of goods i in time zone t, shelf j, and shelf position k in the table as $HK_{t,j,k}^{i}$.

Then, the sales probability $K_{t_1/t_n}^{i,j,k}$ of goods i in the period will be described as follows.

$$K_{t_1/t_n}^{i,j,k} = \sum_{t=1}^{n} HK_{t,j,k}^{i}$$

This can take the value more than 1. For example, the value 2 means that 2 amount of goods were sold during the period. Set Benefit in the sales period from t_1 to t_n as $P_{t_1/t_n}^{i,j,k}$ $(i = 1,\cdots,N)(j = 1,\cdots,m)(k = 1,\cdots,L)$ when goods i is placed at shelf j and shelf position k.

Where Benefit means:

$$Benefit = SalesProbability \times SalesProfit$$

Therefore, this equation is represented as follows.

$$P_{t_1/t_n}^{i,j,k} = K_{t_1/t_n}^{i,j,k} \cdot H^i \tag{1}$$

where $j = 1$ because one shelf case is considered here. Set $x_{i,k}$ as:

$x_{i,k} = 1$: Goods i is placed at shelf position k

$x_{i,k} = 0$: Else

Suppose only one goods can be placed at one shelf position and also suppose that goods is allowed to allocate in multiple shelf positions. Then constraints are described as follows.

$$x_{i,k} = 1,0 (i = 1, \cdots, N)(k = 1, \cdots L) \tag{2}$$

$$\sum_{i=1}^{N} x_{i,k} = 1 (k = 1, \cdots L) \tag{3}$$

Under these constraints,

$$Maximize \quad J = \sum_{k=1}^{L} \sum_{i=1}^{N} P_{t_1/t_n}^{i,j,k} x_{i,k} \tag{4}$$

(2) Case2: The case that there are m shelves

Suppose there are m shelves (Figure 3). Set Benefit as $P_{t_1/t_n}^{i,j,k}$ $(i = 1, \cdots, N)(j = 1, \cdots, m)(k = 1, \cdots, L)$ where goods i is placed at shelf position k of shelf j. The sales period is the same with above stated (1).

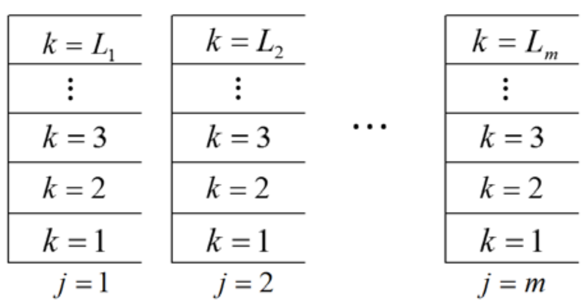

Figure 3: Shelf Position under multiple shelves

Set $x_{i,j,k}$ as:

$x_{i,j,k} = 1$: Goods is placed at shelf position k of shelf j

$x_{i,j,k} = 0$: Else

Suppose only one goods can be placed at one shelf position and also suppose that goods is allowed to allocate in multiple shelf positions. Then constraints are described as follows. The sales period is the same with before.

$$x_{i,j,k} = 1,0(i = 1,\cdots,N)(j = 1,\cdots,m)\left(k = 1,\cdots,L_j\right) \tag{5}$$

$$\sum_{i=1}^{N} x_{i,j,k} = 1(j = 1,\cdots,m)\left(k = 1,\cdots,L_j\right) \tag{6}$$

Under these constraints,

$$Maximize \quad J = \sum_{i=1}^{N}\sum_{j=1}^{m}\sum_{k=1}^{L_j} P_{t_1/t_n}^{i,j,k} x_{i,j,k} \tag{7}$$

3 Algorithm

We can make problem description as stated above, although these are somewhat under restricted cases. As far as only these are considered as they are, there is little difference between these and the conventional optimization problems. However, as soon as the number of involved shelves becomes larger, the number of variables dramatically grows greater, to which the application of Genetic Algorithm solution and Neural Network solutions may be appropriate. There are various means to solve this problem. When that variable takes the value of 0 or 1, the application of genetic algorithm would be a good method. As is well known, the calculation volume reaches numerous or even infinite amounts in these problems when the number of variables increases. It is reported that GA is effective for these problems (Gen et al. (1995), Lin et al. (2005), Zhang et al. (2005)).

A. The Variables

Suppose the number of goods, shelf position, and shelf are 20, 6, 2 respectively. In this paper, shelf position is expanded from 5 to 6. Then the number of variables becomes 240.

$$x_{i,j,k} = 1,0(i = 1,\cdots,20),(j = 1,2),(k = 1,\cdots,6)$$

Therefore, set chromosome as follows.

$$\begin{aligned} X = (&x_{1,1,1}, x_{2,1,1}, x_{3,1,1}, \cdots, x_{20,1,1}, \\ &x_{1,1,2}, x_{2,1,2}, x_{3,1,2}, \cdots, x_{20,1,2}, \\ &\vdots \\ &x_{1,1,6}, x_{2,1,6}, x_{3,1,6}, \cdots, x_{20,1,6}, \\ &x_{1,2,1}, x_{2,2,1}, x_{3,2,1}, \cdots x_{20,2,1}, \\ &x_{1,2,2}, x_{2,2,2}, x_{3,2,2}, \cdots, x_{20,2,2} \\ &\vdots \\ &x_{1,2,6}, x_{2,2,6}, x_{3,2,6}, \cdots, x_{20,2,6}) \end{aligned} \tag{8}$$

B. Initialize population

Initialization of population is executed. The number of initial population is M. Here set $M = 100$. Set gene at random and choose individual which satisfies constraints.

C. Selection

In this paper, we take elitism while selecting. Choose P individuals in the order which take maximum score of objective function. Here, set $P = 20$.

D. Crossover

Here, we take uniform crossover. Set crossover rate as:

$$P_c = 0.7 \tag{9}$$

E. Mutation

Set mutation rate as:

$$P_m = 0.01 \tag{10}$$

Algorithm of GA is exhibited at Table 2.

Table 2: Algorithm of multi-step tournament selection method

Step 1：	Set maximum No. as g_{max}, population size as P, crossover rate as P_c, mutation rate as P_m.
Step 2：	Set $t = 1$ for generation No. and generate initial solution matrix $x_p(t) = \left(x_{ikj}^{p}\right)$ $(p = 1, \cdots, M)$.
Step 3：	Calculate Objective function $J\left(x_p(t)\right)$ for all solution matrix $x_p(t)$ $(p = 1, \cdots, P)$ in generation t.
Step 4：	Set $t = t + 1$ until $t > g_{max}$.
Step 5：	Crossover Generate new individual by crossover utilizing the method of above stated *D*.
Step 6：	Mutation Reproduce by mutation utilizing the method of above stated *E*.
Step 7：	Calculate objective function for reproduction of generation t.
Step 8：	Selection Next generation is selected by elitism. Go to Step 4.

Introducing the variable y_s such that:

$$y_s = i \tag{11}$$

where

$$s = k + (j - 1) \cdot 6 \tag{12}$$

when

$$x_{i,j,k} = 1$$

then (8) is expressed as:

$$Y = (y_1, y_2, \cdots, y_{12}) \tag{13}$$

4 Numerical example

Numerical example is executed in “Case 2” of 2 (2). Suppose the sales period is 5 days for Monday through Friday. Table 3 shows the unit sales profit H^i of each goods.

Table 3: Unit Sales Price and Sales Profit of each goods

Lot i	Sales Price	H^i	
1		6000	For Women
2		5500	
3		5500	
4		5000	
5		4500	
6		4500	
7		4000	
8		3500	
9		3000	
10		3000	For Men
11		6000	
12		5500	
13		5000	
14		4500	
15		4000	
16		3500	
17		3000	
18		4000	For Kids
19		3000	
20		2000	

Supposing a general daytime retail store, we set opening time to be 9 through 18 o’clock. Table 4 shows the sales probabilities of lot i as an example.

Table 5 shows the sales probability by shelf for each shelf position. Table 6 shows the value in which Table 4 and Table 5 are multiplied. Table 7 shows the benefit Table in which accumulated probability of Table 6 and Sales Profit of Table 3 are multiplied

Table 4: Sales Probability of Lot i (Time Zone)

Day of the Week	Time Zone(t)	Sales Probability	Day of the Week	Time Zone(t)	Sales Probability
(Mon.)	9-10		(Thu.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	
(Tue.)	9-10		(Fri.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	
(Wed.)	9-10		(Sat.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	

Table 5: Sales Probability of Lot i (Shelf Position)

Shelf $j=1$						Shelf $j=2$					
Shelf Position						Shelf Position					
k=1	k=2	k=3	k=4	k=5	k=6	k=1	k=2	k=3	k=4	k=5	k=6
0.7	0.9	1.2	1.2	0.9	0.8	0.8	1.0	1.3	1.3	1.0	0.9

In Table 5, shelf $j=2$ is located near the entrance therefore the table value reflects this condition.

Table 6: Sales Probability of Lot i

Day of the Week	Time Zone(t)	Sales Probability											
		Shelf Position						Shelf Position					
		k=1	k=2	k=3	k=4	k=5	k=6	k=1	k=2	k=3	k=4	k=5	k=6
(Mon.)	9-10												
	10-11												
	11-12												
	12-13												
	13-14												
	14-15												
	15-16												
	16-17												
	17-18												
(Tue.)	9-10												
	10-11												
	11-12												
	12-13												
	13-14												
	14-15												
	15-16												
	16-17												
	17-18												
(Sat.)	9-10												
	10-11												
	11-12												
	12-13												
	13-14												
	14-15												
	15-16												
	16-17												
	17-18												

Table 7 shows the benefit when each goods is placed at each shelf position of each shelf.

Table 7: Benefit Table

	Shelf 1						Shelf 2						
	Shelf Position						Shelf Position						
Lot i	1	2	3	4	5	6	1	2	3	4	5	6	
1	2580	4380	5400	5220	4380	4200	5220	7020	7980	7800	7020	6500	
2	2750	4620	5610	5390	4620	4300	5005	6820	7810	7590	6820	6400	For Women
3	3025	4840	5830	5610	4840	4400	4840	6600	7590	7425	6600	6200	
4	3200	5000	6000	5800	5000	4600	4600	6400	7400	7200	6400	6000	
5	3420	5130	6750	6345	5130	4700	4230	6750	7830	7560	6660	6100	
6	3600	5400	6390	6210	5400	4800	4185	5985	7020	6795	5985	5000	
7	3800	5600	6600	6400	5600	4900	4000	5800	6800	6600	5800	5000	
8	3990	5810	6790	6615	5810	5000	3815	5600	6615	6405	5600	5000	
9	4200	6000	6900	6810	6000	5200	3600	5400	6300	6210	5400	4900	
10	4410	6210	7200	6990	6210	5300	3390	5190	6210	6000	5190	4500	
11	2220	4980	5820	6000	5580	4900	4200	7020	7800	7980	7620	6800	For Men
12	2420	5225	5995	6215	5830	5050	4015	6820	7590	7810	7425	6750	
13	2600	5400	6200	6400	6000	5200	3800	6600	7400	7600	7200	6500	
14	2790	5580	6390	6615	6210	5300	3600	6390	7200	7380	7020	6100	
15	3000	5800	6600	6800	6400	5400	3400	6200	7000	7200	6800	6000	
16	3185	5985	6790	7000	6615	5500	3185	5985	6790	7000	6615	5800	
17	3390	6240	6990	7200	6810	5600	3000	5790	6600	6810	6390	5600	
18	6800	6800	3200	2400	200	200	8000	8000	4400	3400	1400	1400	For Kids
19	6990	6990	3390	2610	390	390	7800	7800	4200	3210	1200	1400	
20	7200	7200	3600	2800	600	600	7600	7600	4000	3000	1000	1000	

Experimental results are as follows. The expression Eq. (8) is complicated. Therefore we use expression by Eq. (13). A sample set of initial population is exhibited in Table 6.

Table 8: A Sample Set of Initial Population

$Y_1 =$	(15,	8,	10,	16,	4,	16,	17,	1,	6,	7,	18,	2)
$Y_2 =$	(11,	10,	9,	2,	7,	18,	15,	10,	8,	20,	18,	13)
$Y_3 =$	(12,	4,	14,	11,	9,	2,	18,	15,	7,	12,	8,	11)
						⋮						
$Y_{98} =$	(20,	15,	4,	16,	13,	16,	5,	15,	4,	4,	12,	16)
$Y_{99} =$	(2,	20,	5,	17,	6,	16,	1,	7,	9,	7,	16,	7)
$Y_{100} =$	(8,	7,	7,	8,	11,	7,	18,	20,	8,	10,	6,	11)

Convergence process is exhibited in Figure 4.

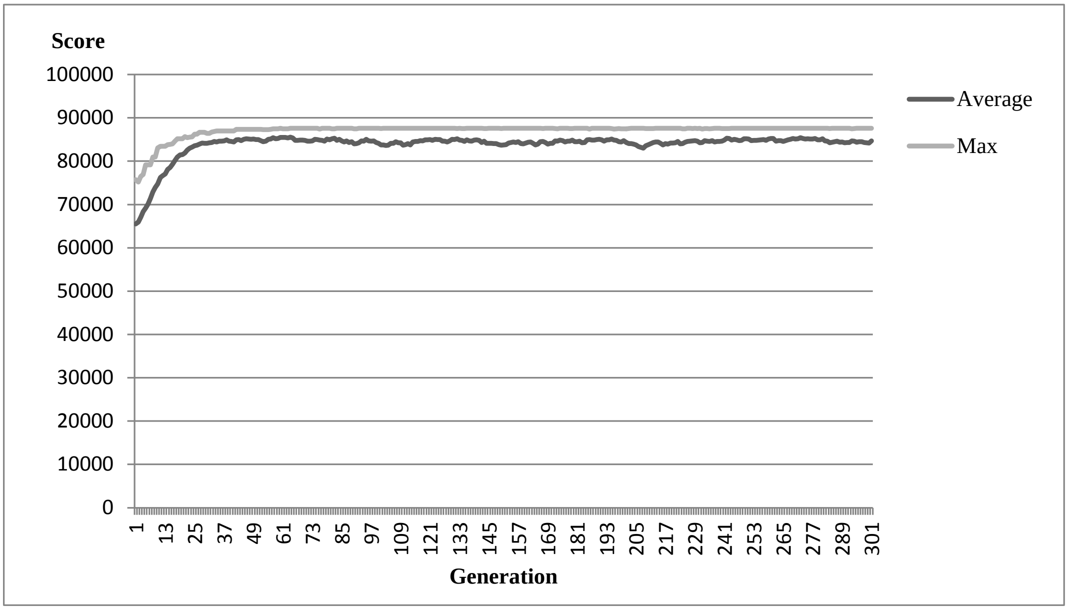

Figure 4: Convergence Process of Case 2

The problem is simple, so combination of genotype for crossover saturates in the 63thgeneration. Genotype in which objective function becomes maximum is as follows.

$Y =$ (20, 20, 10, 17, 17, 17, 18, 18, 1, 11, 11, 11)

This coincides with the result of optimal solution by the calculation of all considerable cases, therefore it coincides with a theoretical optimal solution. We take up simple problem and we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

5 Remarks

As there are few papers made on this theme, we constructed prototype version before (Takeyasu et al.,2008). In this paper, we examined the problem that allowed goods to be allocated in multiple shelves and introduced the concept of sales profits and sales probabilities. An application to the shop with POS sales data was executed. We can see that genetic algorithm is effective for this problem.

In practice, following themes occur.

1. Sales probabilities should be arranged correctly.
2. There are various types of shelves corresponding to goods characteristics (For example, cold storage goods).
3. Furthermore, genotype must be devised in construction when there are huge number of goods and shelves.

For these issues, expanded version of the paper will be built hereafter consecutively. As for 1, constraints are relaxed than those of this paper. As for 2, expansion is easy to make. As for 3, constructing genotype from the shelf side would bear much more simple expression.

6 Conclusion

How to allocate goods in shop shelves makes great influence to sales amount. Searching best fit allocation of goods to shelves is a kind of combinatorial problem. This becomes a problem of integer programming and utilizing genetic algorithm may be an effective method. Reviewing past researches, there were few researches made on this. Formerly, we had presented papers concerning optimization in allocating goods to shop shelves utilizing genetic algorithm. In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examined the problem that allowed goods to be allocated in multiple shelves and introduced the concept of sales profits and sales probabilities. Expansion of shelf position is executed. Optimization in allocating goods to shop shelves was investigated. An application to the shop with POS sales data was executed. Utilizing genetic algorithm, optimum solution was pursued and verified by a numerical example. Various patterns of problems should be examined hereafter.

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