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Estimation of Mean with Non-Response in Double Sampling Using Exponential Technique*

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Abstract

This paper presents exponential type ratio and product estimators for finite population mean of the study variable in presence of non-response when the population mean of the auxiliary variable is unknown. The expressions for Mean Square Error of the proposed estimators have been obtained to the first degree of approximation. These estimators are compared for their precision with Hansen-Hurwitz [2] unbiased estimator and Khare and Srivastava [5] estimators. An empirical study is also carried out to judge the merits of the suggested estimators.

Mathematics Subject Classification: 62D05

Keywords: Auxiliary variable; Study variable; Double sampling; Mean Square Error; Non-response

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1 Introduction

In practice almost all surveys suffer from non-response. The problem of non-response often happens due to the refusal of the subject, absenteeism and sometimes due to the lack of information. The pioneering work of Hansen and Hurwitz [2], assumed that a sub-sample of initial non-respondents is re-contacted with a more expensive method, suggesting the first attempt by mail questionnaire and the second attempt by a personal interview. In estimating population parameters such as the mean, total or ratio, sample survey experts sometimes use auxiliary information to improve precision of the estimates. Sodipo and Obisesan [10] have considered the problem of estimating the population mean in the presence of non-response, in sample survey with full response of an auxiliary character *x*. Other authors such as Cochran [1], Rao [8], Khare and Srivastava [5, 6], Okafor and Lee [7], Khare and Sinha [4], Tabasum and Khan [12, 13] and Khare et al [3] have studied the problem of non-response under double (two-phase) sampling.

Let in a finite population $U = \{U_1, U_2, ..., U_N\}$ of size N, a large first phase sample of size n' is selected by simple random sampling without replacement (SRSWOR). A smaller second phase sample of size n is selected from n' by SRSWOR. Non-response occurs on the second phase sample of size n in which n_1 units respond and n_2 units do not. From the n_2 non-respondents, by SRSWOR a sample of $m = n_2/k$; $k > 1$ units are selected where k is the inverse sampling rate at the second phase sample of size n. Here we assume that the response is obtained for all the m units. This method of double sampling can be applied in a household survey where the household size is used as an auxiliary variable for the estimation of family expenditure. Information can be obtained completely on the family size, while there may be some non-response on the household expenditure. The whole population is divided into two classes, one consists of N_1 units, which would respond on the first attempt at the second phase and the other consists of $N_2 (= N - N_1)$ units, which would not respond on the first attempt at the second phase of sampling but will respond on the second attempt. Hansen and Hurwitz [2] suggested an unbiased estimator for population mean $\bar{Y}$ of the study variable y, is defined by

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{2m} \tag{1}$$

The variance of unbiased estimator $\bar{y}^*$ is given by

$$V(\bar{y}^*) = \frac{(N-n)}{nN} S_y^2 + \frac{N_2(k-1)}{nN} S_{y(2)}^2 \tag{2}$$

where $w_1 = n_1/n$, $w_2 = n_2/n$, $\bar{y}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i$, $\bar{y}_{2m} = \frac{1}{m}\sum_{i=1}^{m} y_i$,

$$S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \bar{Y})^2, \quad S_{y(2)}^2 = \frac{1}{(N_2-1)}\sum_{i=1}^{N_2}(Y_i - \bar{Y}_2)^2,$$

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} Y_{2i}, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \bar{y}_{2m} = \frac{1}{m}\sum_{i=1}^{m} y_i .$$

It is well known that in estimating the population mean, sample survey experts sometimes use auxiliary information to improve the precision of the estimates. Let x denote an auxiliary variable with population mean $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$.

Let $\bar{X}_1 = \frac{1}{N_1}\sum_{i=1}^{N_1} X_i$ and $\bar{X}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} X_{2i}$ denote the population means of the response and non-response groups (or strata). Let $\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ and $\bar{x}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_i$ denote the means of the n_1 responding units and n_2 non-responding. Further, let $\bar{x}_{2m} = \frac{1}{m}\sum_{i=1}^{m} x_i$ denote the mean of the $m = n_2/k$; $k > 1$ sub-sampled units. Keeping with this background we define an unbiased estimator of population mean $\bar{X}$ as

$$\bar{x}^* = w_1\,\bar{x}_1 + w_2\,\bar{x}_{2m} \tag{3}$$

The variance of $\bar{x}^*$ is given by

$$V\left(\bar{x}^*\right) = \frac{(N-n)}{n\,N} S_x^2 + \frac{N_2(k-1)}{n\,N} S_{x(2)}^2 \tag{4}$$

where $S_x^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2$, $S_{x(2)}^2 = \frac{1}{(N_2-1)} \sum_{i=1}^{N_2} \left(X_i - \bar{X}_2\right)^2$.

The objective of this paper is to propose exponential estimators for population mean using auxiliary character in the presence of non-response under double sampling. The properties of the suggested estimators are given under a large sample approximation. An empirical study is carried out to demonstrate the performance of the suggested estimators over others.

2 Proposed Ratio and Product Estimators

Let $\bar{x}' = \frac{1}{n'} \sum_{i=1}^{n'} x_i$ (based on the first phase sample) be the sample mean estimator of population mean $\bar{X}$, and let $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ (based on the second phase sample) be the sample mean estimator of population means $\bar{X}$, $\bar{Y}$ respectively. Then the double sampling version of the ratio $\bar{y}_{Rd}$ and product $\bar{y}_{Pd}$ estimators of population mean $\bar{Y}$ are respectively given by

$$\bar{y}_{Rd} = \bar{y}\,\frac{\bar{x}'}{\bar{x}}$$

and

$$\bar{y}_{Pd} = \bar{y}\,\frac{\bar{x}}{\bar{x}'}$$

Singh and Vishwakarma [9] suggested the exponential ratio and product estimators for population mean $\bar{Y}$ as

$$\bar{y}_{\mathrm{Re}} = \bar{y}\,\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right)$$

and

$$\bar{y}_{Pe} = \bar{y}\,\exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right)$$

Khare and Srivastava [5] proposed the conventional ratio and product estimators in the presence of non-response two phase sampling as

Case I: non- response is only on study variable

$$\bar{y}^*_{IRd} = \bar{y}^*\,\frac{\bar{x}'}{\bar{x}} \tag{5}$$

and

$$\bar{y}^*_{IPd} = \bar{y}^*\,\frac{\bar{x}}{\bar{x}'} \tag{6}$$

Case II: non- response is both on study and auxiliary variables

$$\bar{y}^*_{IIRd} = \bar{y}^*\,\frac{\bar{x}'}{\bar{x}^*} \tag{7}$$

and

$$\bar{y}^*_{IIPd} = \bar{y}^*\,\frac{\bar{x}^*}{\bar{x}'} \tag{8}$$

To the first degree of approximation, the Mean Square Error (MSE) of the estimators $\bar{y}^*_{IRd}$, $\bar{y}^*_{IPd}$, $\bar{y}^*_{IIRd}$ and $\bar{y}^*_{IIPd}$ are given by

$$MSE(\bar{y}^*_{IRd}) = \left(\frac{N-n}{nN}\right)S_y^2 + R\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 - 2\rho_{yx}S_yS_x\right\} + \left(\frac{N_2(k-1)}{nN}\right)S_{y(2)}^2 \tag{9}$$

$$MSE(\bar{y}^*_{IPd}) = \left(\frac{N-n}{nN}\right)S_y^2 + R\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 + 2\rho_{yx}S_yS_x\right\} + \left(\frac{N_2(k-1)}{nN}\right)S_{y(2)}^2 \tag{10}$$

$$\begin{aligned} MSE(\bar{y}^*_{IIRd}) = {} & \left(\frac{N-n}{nN}\right)S_y^2 + R\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 - 2\rho_{yx}S_yS_x\right\} \\ & + \left(\frac{N_2(k-1)}{nN}\right)\left\{S_{y(2)}^2 + R^2S_{x(2)}^2 - 2R\,\rho_{yx(2)}S_{y(2)}S_{x(2)}\right\} \end{aligned} \tag{11}$$

$$MSE(\bar{y}^{*}_{IIPd}) = \left(\frac{N-n}{nN}\right)S_y^2 + R\left(\frac{n'-n}{nn'}\right)\left\{R S_x^2 + 2\rho_{yx}S_yS_x\right\} + \left(\frac{N_2(k-1)}{nN}\right)\left\{S_{y(2)}^2 + R^2S_{x(2)}^2 + 2R\,\rho_{yx(2)}S_{y(2)}S_{x(2)}\right\} \quad (12)$$

where, $R = \bar{Y}/\bar{X}$. Motivated by Singh and Vishwakarma [9], we propose exponential ratio and product estimators for population mean $\bar{Y}$ using auxiliary character in the presence of non-response as

Case I: non- response is only on study variable

$$\bar{y}^{*}_{I\,\mathrm{Re}} = \bar{y}^{*}\ \exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \quad (13)$$

and

$$\bar{y}^{*}_{I\,Pe} = \bar{y}^{*}\ \exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right) \quad (14)$$

Case II: non- response is both on study and auxiliary variables

$$\bar{y}^{*}_{II\,\mathrm{Re}} = \bar{y}^{*}\ \exp\left(\frac{\bar{x}' - \bar{x}^{*}}{\bar{x}' + \bar{x}^{*}}\right) \quad (15)$$

and

$$\bar{y}^{*}_{II\,Pe} = \bar{y}^{*}\ \exp\left(\frac{\bar{x}^{*} - \bar{x}'}{\bar{x}^{*} + \bar{x}'}\right) \quad (16)$$

Remark: There are many ways to utilize the available information on estimation stage in terms of the formulation of suitable estimators. Motivated with this argument, we have used the information on auxiliary variable under Hansen-Hurwitz techniques which results precise the estimation procedure and the subsequent estimators given in Eq. (15) and Eq. (16).

It is easily observed that $\bar{y}^{*}_{I\,\mathrm{Re}}$, $\bar{y}^{*}_{I\,Pe}$, $\bar{y}^{*}_{II\,\mathrm{Re}}$ and $\bar{y}^{*}_{II\,Pe}$ are biased estimators, but the bias being of the order n^{-1}, can be assumed negligible in large samples. It is assumed that the sample size n is large enough so that the biases of these estimators are negligible and the Mean Square Errors (MSEs) of all biased estimators are obtained up to the terms of order n^{-1}.

3 Mean Square Error (MSE) of the Proposed Estimators

To obtain the MSE of the proposed estimators $\bar{y}^*_{I\,\mathrm{Re}}$, $\bar{y}^*_{I\,Pe}$, $\bar{y}^*_{II\,\mathrm{Re}}$ and $\bar{y}^*_{II\,Pe}$, we assume $\bar{y}^* = \bar{Y}(1+e_0)$, $\bar{x}^* = \bar{X}(1+e_1)$, $\bar{x}' = \bar{X}(1+e_1')$ and $\bar{x} = \bar{X}(1+e_2)$ such that $E(e_0) = E(e_1) = E(e_1') = E(e_2) = 0$ and under SRSWOR sampling scheme,

$$\left.\begin{aligned}
E\left(e_0^2\right) &= \frac{1}{\bar{Y}^2}\left[\frac{(N-n)}{nN}S_y^2 + \frac{N_2(k-1)}{nN}S_{y(2)}^2\right],\\
E\left(e_1^2\right) &= \frac{1}{\bar{X}^2}\left[\frac{(N-n)}{nN}S_x^2 + \frac{N_2(k-1)}{nN}S_{x(2)}^2\right],\\
E\left(e_1'^2\right) &= E\left(e_1e_1'\right) = E\left(e_2e_1'\right) = \frac{(N-n')}{n'N}\frac{S_x^2}{\bar{X}^2},\\
E\left(e_2^2\right) &= E\left(e_1e_2\right) = \frac{(N-n)}{nN}\frac{S_x^2}{\bar{X}^2},\\
E\left(e_0e_1\right) &= \frac{1}{\bar{Y}\,\bar{X}}\left[\frac{(N-n)}{nN}\rho_{yx}\,S_yS_x + \frac{N_2(k-1)}{nN}\rho_{yx(2)}\,S_{y(2)}S_{x(2)}\right],\\
E\left(e_0e_2\right) &= \frac{(N-n)}{nN}\rho_{yx}\frac{S_yS_x}{\bar{Y}\,\bar{X}},\\
E\left(e_0e_1'\right) &= \frac{(N-n')}{n'N}\rho_{yx}\frac{S_yS_x}{\bar{Y}\,\bar{X}}
\end{aligned}\right\} \tag{17}$$

where, ρ_{yx} and $\rho_{yx(2)}$ are respectively the correlation coefficients of response and non-response groups between study variable y and auxiliary variable x. We can reasonably assume that the sample sizes are large enough to make $|e_0|$, $|e_1|$, $|e_2|$ and $|e_1'| << 1$. Now we have

$$\begin{aligned}
\bar{y}^*_{I\,\mathrm{Re}} &= \bar{Y}(1+e_0)\exp\left\{\frac{e_1'-e_2}{2+e_1'+e_2}\right\}\\
&= \bar{Y}(1+e_0)\exp\left[\frac{1}{2}(e_1'-e_2)\left\{1+\frac{1}{2}(e_1'+e_2)\right\}^{-1}\right]
\end{aligned} \tag{18}$$

$$\bar{y}^*_{I\,Pe} = \bar{Y}(1+e_0)\exp\left\{\frac{e_2 - e_1'}{2+e_1'+e_2}\right\}$$

$$= \bar{Y}(1+e_0)\exp\left[-\frac{1}{2}(e_1'-e_2)\left\{1+\frac{1}{2}(e_1'+e_2)\right\}^{-1}\right] \tag{19}$$

$$\bar{y}^*_{II\,\mathrm{Re}} = \bar{Y}(1+e_0)\exp\left\{\frac{e_1' - e_1}{2+e_1'+e_1}\right\}$$

$$= \bar{Y}(1+e_0)exp\left[\frac{1}{2}(e_1'-e_1)\left\{1+\frac{1}{2}(e_1'+e_1)\right\}^{-1}\right] \tag{20}$$

$$\bar{y}^*_{II\,Pe} = \bar{Y}(1+e_0)\exp\left\{\frac{e_1 - e_1'}{2+e_1'+e_1}\right\}$$

$$= \bar{Y}(1+e_0)exp\left[-\frac{1}{2}(e_1'-e_1)\left\{1+\frac{1}{2}(e_1'+e_1)\right\}^{-1}\right] \tag{21}$$

Expanding the right hand sides of equations Eqs. (18), (19), (20) and (21), multiplying out and neglecting the terms of e's involving degree greater than or equal to two, we get

$$\bar{y}^*_{I\,\mathrm{Re}} \cong \bar{Y}\left[1+e_0+\frac{1}{2}(e_1'-e_2)\right]$$

or, $$\left(\bar{y}^*_{I\,\mathrm{Re}} - \bar{Y}\right) = \bar{Y}\left[e_0+\frac{1}{2}(e_1'-e_2)\right] \tag{22}$$

$$\bar{y}^*_{I\,Pe} \cong \bar{Y}\left[1+e_0-\frac{1}{2}(e_1'-e_2)\right]$$

or, $$\left(\bar{y}^*_{I\,Pe} - \bar{Y}\right) = \bar{Y}\left[e_0-\frac{1}{2}(e_1'-e_2)\right] \tag{23}$$

$$\bar{y}^*_{II\,\mathrm{Re}} \cong \bar{Y}\left[1+e_0+\frac{1}{2}(e_1'-e_1)\right]$$

or $$\left(\bar{y}^*_{II\,\mathrm{Re}} - \bar{Y}\right) = \bar{Y}\left[e_0+\frac{1}{2}(e_1'-e_1)\right] \tag{24}$$

$$\bar{y}^*_{II\,Pe} \cong \bar{Y}\left[1 + e_0 - \frac{1}{2}(e_1' - e_1)\right]$$

or
$$\left(\bar{y}^*_{II\,Pe} - \bar{Y}\right) = \bar{Y}\left[e_0 - \frac{1}{2}(e_1' - e_1)\right] \tag{25}$$

Squaring both sides of equations Eqs. (22), (23), (24) and (25), taking expectations and using the results in Eq. (17), we get the MSEs of $\bar{y}^*_{I\,\mathrm{Re}}$, $\bar{y}^*_{I\,Pe}$, $\bar{y}^*_{II\,\mathrm{Re}}$ and $\bar{y}^*_{II\,Pe}$ to the first degree of approximation, respectively as

$$MSE\left(\bar{y}^*_{I\,\mathrm{Re}}\right) = \bar{Y}^2 E\left[e_0 + \frac{1}{2}(e_1' - e_2)\right]^2$$

$$= \bar{Y}^2 E\left[e_0^2 + \frac{1}{4}\left(e_1'^2 - 2e_1'e_2 + e_2^2\right) + \left(e_0e_1' - e_0e_2\right)\right]$$

$$= \left(\frac{N-n}{nN}\right)S_y^2 + \frac{R}{4}\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 - 4\rho_{yx}S_yS_x\right\} + \left(\frac{N_2(k-1)}{nN}\right)S_{y(2)}^2 \tag{26}$$

$$MSE\left(\bar{y}^*_{I\,Pe}\right) = \bar{Y}^2 E\left[e_0 - \frac{1}{2}(e_1' - e_2)\right]^2$$

$$= \bar{Y}^2 E\left[e_0^2 + \frac{1}{4}\left(e_1'^2 - 2e_1'e_2 + e_2^2\right) - \left(e_0e_1' - e_0e_2\right)\right]$$

$$= \left(\frac{N-n}{nN}\right)S_y^2 + \frac{R}{4}\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 + 4\rho_{yx}S_yS_x\right\} + \left(\frac{N_2(k-1)}{nN}\right)S_{y(2)}^2 \tag{27}$$

$$MSE\left(\bar{y}^*_{II\,\mathrm{Re}}\right) = \bar{Y}^2 E\left[e_0 + \frac{1}{2}(e_1' - e_1)\right]^2$$

$$= \bar{Y}^2 E\left[e_0^2 + \frac{1}{4}\left(e_1'^2 - 2e_1'e_1 + e_1^2\right) + \left(e_0e_1' - e_0e_1\right)\right]$$

$$= \left(\frac{N-n}{nN}\right)S_y^2 + \frac{R}{4}\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2 - 4\rho_{yx}S_yS_x\right\}$$

$$+ \left(\frac{N_2(k-1)}{nN}\right)\left\{S_{y(2)}^2 + \frac{R^2}{4}S_{x(2)}^2 - R\,\rho_{yx(2)}S_{y(2)}S_{x(2)}\right\} \tag{28}$$

$$MSE\left(\bar{y}^{*}_{II\,Pe}\right)=\bar{Y}^{2}E\left[e_0-\frac{1}{2}\left(e'_1-e_1\right)\right]^2$$

$$=\bar{Y}^{2}E\left[e_0^2+\frac{1}{4}\left(e'^2_1-2e'_1e_1+e_1^2\right)-\left(e_0e'_1-e_0e_1\right)\right]$$

$$=\left(\frac{N-n}{nN}\right)S_y^2+\frac{R}{4}\left(\frac{n'-n}{n\,n'}\right)\left\{R\,S_x^2+4\rho_{yx}S_yS_x\right\}$$

$$+\left(\frac{N_2(k-1)}{nN}\right)\left\{S_{y(2)}^2+\frac{R^2}{4}S_{x(2)}^2+R\,\rho_{yx(2)}S_{y(2)}S_{x(2)}\right\} \tag{29}$$

4 Efficiency Comparisons

It is well known that under SRSWOR sampling scheme, we have

$$V\left(\bar{y}^{*}\right)=\frac{(N-n)}{n\,N}S_y^2+\frac{N_2(k-1)}{n\,N}S_{y(2)}^2 \tag{30}$$

From Eqs. (9) to (12), (26) to (29), and (30), we have

(i) $MSE(\bar{y}^{*}_{I\,\mathrm{Re}})<V(\bar{y}^{*})$ if

$$\rho_{yx}\frac{C_y}{C_x}>\frac{1}{4} \tag{31}$$

(ii) $MSE(\bar{y}^{*}_{I\,\mathrm{Pe}})<V(\bar{y}^{*})$ if

$$\rho_{yx}\frac{C_y}{C_x}<-\frac{1}{4} \tag{32}$$

(iii) $MSE(\bar{y}^{*}_{II\,\mathrm{Re}})<V(\bar{y}^{*})$ if

$$\frac{RS_x^2-4\rho_{yx}S_yS_x}{RS_{x(2)}^2-4\rho_{yx(2)}S_{y(2)}S_{x(2)}}<-\frac{N_2(k-1)}{(n'-n)}\frac{n'}{N} \tag{33}$$

(iv) $MSE(\bar{y}^{*}_{II\,\mathrm{Pe}})<V(\bar{y}^{*})$ if

$$\frac{RS_x^2 + 4\rho_{yx}S_yS_x}{RS_{x(2)}^2 + 4\rho_{yx(2)}S_{y(2)}S_{x(2)}} < -\frac{N_2(k-1)}{(n'-n)}\frac{n'}{N} \quad (34)$$

(v) $MSE(\overline{y}_{I\,\mathrm{Re}}^{*}) < MSE(\overline{y}_{I\,Rd}^{*})$ if

$$\frac{RS_x^2 - 4\rho_{yx}S_yS_x}{RS_x^2 - 2\rho_{yx}S_yS_x} < 4 \quad (35)$$

(vi) $MSE(\overline{y}_{I\,P\mathrm{e}}^{*}) < MSE(\overline{y}_{I\,Pd}^{*})$ if

$$\frac{RS_x^2 + 4\rho_{yx}S_yS_x}{RS_x^2 + 2\rho_{yx}S_yS_x} < 4 \quad (36)$$

(vii) $MSE(\overline{y}_{II\,\mathrm{Re}}^{*}) < MSE(\overline{y}_{II\,Rd}^{*})$ if

$$\frac{3RS_x^2 - 4\rho_{yx}S_yS_x}{3RS_{x(2)}^2 - 4\rho_{yx(2)}S_{y(2)}S_{x(2)}} < -\frac{N_2(k-1)}{(n'-n)}\frac{n'}{N} \quad (37)$$

(viii) $MSE(\overline{y}_{II\,Pe}^{*}) < MSE(\overline{y}_{II\,Pd}^{*})$ if

$$\frac{3RS_x^2 + 4\rho_{yx}S_yS_x}{3RS_{x(2)}^2 + 4\rho_{yx(2)}S_{y(2)}S_{x(2)}} < -\frac{N_2(k-1)}{(n'-n)}\frac{n'}{N} \quad (38)$$

5 Empirical Study

To illustrate the properties of the proposed estimators of the population mean $\overline{Y}$, we consider a real data set considered by Srivastava [11]. The description of the population is given below.

A list of seventy villages in a Tehsil of India along with their population in 1981 and cultivated area (in acres) in the same year is taken into consideration. Here the cultivated area (in acres) is taken as main study character and the population of village is taken as auxiliary character. The values of the parameters are as follows:

$\bar{Y} = 981.29,\quad \bar{Y}_2 = 597.29,\quad \bar{X} = 1755.53,\quad \bar{X}_2 = 1100.24,\quad N = 70,\quad W_2 = 0.20,$
$n' = 40,\quad n = 25,\quad S_y = 613.66,\quad S_x = 1406.13,\quad S_{y(2)} = 241.11,\quad S_{x(2)} = 631.51,$
$\rho_{yx} = 0.778,\quad \rho_{yx(2)} = 0.445$

Table 1: Percentage Relative Efficiencies (PREs) of different estimators of $\bar{Y}$ with respect to $\bar{y}^*$

Value of k	Estimators				
	$\bar{y}^*$	$\bar{y}^*_{I\,Rd}$	$\bar{y}^*_{II\,Rd}$	$\bar{y}^*_{I\,Re}$	$\bar{y}^*_{II\,Re}$
2	100.00	124.37	118.79	148.37	149.62
3	100.00	123.03	113.01	145.22	147.54
4	100.00	121.82	108.19	142.46	145.68
5	100.00	120.74	104.11	140.01	144.02

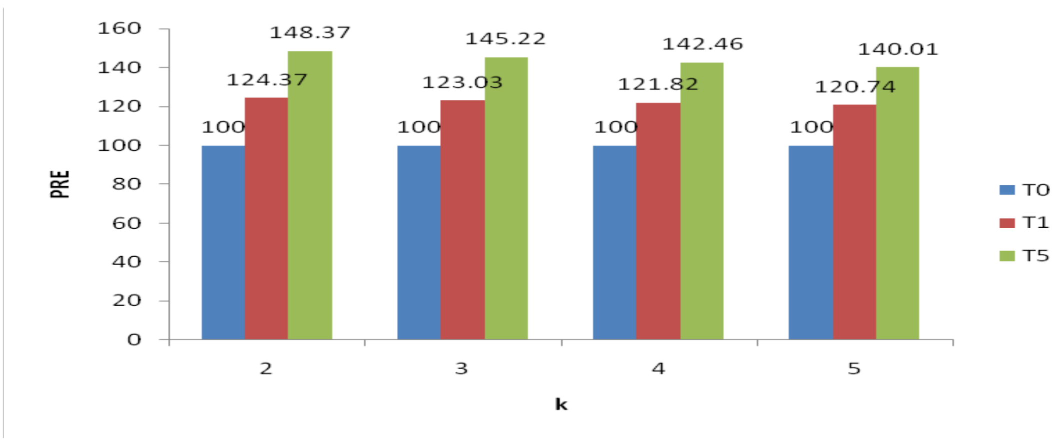

Figure 1: Percentage Relative Efficiencies (PREs) of different estimators of $\bar{Y}$ with respect to $\bar{y}^*$ under case *I*

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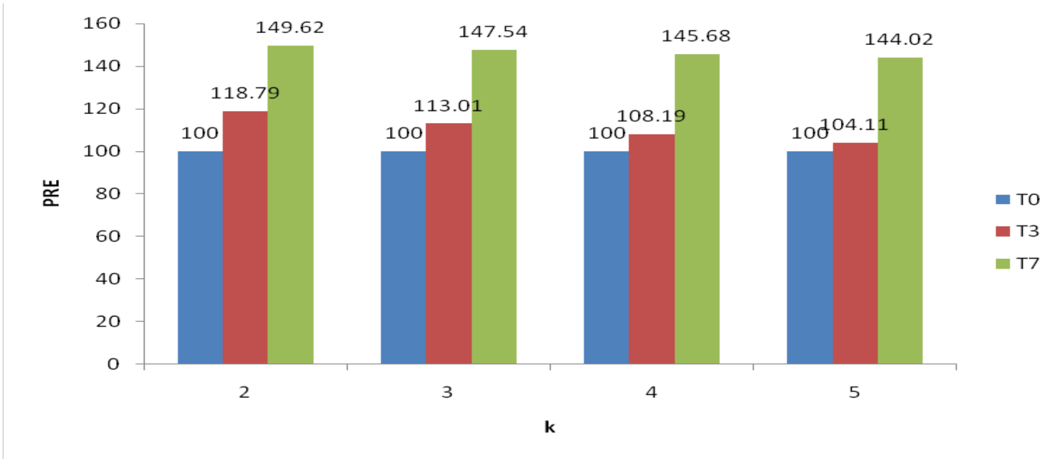

Figure 2: Percentage Relative Efficiencies (PREs) of different estimators of $\bar{Y}$ with respect to $\bar{y}^*$ under case *II*

6 Conclusion

Table 1 exhibits that

(i) For the given population data set, the PREs of various estimators decreases as the value of k increases.

(ii) The proposed estimators $\bar{y}^*_{I\,\mathrm{Re}}$ and $\bar{y}^*_{II\,\mathrm{Re}}$ for the two cases of non-response shows the maximum gain in efficiency as compared to the Hansen-Hurwitz [2] unbiased estimator and Khare and Srivastava [5] estimators.

Figures 1 and 2, the graphical representations of PREs for both the cases *I* (non-response is only on study variable) and *II* (non- response is both on study and auxiliary variables) respectively are also shown the improvement in percentage relative efficiencies of the proposed estimators as compared to the Hansen-Hurwitz [2] unbiased estimator and Khare and Srivastava [5] estimators.

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