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# A Hybrid Method to Improve Forecasting Accuracy In the Case of Japanese Food Restaurant

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## Abstract

Japanese food restaurant's sales forecasting is an important factor for the manager in order to keep the shop in surplus. He/she manages the shop by increasing/decreasing the employee/part-timer on the forecasting result. In this paper, we propose a new method to improve forecasting accuracy and confirm them by the numerical example. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Combining the trend removing

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method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original restaurant's sales data. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. Good result is obtained.

**Mathematics Subject Classification:** 62M10

**Keywords:** minimum variance; exponential smoothing method; forecasting; trend

## 1 Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM)[1]−[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he couldn't grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as shipping data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original restaurant's sales data. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3.The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

## 2 Description of ESM using ARMA model[13]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha\left(x_t - \hat{x}_t\right)$$

$$= \alpha x_t + \left(1-\alpha\right)\hat{x}_t$$

Here,

$\hat{x}_{t+1}$ : forecasting at $t+1$

$x_t$ : realized value at $t$

$\alpha$ : smoothing constant $\left(0 < \alpha < 1\right)$

(2) is re-stated as follows.

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1}$$

Generally, $(p,q)$ order ARMA model is stated as follows.

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$

$t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean, $\sigma_e^2$ variance

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

We get the following equation from (4) :

$$\hat{x} = x_{t-1} - \beta e_{t-1}$$

Operating this scheme on $t+1$, we finally get the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + (1-\beta)e_t$$

$$= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t)$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1[1] [3] . Comparing with (4) and (5), we obtain the following equations.

$$a_1 = -1$$
$$b_1 = -\beta = \alpha - 1$$

From (1), (7),

$$\alpha = 1 - \beta$$

Therefore, we get the following equations.

$$a_1 = -1$$
$$b_1 = -\beta = \alpha - 1$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below. Let (5) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i}$$

Then (9) is expressed as follows.

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known[3].

$$\tilde{r}_k = \left\{ \begin{array}{ll} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{array} \right\} \tag{11}$$

$$\tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only $b_1$, so it can be solved in the following way. From (4) (5) (8) (11), we get the following equations.

$$\left.\begin{aligned} &q = 1 \\ &a_1 = -1 \\ &b_1 = -\beta = \alpha - 1 \\ &\tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ &\tilde{r}_1 = b_1 \sigma_e^2 \end{aligned}\right\}$$

If we set :

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2}$$

We can get $b_1$ as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1}$$

In order to have real roots, $\rho_1$ must satisfy :

$$\left|\rho_1\right| \le \frac{1}{2}$$

From invertibility condition, $b_1$ must satisfy :

$$|b_1| < 1$$

From (14), using the next relation :

$$(1 - b_1)^2 \geq 0$$

$$(1 + b_1)^2 \geq 0$$

(16) always holds.

As

$$\alpha = b_1 + 1$$

$b_1$ is within the range of

$$-1 < b_1 < 0$$

Finally we get the following equations

$$\left. \begin{aligned} b_1 &= \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \end{aligned} \right\}$$

which satisfy the above conditions. Thus we can obtain a theoretical solution by a simple method. Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating the 0th and the 1st order autocorrelation function.

# 3 Trend removal method

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set :

$$y = a_1 x + b_1$$

as a linear function.

[2] Non-linear function

We set :

$$y = a_2 x^2 + b_2 x + c_2$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function

We set :

$$y = \alpha_1 \left( a_1 x + b_1 \right) + \alpha_2 \left( a_2 x^2 + b_2 x + c_2 \right)$$

$$y = \beta_1 \left( a_1 x + b_1 \right) + \beta_2 \left( a_3 x^3 + b_3 x^2 + c_3 x + d_3 \right)$$

$$y = \gamma_1 \left( a_1 x + b_1 \right) + \gamma_2 \left( a_2 x^2 + b_2 x + c_2 \right) + \gamma_3 \left( a_3 x^3 + b_3 x^2 + c_3 x + d_3 \right)$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, $\alpha_2 = 1 - \alpha_1$, $\beta_2 = 1 - \beta_1$, $\gamma_3 = 1 - (\gamma_1 + \gamma_2)$. Comparative discussion concerning (21), (22) and (23) are described in section 5.

# 4 Monthly ratio

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\}\ (i = 1, \cdots, L)\ (j = 1, \cdots, 12)$$

where, $x_{ij} \in R$ in which $j$ means month and $i$ means year and $x_{ij}$ is a shipping data of i-th year, j-th month, then, monthly ratio $\tilde{x}_j\ (j = 1, \cdots, 12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \cdot \frac{1}{12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij}}$$

Monthly trend is removed by dividing the data by (24). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

# 5 Forecasting the sales data

## 5.1 Analysis Procedure

The original restaurant's sales data for 3 cases from January 1999 to December 2001 are analyzed. First of all, graphical charts of these time series data are exhibited in Figures 5-1,5-2,5-3.

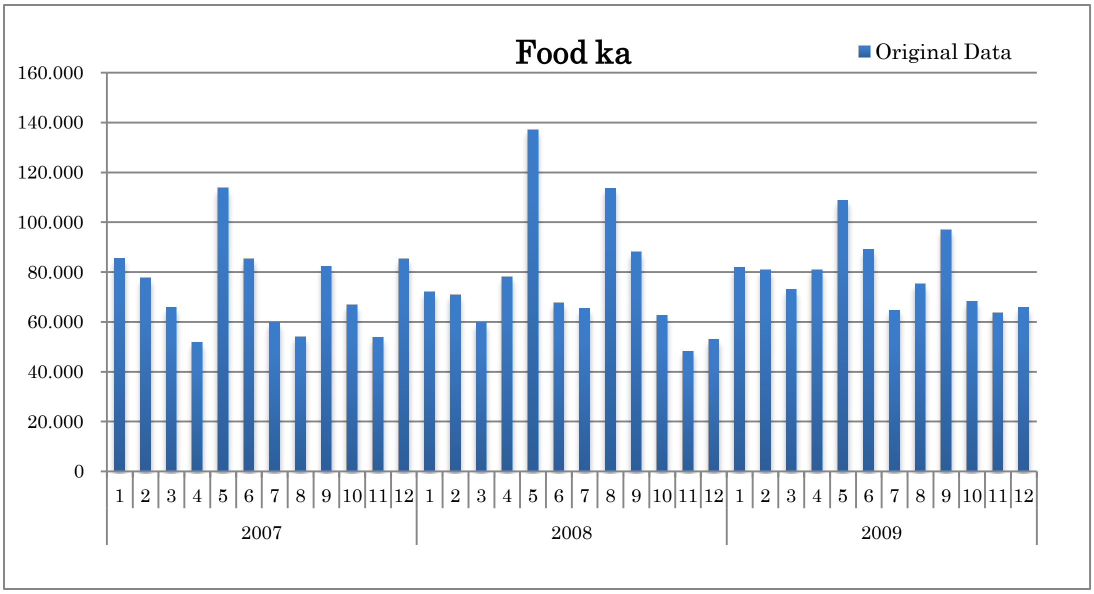


Figure 5-1: Sales Data of Food ka

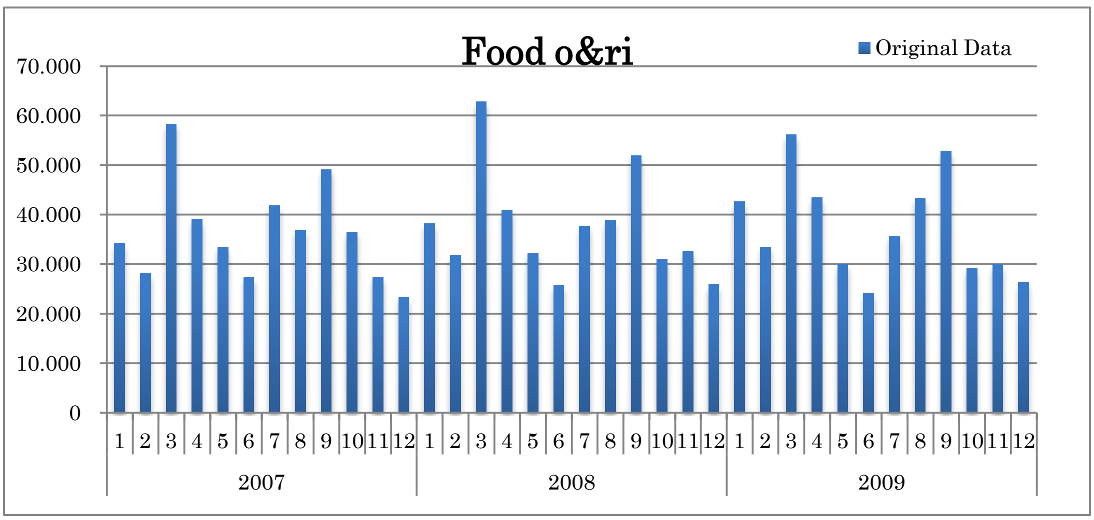


Figure 5-2: Sales Data of Food o&ri

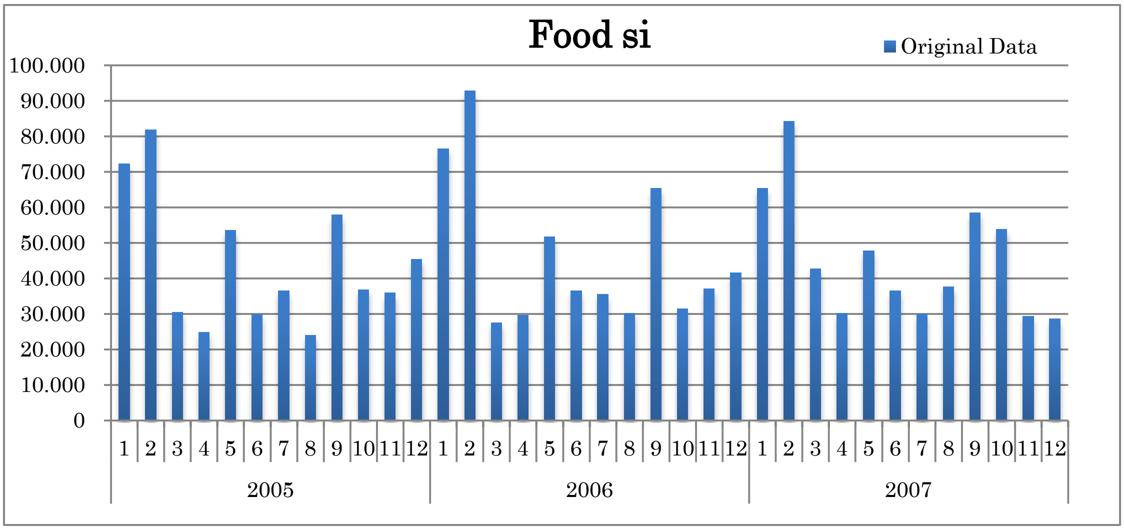


Figure 5-3: Sales Data of Food si

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data(1 to 24) and remove trend by the method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i$$

Variance of forecasting error is calculated by:

$$\sigma_{\varepsilon}^{2} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^{2}$$

## 5.2 Trend Removing

Trend is removed by dividing original data by,(21),(22),(23). The patterns of trend removal are exhibited in Table5-1.

Table 5-1: The patterns of trend removal

| | |
|---|---|
| Pattern1 | $\alpha_1, \alpha_2$ are set 0.5 in the equation (21) |
| Pattern2 | $\beta_1, \beta_2$ are set 0.5 in the equation (22) |
| Pattern3 | $\alpha_1$ is shifted by 0.01 increment in (21) |
| Pattern4 | $\beta_1$ is shifted by 0.01 increment in (22) |
| Pattern5 | $\gamma_1$ and $\gamma_2$ are shifted by 0.01 increment in (23) |

In pattern1 and 2, the weight of $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$ are set 0.5 in the equation (21),(22). In pattern3, the weight of $\alpha_1$ is shifted by 0.01 increment in (21) which satisfy the range $0 \le \alpha_1 \le 1.00$. In pattern4, the weight of $\beta_1$ is shifted in the same way which satisfy the range $0 \le \beta_1 \le 1.00$. In pattern5, the weight of $\gamma_1$ and $\gamma_2$ are shifted by 0.01 increment in (23) which satisfy the range $0 \le \gamma_1 \le 1.00$, $0 \le \gamma_2 \le 1.00$ .The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 5-2. Estimation results of weights of (21), (22) and (23) are exhibited in Table 5-3.

Table 5-2: Coefficient of (18), (19) and (20)

| | 1st | | 2nd | | | 3rd | | | |
|---|---|---|---|---|---|---|---|---|---|
| | a1 | b1 | a2 | b2 | c2 | a3 | b3 | c3 | d3 |
| Food ka | -114.990 | 76374.000 | -52.781 | 1204.528 | 70656.087 | -30.938 | 1107.399 | -10635.491 | 97804.291 |
| Food o&ri | -129.748 | 38519.348 | -22.582 | 434.793 | 36073.004 | -5.073 | 167.673 | -1506.815 | 40524.954 |
| Food si | -437.520 | 50634.667 | 29.544 | -1176.117 | 53835.253 | -22.759 | 883.021 | -9886.140 | 73806.628 |

Table 5-3: Weights of (21), (22) and (23)

| | monthly ratio | Pattern 1 | | Pattern 2 | | Pattern 3 | | Pattern 4 | | Pattern5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
| Food ka | Used | 0.5 | 0.5 | 0.5 | 0.5 | 0.4 | 0.6 | 1.0 | 0.0 | 0.4 | 0.6 | 0.0 |
| | Not used | 0.5 | 0.5 | 0.5 | 0.5 | 0.0 | 1.0 | 1.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| Food o&ri | Used | 0.5 | 0.5 | 0.5 | 0.5 | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 0.0 |
| | Not used | 0.5 | 0.5 | 0.5 | 0.5 | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 0.0 |
| Food si | Used | 0.5 | 0.5 | 0.5 | 0.5 | 1.0 | 0.0 | 1.0 | 0.0 | 0.7 | 0.2 | 0.1 |
| | Not used | 0.5 | 0.5 | 0.5 | 0.5 | 0.7 | 0.3 | 1.0 | 0.0 | 0.7 | 0.3 | 0.0 |

Graphical chart of trend is exhibited in Figure 5-4, 5-5, 5-6 for the cases that monthly ratio is used.

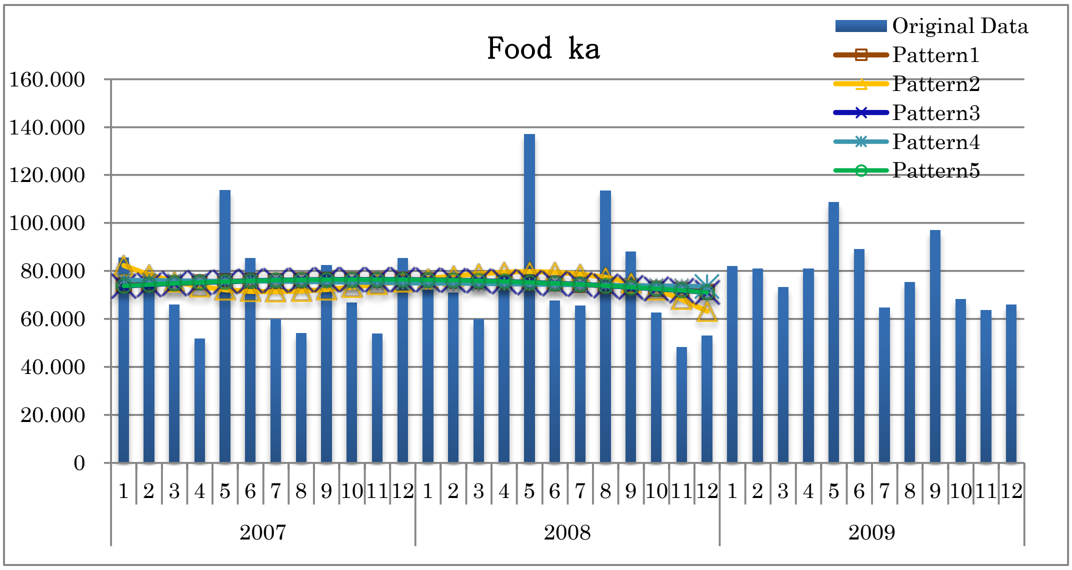


Figure 5-4: Trend of Food ka

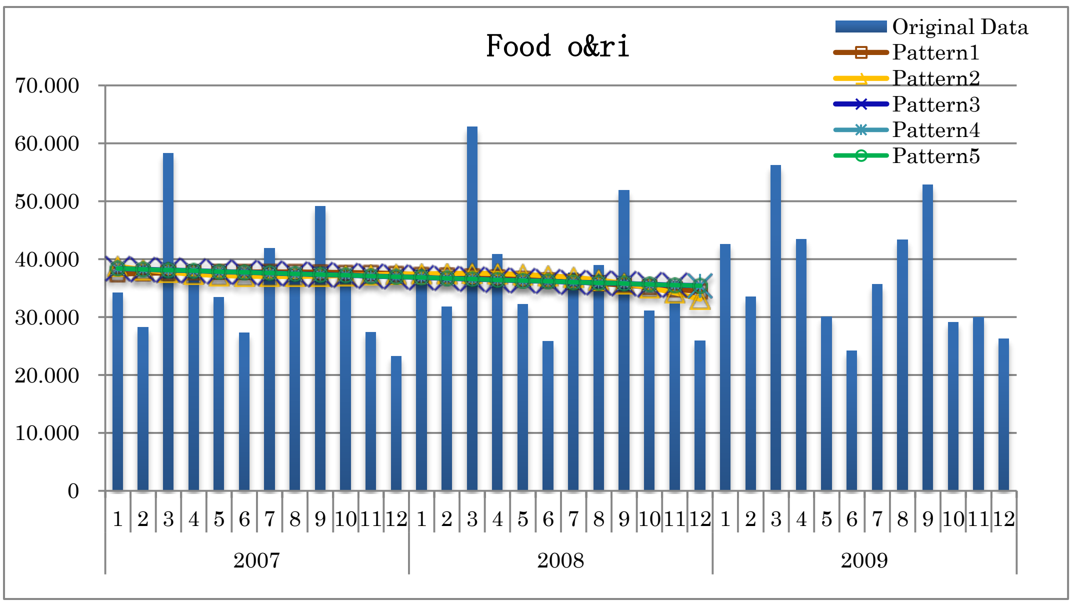


Figure 5-5: Trend of Food o&ri

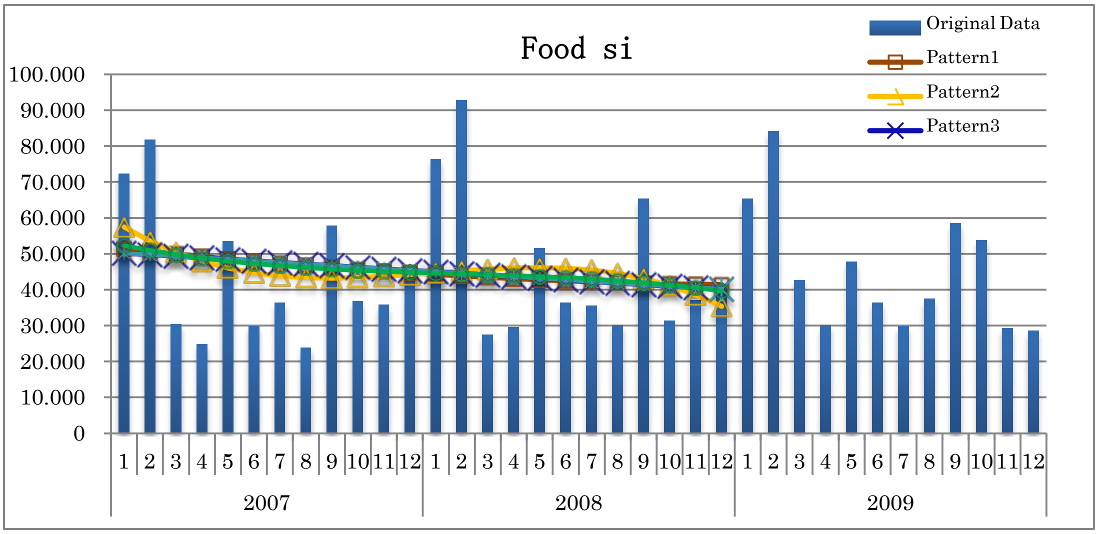


Figure 5-6: Trend of Food si

## 5.3 Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 4. Calculation result for 1st to 24th data is exhibited in Tables 5-4,5-5,5-6,5-7,5-8.

Table 5-4: Monthly ratio (Pattern1)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 1.05 | 0.99 | 0.83 | 0.86 | 1.67 | 1.02 | 0.83 | 1.12 | 1.14 | 0.87 | 0.69 | 0.93 |
| Food o&ri | 0.97 | 0.80 | 1.62 | 1.07 | 0.88 | 0.72 | 1.08 | 1.03 | 1.38 | 0.93 | 0.83 | 0.69 |
| Food si | 1.57 | 1.87 | 0.62 | 0.59 | 1.15 | 0.74 | 0.81 | 0.61 | 1.41 | 0.78 | 0.84 | 1.01 |

Table 5-4: Monthly ratio (Pattern1)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 1.05 | 0.99 | 0.83 | 0.86 | 1.67 | 1.02 | 0.83 | 1.12 | 1.14 | 0.87 | 0.69 | 0.93 |
| Food o&ri | 0.97 | 0.80 | 1.62 | 1.07 | 0.88 | 0.72 | 1.08 | 1.03 | 1.38 | 0.93 | 0.83 | 0.69 |
| Food si | 1.57 | 1.87 | 0.62 | 0.59 | 1.15 | 0.74 | 0.81 | 0.61 | 1.41 | 0.78 | 0.84 | 1.01 |

Table 5-5: Monthly ratio (Pattern2)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 0.99 | 0.95 | 0.82 | 0.85 | 1.65 | 1.03 | 0.84 | 1.11 | 1.16 | 0.89 | 0.72 | 0.99 |
| Food o&ri | 0.95 | 0.79 | 1.61 | 1.07 | 0.88 | 0.72 | 1.08 | 1.03 | 1.39 | 0.93 | 0.84 | 0.70 |
| Food si | 1.49 | 1.79 | 0.60 | 0.58 | 1.14 | 0.73 | 0.81 | 0.62 | 1.43 | 0.81 | 0.89 | 1.10 |

Table 5-6: Monthly ratio (Pattern3)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 1.05 | 0.99 | 0.83 | 0.86 | 1.66 | 1.02 | 0.83 | 1.12 | 1.14 | 0.87 | 0.69 | 0.93 |
| Food o&ri | 0.96 | 0.80 | 1.62 | 1.08 | 0.89 | 0.72 | 1.08 | 1.03 | 1.38 | 0.93 | 0.83 | 0.68 |
| Food si | 1.57 | 1.86 | 0.62 | 0.59 | 1.15 | 0.74 | 0.80 | 0.61 | 1.41 | 0.78 | 0.85 | 1.02 |

Table 5-7: Monthly ratio (Pattern4)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 1.04 | 0.98 | 0.83 | 0.86 | 1.67 | 1.02 | 0.84 | 1.12 | 1.14 | 0.87 | 0.69 | 0.93 |
| Food o&ri | 0.96 | 0.80 | 1.62 | 1.08 | 0.89 | 0.72 | 1.08 | 1.03 | 1.38 | 0.93 | 0.83 | 0.68 |
| Food si | 1.57 | 1.86 | 0.62 | 0.59 | 1.15 | 0.74 | 0.80 | 0.61 | 1.41 | 0.78 | 0.85 | 1.02 |

Table 5-8: Monthly ratio (Pattern5)

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Food ka | 1.05 | 0.99 | 0.83 | 0.86 | 1.66 | 1.02 | 0.83 | 1.12 | 1.14 | 0.87 | 0.69 | 0.93 |
| Food o&ri | 0.96 | 0.80 | 1.62 | 1.08 | 0.89 | 0.72 | 1.08 | 1.03 | 1.38 | 0.93 | 0.83 | 0.68 |
| Food si | 1.55 | 1.85 | 0.62 | 0.59 | 1.15 | 0.74 | 0.81 | 0.61 | 1.41 | 0.79 | 0.86 | 1.03 |

## 5.4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (17). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (16). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 5-9.

Table 5-9: Estimated Smoothing Constant with Minimum Variance

| | Monthly ratio | Pattern1 | | Pattern2 | | Pattern3 | | Pattern4 | | Pattern5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ρ1 | α | ρ1 | α | ρ1 | α | ρ1 | α | ρ1 | α |
| Food ka | Used | −0.226 | 0.761 | −0.025 | 0.975 | −0.226 | 0.761 | −0.225 | 0.763 | −0.226 | 0.761 |
| | Not used | −0.299 | 0.668 | −0.045 | 0.955 | −0.298 | 0.670 | −0.299 | 0.667 | −0.298 | 0.670 |
| Food o&ri | Used | −0.356 | 0.582 | −0.289 | 0.681 | −0.362 | 0.571 | −0.362 | 0.571 | −0.362 | 0.571 |
| | Not used | −0.011 | 0.989 | −0.021 | 0.979 | −0.008 | 0.992 | −0.008 | 0.992 | −0.008 | 0.992 |
| Food si | Used | * | 0.120 | * | 0.120 | * | 0.120 | * | 0.120 | * | 0.180 |
| | Not used | −0.326 | 0.629 | −0.345 | 0.600 | −0.327 | 0.628 | −0.327 | 0.627 | −0.327 | 0.628 |

*: Out of range (i.e. do not satisfy (16))

## 5.5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (27). Forecasting results are exhibited in Figures 5-7,5-8,5-9 for the cases that monthly ratio is not used.

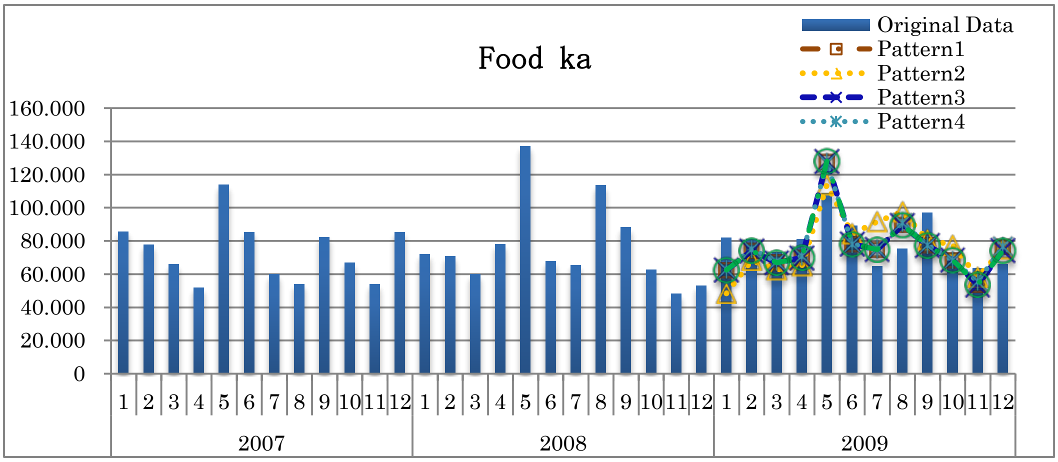


Figure 5-7: Forecasting Results of Food ka

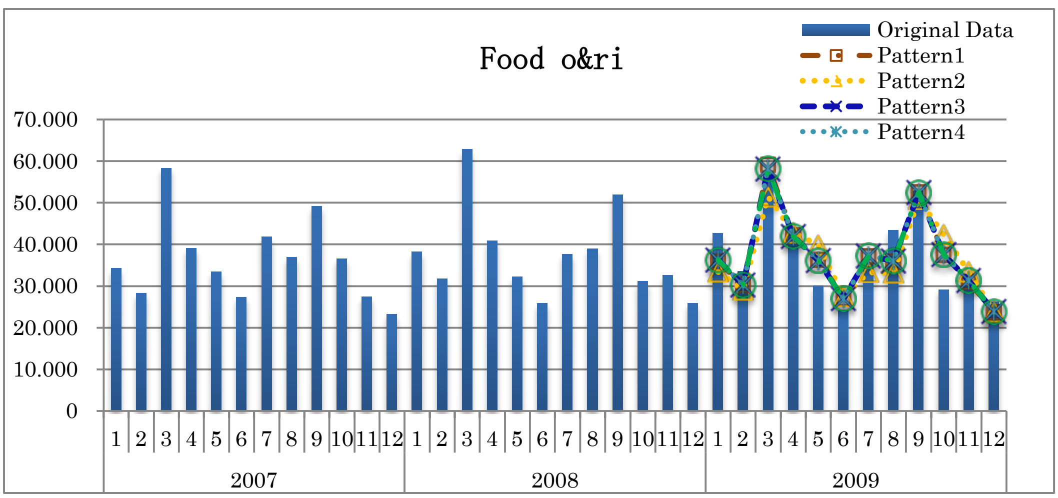


Figure 5-8: Forecasting Results of Food o&ri

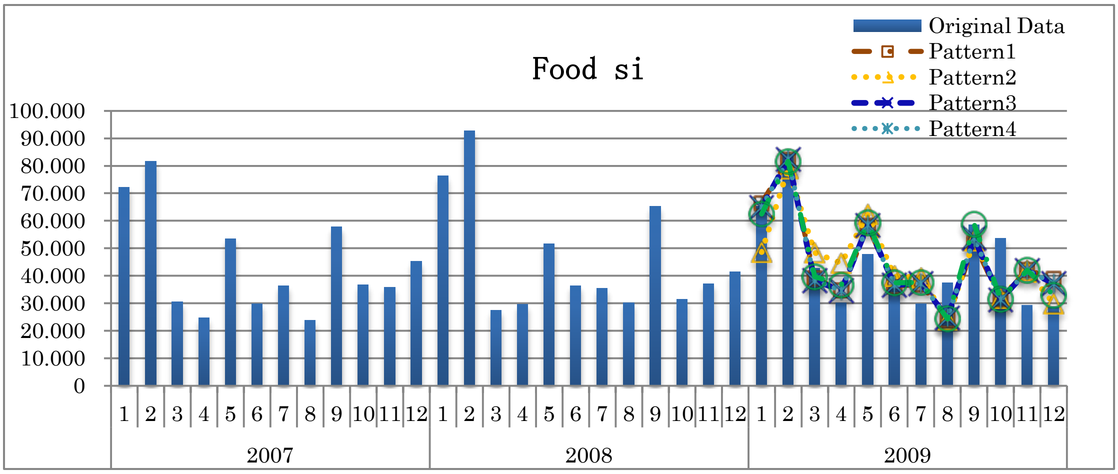


Figure 5-9: Forecasting Results of Food si

Variance of forecasting error is exhibited in Table 5-10.

Table 5-10: Variance of Forecasting Error

| | Monthly ratio | Pattern1 | Pattern2 | Pattern3 | Pattern4 | Pattern5 |
|---|---|---|---|---|---|---|
| Food ka | Used | 172,932,260.205 | 301,592,502.528 | 167,971,741.555 | 169,098,774.307 | 167,971,741.555 |
| | Not used | 351,059,244.620 | 471,462,796.008 | 349,532,477.032 | 356,731,154.337 | 349,532,477.032 |
| Food o&ri | Used | 22,897,634.692 | 48,712,253.600 | 21,448,538.914 | 21,448,538.914 | 21,448,538.914 |
| | Not used | 171,973,460.678 | 213,904,259.203 | 170,534,225.027 | 170,534,225.027 | 170,534,225.027 |
| Food si | Used | 109,168,061.297 | 155,803,858.113 | 103,082,238.108 | 103,082,238.108 | 101,002,598.354 |
| | Not used | 401,139,446.151 | 452,440,180.378 | 370,618,787.684 | 370,694,289.247 | 370,618,787.684 |

### 5.6 Remarks

In all cases, that monthly ratio was used had a better forecasting accuracy. Food ka and Food o&ri had a good result in Pattern3($1^{st}+2^{nd}$ order), Food si in pattern5($1^{st}+2^{nd}+3^{rd}$ order).

## 6 Conclusion

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrary. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to increase forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original restaurant's sales data. The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting was executed on these data. The calculation results show that the case monthly ratio was used had a better forecasting accuracy. Various cases should be examined hereafter.

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