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Brand bag purchasing selection and its matrix structure

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Abstract

It is often observed that consumers select the upper class brand when they buy the next time. Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix becomes an upper triangle matrix under the supposition that the former buying variables are set input and the current buying variables are set output. If the top brand were selected from the lower brand in jumping way, corresponding part in the upper triangle matrix would be 0. A questionnaire investigation for automobile purchasing case is executed and the above structure is confirmed. If the transition matrix is identified, a S-step forecasting can be

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executed. Generalized forecasting matrix components' equations are introduced. In this paper, brand bag purchasing case is considered and a method of building the ranking table by utilizing correspondence analysis is newly proposed. Unless planner for products does not notice its brand position whether it is upper or lower than another products, matrix structure make it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Keywords: brand selection; matrix structure; brand position; brand bag; correspondence analysis

1 Introduction

It is often observed that consumers select the upper class brand when they buy the next time. Focusing the transition matrix structure of brand selection, their activities may be analyzed. In the past, there are many researches about brand selection [1-5]. But there are few papers concerning the analysis of the transition matrix structure of brand selection. In this paper, we make analysis of the preference shift of customer brand selection and confirm them by the questionnaire investigation for automobile purchasing case. If we can identify the feature of the matrix structure of brand selection, it can be utilized for the marketing strategy.

Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix becomes an upper triangular matrix under the supposition that the former buying variables are set input and the current buying variables are set output. If the top brand were selected from the lower brand in jumping way, corresponding part in the upper triangular matrix would be 0. These are verified by the numerical examples with simple models.

If the transition matrix is identified, a S-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planner for products does not notice its brand position whether it is upper or lower than another products, matrix structure make it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

A quantitative analysis concerning brand selection has been executed by [4, 5]. [5] examined purchasing process by Markov Transition Probability with the input of advertising expense. [4] made analysis by the Brand Selection Probability model using logistics distribution.

Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. In Takeyasu et al. (2007) [6], matrix structure was analyzed for the case brand selection was executed for upper class in an automobile purchasing case. Ranking Table is required before making ranking of the brand. Furthermore, it was hard to make ranking table because there were plural standards to assort. In this paper, a new method to arrange ranking table of brand is proposed, which utilizes correspondence analysis. Brand selection process is calculated by using above table and the effectiveness of this method is clarified through the total calculation process. Such research cannot be found as long as searched.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. A block matrix structure is analyzed when brands are handled in a group and a s -step forecasting is formulated in section 3. The method of building the brand ranking table by utilizing correspondence analysis is introduced in section 4. Numerical calculation is executed in section 5. Application of this method is extended in section 6. Section 7 is a summary.

2 Brand selection and its matrix structure

2.1 Upper shift of Brand selection

It is often observed that consumers select the upper class brand when they buy the next time. Now, suppose that x is the most upper class brand, y is the second upper brand, and z is the lowest brand. Consumer's behavior of selecting brand would be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few. Suppose that x is the current buying variable, and x_b is the previous buying variable. Shift to x is executed from x_b, y_b, or z_b.

Therefore, x is stated in the following equation.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

and

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{AX_b}$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix.

To examine this, generating the following data, which are all consisted by the upper brand shift data,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$i = 1, \qquad 2 \qquad \cdots \qquad N$$

parameter can be estimated using least square method.

Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i$$

Where

$$\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1,2,\cdots,N$$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT}\boldsymbol{\varepsilon}^i \to Min$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT}\right)\left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT}\right)^{-1}$$

In the data group of the upper shift brand, estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix.

If the following data, that have the lower shift brand, are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{X}_\mathbf{b}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part triangle.

2.2 Sorting Brand Ranking by Re-arranging Row

In a general data, variables may not be in order as x, y, z. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in an upper triangular matrix, and the small value parts are gathered in a lower triangular matrix.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix}} \quad \Longleftarrow \quad \begin{pmatrix} z \\ x \\ y \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix}}$$

2.3 In the Case that Brand Selection Shifts in Jump

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand.

We suppose v, w, x, y, z brands (suppose they are laid from the upper position to the lower position as $v > w > x > y > z$).

In the above case, the selection shifts would be

$v \leftarrow z$

$v \leftarrow y$

Suppose there is no shift from z to y, corresponding part of the transition matrix is 0 (i.e. a_{45}=0). Similarly, if there is no shift from z to y, from z to w, from y to x, form y to w, from x to w, then the matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix}$$

3 Brock matrix structure in Brand group and s-step forecasting

Next, we examine the case in brand groups. Matrices are composed by the Block Matrix.

3.1 In the Case that Brand Selection Shifts in Jump

Suppose the brand selection shifts from Corolla class to MarkⅡ class in a car. In this case, it does not matter which company's car they choose. Thus, selection of cars is executed in a group and the brand shift is considered to be done from group to group. Suppose brand groups at time n are as follows. $\mathbf{X}$ consists of p varieties of goods, and $\mathbf{Y}$ consists of q varieties of goods.

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \quad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}} \\ \mathbf{0}, & \mathbf{A_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \end{pmatrix}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n=1,2,\cdots),\quad \mathbf{Y_n} \in \mathbf{R}^q \ (n=1,2,\cdots),\quad \mathbf{A_{11}} \in \mathbf{R}^{p\times p},\quad \mathbf{A_{12}} \in \mathbf{R}^{p\times q},\quad \mathbf{A_{22}} \in \mathbf{R}^{q\times q}$$

Make one more step of shift, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^2, & \mathbf{A_{11}}\mathbf{A_{12}} + \mathbf{A_{12}}\mathbf{A_{22}} \\ \mathbf{0}, & \mathbf{A_{22}}^2 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-2}} \\ \mathbf{Y_{n-2}} \end{pmatrix}$$

Make one more step of shift again, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^3, & \mathbf{A_{11}}^2\mathbf{A_{12}} + \mathbf{A_{11}}\mathbf{A_{12}}\mathbf{A_{22}} + \mathbf{A_{12}}\mathbf{A_{22}}^2 \\ \mathbf{0}, & \mathbf{A_{22}}^3 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-3}} \\ \mathbf{Y_{n-3}} \end{pmatrix}$$

Similarly,

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^4, & \mathbf{A_{11}}^3\mathbf{A_{12}} + \mathbf{A_{11}}^2\mathbf{A_{12}}\mathbf{A_{22}} + \mathbf{A_{11}}\mathbf{A_{12}}\mathbf{A_{22}}^2 + \mathbf{A_{12}}\mathbf{A_{22}}^3 \\ \mathbf{0}, & \mathbf{A_{22}}^4 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-4}} \\ \mathbf{Y_{n-4}} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^5, & \mathbf{A_{11}}^4\mathbf{A_{12}} + \mathbf{A_{11}}^3\mathbf{A_{12}}\mathbf{A_{22}} + \mathbf{A_{11}}^2\mathbf{A_{12}}\mathbf{A_{22}}^2 + \mathbf{A_{11}}\mathbf{A_{12}}\mathbf{A_{22}}^3 + \mathbf{A_{12}}\mathbf{A_{22}}^4 \\ \mathbf{0}, & \mathbf{A_{22}}^5 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-5}} \\ \mathbf{Y_{n-5}} \end{pmatrix}$$

Finally, we get the generalized equation for a s-step shift as follows.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^s, & \mathbf{A_{11}}^{s-1}\mathbf{A_{12}} + \sum_{k=2}^{s-1}\mathbf{A_{11}}^{s-k}\mathbf{A_{12}}\mathbf{A_{22}}^{k-1} + \mathbf{A_{12}}\mathbf{A_{22}}^{s-1} \\ \mathbf{0}, & \mathbf{A_{22}}^s \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \end{pmatrix}$$

If we replace $n-s \to n, n \to n+s$ in equation (15), we can make a s-step forecast.

3.2 Brand Shift Group for the Case of Three Groups

Suppose the brand selection is executed in the same group or to the upper group, and also suppose that the brand position is $x > y > z$ (x is upper position). Then the brand selection transition matrix would be expressed as

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}}, & \mathbf{A_{13}} \\ \mathbf{0}, & \mathbf{A_{22}}, & \mathbf{A_{23}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix}$$

Where,

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \quad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}, \quad \mathbf{Z_n} = \begin{pmatrix} z_1^n \\ z_2^n \\ \vdots \\ z_r^n \end{pmatrix}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots), \quad \mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots), \quad \mathbf{Z_n} \in \mathbf{R}^r \ (n = 1,2,\cdots),$$

$$\mathbf{A_{11}} \in \mathbf{R}^{p \times p}, \quad \mathbf{A_{12}} \in \mathbf{R}^{p \times q},$$

$$\mathbf{A_{13}} \in \mathbf{R}^{p \times r}, \quad \mathbf{A_{22}} \in \mathbf{R}^{q \times q}, \quad \mathbf{A_{23}} \in \mathbf{R}^{q \times r}, \quad \mathbf{A_{33}} \in \mathbf{R}^{r \times r}$$

These are re-stated as :

$$\mathbf{W_n} = \mathbf{A}\mathbf{W_{n-1}}$$

where,

$$\mathbf{W_n} = \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}}, & \mathbf{A_{13}} \\ \mathbf{0}, & \mathbf{A_{22}}, & \mathbf{A_{23}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}} \end{pmatrix}, \quad \mathbf{W_{n-1}} = \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix}$$

Hereinafter, we shift steps as is done in previous section.

In the general description, we state as :

$$\mathbf{W_n} = \mathbf{A}^{(\mathbf{s})}\mathbf{W_{n-s}}$$

Here,

$$\mathbf{A}^{(\mathbf{s})}=\begin{pmatrix}\mathbf{A}_{11}{}^{(\mathbf{s})}, & \mathbf{A}_{12}{}^{(\mathbf{s})}, & \mathbf{A}_{13}{}^{(\mathbf{s})}\\ \mathbf{0}, & \mathbf{A}_{22}{}^{(\mathbf{s})}, & \mathbf{A}_{23}{}^{(\mathbf{s})}\\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}{}^{(\mathbf{s})}\end{pmatrix},\quad \mathbf{W}_{\mathbf{n-s}}=\begin{pmatrix}\mathbf{X}_{\mathbf{n-s}}\\ \mathbf{Y}_{\mathbf{n-s}}\\ \mathbf{Z}_{\mathbf{n-s}}\end{pmatrix}$$

From definition,

$$\mathbf{A}^{(1)}=\mathbf{A}$$

In the case s=2, we obtain :

$$\begin{aligned}\mathbf{A}^{(2)}&=\begin{pmatrix}\mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13}\\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23}\\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}\end{pmatrix}\begin{pmatrix}\mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13}\\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23}\\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}\end{pmatrix}\\ &=\begin{pmatrix}\mathbf{A}_{11}^2, & \mathbf{A}_{11}\mathbf{A}_{12}+\mathbf{A}_{12}\mathbf{A}_{22}, & \mathbf{A}_{11}\mathbf{A}_{13}+\mathbf{A}_{12}\mathbf{A}_{23}+\mathbf{A}_{13}\mathbf{A}_{33}\\ \mathbf{0}, & \mathbf{A}_{22}^2, & \mathbf{A}_{22}\mathbf{A}_{23}+\mathbf{A}_{23}\mathbf{A}_{33}\\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^2\end{pmatrix}\end{aligned}$$

Next, in the case s=3, we obtain :

$$\mathbf{A}^{(3)}=\begin{pmatrix}\mathbf{A}_{11}^3, & \mathbf{A}_{11}^2\mathbf{A}_{12}+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}+\mathbf{A}_{12}\mathbf{A}_{22}^2, & P\\ \mathbf{0}, & \mathbf{A}_{22}^3, & \mathbf{A}_{22}^2\mathbf{A}_{23}+\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}+\mathbf{A}_{23}\mathbf{A}_{33}^2\\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^3\end{pmatrix}$$

Here,

$$P=\mathbf{A}_{11}^2\mathbf{A}_{13}+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}+\mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}+\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}+\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}+\mathbf{A}_{13}\mathbf{A}_{33}^2$$

In the case s=4, equations become wide-spread, so we express each Block Matrix as follows.

$$\mathbf{A}_{11}^{(4)}=\mathbf{A}_{11}^4$$

$$\mathbf{A}_{12}^{(4)}=\mathbf{A}_{11}^3\mathbf{A}_{12}+\mathbf{A}_{11}^2\mathbf{A}_{12}\mathbf{A}_{22}+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^2+\mathbf{A}_{12}\mathbf{A}_{22}^3$$

$$\begin{aligned}\mathbf{A}_{13}^{(4)}=&\ \mathbf{A}_{11}^3\mathbf{A}_{13}+\mathbf{A}_{11}^2\mathbf{A}_{12}\mathbf{A}_{23}+\mathbf{A}_{11}^2\mathbf{A}_{13}\mathbf{A}_{33}+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\\ &+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}+\mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^2+\mathbf{A}_{12}\mathbf{A}_{22}^2\mathbf{A}_{23}\\ &+\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}+\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^2+\mathbf{A}_{13}\mathbf{A}_{33}^3\end{aligned}$$

$$\mathbf{A}_{22}^{(4)} = \mathbf{A}_{22}^{4}$$

$$\mathbf{A}_{23}^{(4)} = \mathbf{A}_{22}^{3}\mathbf{A}_{23} + \mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{23}\mathbf{A}_{33}^{3}$$

$$\mathbf{A}_{33}^{(4)} = \mathbf{A}_{33}^{4}$$

In the case s=5, we obtain the following equations similarly.

$$\mathbf{A}_{11}^{(5)} = \mathbf{A}_{11}^{5}$$

$$\mathbf{A}_{12}^{(5)} = \mathbf{A}_{11}^{4}\mathbf{A}_{12} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{2} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{3} + \mathbf{A}_{12}\mathbf{A}_{22}^{4}$$

$$\begin{aligned}\mathbf{A}_{13}^{(5)} = {} & \mathbf{A}_{11}^{4}\mathbf{A}_{13} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}^{3}\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} \\ & + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{13}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23} \\ & + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^{3} \\ & + \mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{2} \\ & + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{13}\mathbf{A}_{33}^{4}\end{aligned}$$

$$\mathbf{A}_{22}^{(5)} = \mathbf{A}_{22}^{5}$$

$$\mathbf{A}_{23}^{(5)} = \mathbf{A}_{22}^{4}\mathbf{A}_{23} + \mathbf{A}_{22}^{3}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{23}\mathbf{A}_{33}^{4}$$

$$\mathbf{A}_{33}^{(5)} = \mathbf{A}_{33}^{5}$$

In the case s=6, we obtain :

$$\mathbf{A}_{11}^{(6)} = \mathbf{A}_{11}^{6}$$

$$\begin{aligned}\mathbf{A}_{12}^{(6)} = {} & \mathbf{A}_{11}^{5}\mathbf{A}_{12} + \mathbf{A}_{11}^{4}\mathbf{A}_{12}\mathbf{A}_{22} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22}^{2} \\ & + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{3} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{4} + \mathbf{A}_{12}\mathbf{A}_{22}^{5}\end{aligned}$$

$$
\begin{aligned}
\mathbf{A}_{13}^{(6)} &= \mathbf{A}_{11}^{5}\mathbf{A}_{13} + \mathbf{A}_{11}^{4}\mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{11}^{4}\mathbf{A}_{13}\mathbf{A}_{33} + \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23} \\
&+ \mathbf{A}_{11}^{3}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{3}\mathbf{A}_{13}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23} \\
&+ \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{11}^{2}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}^{2}\mathbf{A}_{13}\mathbf{A}_{33}^{3} \\
&+ \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33} \\
&+ \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{2} + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{11}\mathbf{A}_{13}\mathbf{A}_{33}^{4} \\
&+ \mathbf{A}_{12}\mathbf{A}_{22}^{4}\mathbf{A}_{23} + \mathbf{A}_{12}\mathbf{A}_{22}^{3}\mathbf{A}_{23}\mathbf{A}_{33} + \mathbf{A}_{12}\mathbf{A}_{22}^{2}\mathbf{A}_{23}\mathbf{A}_{33}^{2} \\
&+ \mathbf{A}_{12}\mathbf{A}_{22}\mathbf{A}_{23}\mathbf{A}_{33}^{3} + \mathbf{A}_{12}\mathbf{A}_{23}\mathbf{A}_{33}^{4} + \mathbf{A}_{13}\mathbf{A}_{33}^{5}
\end{aligned}
$$

We get the generalized equations for a *s*-step shift as follows.

$$\mathbf{A}_{11}^{(s)} = \mathbf{A}_{11}^{s}$$

$$\mathbf{A}_{12}^{(s)} = \mathbf{A}_{11}^{s-1}\mathbf{A}_{12} + \sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1} + \mathbf{A}_{12}\mathbf{A}_{22}^{s-1}$$

$$
\begin{aligned}
\mathbf{A}_{13}^{(s)} &= \mathbf{A}_{11}^{s-1}\mathbf{A}_{13} + \mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{\mathbf{1(k+1)}}\mathbf{A}_{\mathbf{(k+1)3}}\right) \\
&+ \sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{S-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right) + \mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right]
\end{aligned}
$$

$$\mathbf{A}_{22}^{(s)} = \mathbf{A}_{22}^{s}$$

$$\mathbf{A}_{23}^{(s)} = \sum_{K-1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}$$

$$\mathbf{A}_{33}^{(s)} = \mathbf{A}_{33}^{s}$$

Expressing them in matrix, it follows :

$$
\mathbf{A}^{(S)} = \begin{pmatrix}
\mathbf{A}_{11}^{s}, & \mathbf{A}_{11}^{s-1}\mathbf{A}_{12} + \sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1} + \mathbf{A}_{12}\mathbf{A}_{22}^{s-1}, & Q \\
\mathbf{0}, & \mathbf{A}_{22}^{s}, & \sum_{k=1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1} \\
\mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^{s}
\end{pmatrix}
$$

$$Q = \mathbf{A}_{11}^{s-1}\mathbf{A}_{13} + \mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{1(k+1)}\mathbf{A}_{(K+1)3}\right)$$
$$+\sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{s-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right)+\mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right]$$

Generalizing them to m groups, they are expressed as :

$$\begin{pmatrix} \mathbf{X}_{\mathbf{n}}^{(1)} \\ \mathbf{X}_{\mathbf{n}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n}}^{(m)} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1m} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2m} \\ \vdots & \vdots & & \vdots \\ \mathbf{A}_{m1} & \mathbf{A}_{m2} & \cdots & \mathbf{A}_{mm} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{\mathbf{n-1}}^{(1)} \\ \mathbf{X}_{\mathbf{n-1}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n-1}}^{(m)} \end{pmatrix}$$

$$\mathbf{X}_{\mathbf{n}}^{(1)} \in R^{k_1}\ ,\quad \mathbf{X}_{\mathbf{n}}^{(2)} \in R^{k_2}\ ,\quad \mathbf{X}_{\mathbf{n}}^{(m)} \in R^{k_m}\ ,\quad \mathbf{A}_{\mathbf{ij}} \in R^{k_i \times k_j}\ \left(i = 1,\cdots,m\right)\left(j = 1,\cdots,m\right)$$

4 Bulding the ranking table by utilizing correspondence analysis

In this section, the method to build the ranking table by utilizing correspondence analysis is described. Formerly, we have presented the paper, where the matrix structure was clarified when brand selection was executed toward higher grade brand in an automobile purchasing case[6]. The ranking table for automobile was arranged from the interview result to the car dealers and it was rectified by the target zone etc. We cannot deny the possibility that the gap may arise between dealer’s ranking table and consumer’s ranking table.

In this paper, the ranking table is established by accepting the consumer’s thought for the brand ranking. In particular, we collect the data of consumer’s perceived quality by executing questionnaire investigation. Next, by utilizing correspondence analysis, we classify some categories depending on these results. It becomes possible to rank brands depending on suitable factors except for price factor by this method.

Suppose take following 4 brands for example. Table 1 shows customers' evaluation to each brand by 5-point scale (highest grade: 5 to lowest grade: 1) from the view point of high grade or not. These are the assumed data.

Table 1: Evaluated Score of the Brand Bags (Ex.)

		Brand Name			
		Gucci	Vuitton	HELMES	COACH
Evaluated Class	5	22	40	48	14
	4	16	25	37	13
	3	33	23	18	28
	2	18	15	6	35
	1	12	1	2	15
Total		101	104	111	105

Based on these data, the brands are ranked by utilizing correspondence analysis. Correspondence analysis is calculated by using SPSS software. Based on the result of analysis, we obtained the following figure.

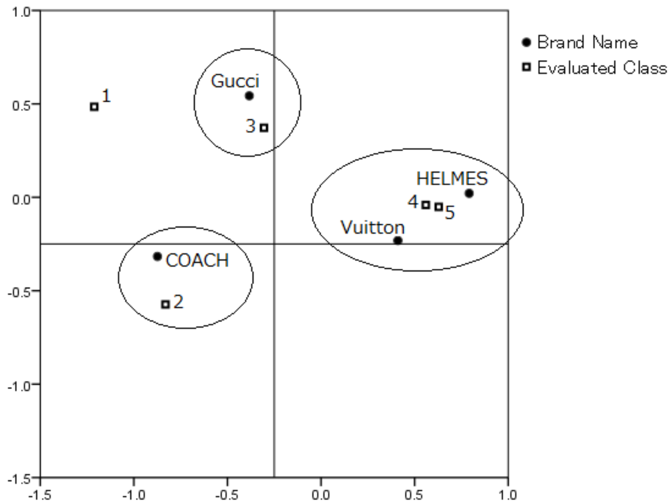

Figure 1: Principal Components' Score Map of the Brand Bags (Ex.)

From this result, HELMES and Vuitton are ranked as the most upper class. Gucci is ranked in the middle class, and COACH is ranked at the lowest class. Based on this ranking table, matrix structure is analyzed hereafter.

5 Numerical calculation

5.1 Numerical Example

Now, the proposed method is examined under the supposition that the data sets (Evaluated class, Purchase history) are obtained. We have 5 brands (A-E), and they are evaluated by 5-point scale (highest grade: 5 to lowest grade: 1) individually from the viewpoint of high grade or not. In this paper, these scores are generated in a random manner. According to this, purchasing data $(\mathbf{X_b}, \mathbf{X})$ is generated by the probability rate shown in Table 2.

Table 2: Brand Shift Probability Rate

%	5 point Brand	4 point Brand	3 point Brand	2 point Brand	1 point Brand	Total
$\mathbf{X_b}$	15%	15%	20%	25%	25%	100%
X	25%	25%	20%	15%	15%	100%

Following 100 cases are generated.

Table 3: Evaluation Score and Data Shift

	Evaluation Score					Data Shift				Evaluation Score					Data Shift		
	A	B	C	D	E	$\mathbf{X_b}$		**X**		A	B	C	D	E	$\mathbf{X_b}$		**X**
1.	3	4	5	2	1	B	→	C	51.	4	2	1	5	3	E	→	A
2.	5	3	4	1	2	D	→	E	52.	2	1	4	3	5	B	→	B
3.	5	2	3	4	1	B	→	C	53.	5	1	3	4	2	E	→	C
4.	2	5	4	3	1	E	→	A	54.	5	4	1	2	3	E	→	B
5.	5	1	3	2	4	D	→	C	55.	5	2	3	4	1	C	→	D
6.	2	1	3	4	5	B	→	B	56.	3	2	4	5	1	C	→	C
7.	2	1	3	4	5	A	→	A	57.	5	1	4	3	2	E	→	D
8.	4	1	5	3	2	D	→	A	58.	1	5	2	3	4	C	→	D
9.	3	2	5	4	1	D	→	C	59.	4	1	3	2	5	C	→	A
10.	3	5	1	4	2	C	→	C	60.	4	3	1	5	2	E	→	B
11.	4	1	5	2	3	B	→	D	61.	5	2	3	1	4	B	→	C

12.	2	4	5	3	1	A → D
13.	5	1	3	4	2	B → E
14.	3	2	5	1	4	E → C
15.	4	1	5	3	2	D → A
16.	1	3	2	4	5	B → D
17.	5	3	4	2	1	D → B
18.	3	4	5	2	1	E → E
19.	3	1	4	5	2	E → A
20.	1	4	2	3	5	A → A
21.	4	3	1	5	2	A → D
22.	4	3	2	1	5	A → E
23.	1	4	3	2	5	B → E
24.	4	1	2	5	3	B → B
25.	3	1	5	2	4	D → A
26.	4	1	3	5	2	B → E
27.	3	5	1	2	4	B → B
28.	4	2	1	5	3	C → C
29.	2	3	4	5	1	E → E
30.	3	5	1	2	4	E → E
31.	4	5	2	3	1	D → A
32.	5	3	2	4	1	C → B
33.	3	2	4	1	5	B → A

62.	4	3	2	5	1	D → D
63.	1	5	3	2	4	A → A
64.	4	1	2	5	3	A → A
65.	3	4	5	1	2	B → C
66.	3	5	2	4	1	A → D
67.	5	3	2	4	1	B → D
68.	3	2	4	5	1	B → A
69.	2	4	5	1	3	B → C
70.	3	1	5	2	4	A → E
71.	2	4	3	1	5	A → C
72.	1	5	4	2	3	E → C
73.	5	4	1	2	3	C → D
74.	1	3	2	4	5	E → E
75.	2	1	5	4	3	D → C
76.	5	1	3	2	4	A → A
77.	2	4	5	3	1	C → C
78.	4	5	2	1	3	D → C
79.	4	1	5	3	2	D → A
80.	1	5	2	4	3	B → B
81.	4	2	3	1	5	D → B
82.	4	2	5	1	3	B → E
83.	1	3	4	2	5	A → D

34.	2	3	5	1	4	A → B	84.	3	2	5	1	4	C → C
35.	1	5	4	3	2	A → E	85.	2	1	3	5	4	D → D
36.	4	1	3	2	5	E → E	86.	1	4	5	2	3	A → A
37.	5	3	2	4	1	B → D	87.	4	2	1	5	3	A → D
38.	1	4	2	3	5	E → E	88.	5	3	4	2	1	D → D
39.	3	1	5	2	4	D → A	89.	3	4	2	5	1	A → B
40.	4	5	3	1	2	E → C	90.	5	1	4	3	2	B → B
41.	2	1	3	4	5	D → E	91.	3	4	5	2	1	D → A
42.	1	4	5	2	3	E → B	92.	1	3	2	5	4	E → E
43.	5	3	4	2	1	D → B	93.	1	4	2	3	5	C → D
44.	1	5	2	4	3	B → B	94.	3	4	1	5	2	C → C
45.	4	3	5	2	1	B → A	95.	2	5	1	4	3	E → D
46.	5	4	2	1	3	D → C	96.	5	1	2	4	3	E → D
47.	2	1	3	4	5	B → B	97.	1	3	4	2	5	E → E
48.	2	3	5	1	4	D → A	98.	4	5	1	2	3	D → E
49.	5	1	4	3	2	B → B	99.	4	2	1	5	3	D → D
50.	4	5	1	3	2	E → D	100.	1	2	5	3	4	C → C

Total 100 cases

5.2 Correspondence Analysis

Now, the proposed method is examined to the data sets shown in Table 3. We obtained Table 4.

Table 4: Summary for Evaluated Score

		Brand name				
		A	B	C	D	E
Evaluated Score	5	21	17	25	18	19
	4	25	19	18	21	17
	3	20	20	20	18	22
	2	16	16	22	27	19
	1	18	28	15	16	23
Total		100	100	100	100	100

We obtained Figure 2.

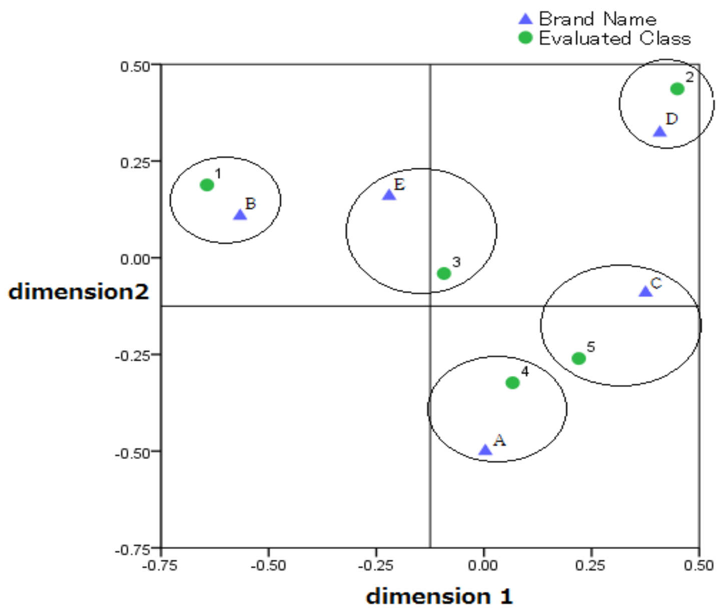

Figure 2: Principal Components' Score Map of The Brand Bags

From these results, the brands are ranked as follows (Table 5).

Table 5: Evaluation of the Brand Rank

Brand Name	Evaluated Rank
A	4(Upper Middle)
B	1(Low)
C	5(High)
D	2(Lower Middle)
E	3(Middle)

5.3 Matrix Calculation

In this section, the shift data is transformed into vectors based on Table 5. Vector sets $\mathbf{X}, \mathbf{X_b}$ in the cases for 1 through 10 in Table 3 are expressed as follows.

1. $\mathbf{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{X_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ 2. $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{X_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ 3. $\mathbf{X} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{X_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

4. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$ 5. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ 6. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$

7. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ 8. $\mathbf{X}=\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ 9. $\mathbf{X}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$

10. $\mathbf{X}=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}$ $\mathbf{X_b}=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$

Shift data in the cases for 11 through 100 are omitted. Substituting these to Eq.(7), we obtain the following equation.

$$\hat{\mathbf{A}}=\begin{pmatrix}7&1&4&5&5\\1&6&3&8&3\\0&3&8&3&4\\4&5&4&4&4\\1&2&3&3&9\end{pmatrix}\begin{pmatrix}13&0&0&0&0\\0&17&0&0&0\\0&0&22&0&0\\0&0&0&23&0\\0&0&0&0&25\end{pmatrix}^{-1}=\begin{pmatrix}\frac{2}{5}&\frac{17}{126}&\frac{12}{91}&\frac{4}{97}&\frac{10}{167}\\\frac{1}{7}&\frac{55}{126}&\frac{4}{13}&\frac{20}{97}&\frac{29}{167}\\\frac{1}{7}&\frac{5}{21}&\frac{22}{91}&\frac{24}{97}&\frac{26}{167}\\\frac{1}{7}&\frac{1}{14}&\frac{2}{13}&\frac{30}{97}&\frac{31}{167}\\\frac{6}{35}&\frac{5}{42}&\frac{15}{91}&\frac{19}{97}&\frac{71}{167}\end{pmatrix}$$

Eq.(32) becomes to be an upper triangular matrix on the whole. From this result, it becomes clear that the ranking table utilizing correspondence analysis is effective.

40 cases are the upper shifts, 34 cases are the same rank movement, and 26 cases are the lower shifts.

6 Remarks

Applications of this method are considered to be as follows. Consumers' behavior may converges by repeating forecast under the above method and the total volume of sales of all brands may be reduced. Therefore, the analysis results suggest when and what to put the new brand into the market which contribute to the expansion of the market. There may arise following cases. Consumers and producers do not recognize the brand position clearly. But the analysis of consumers' behavior let them know their brand position in the market. In such a case, strategic marketing guidance to select the brand would be introduced. Setting in order the brand position of various goods and taking suitable marketing policy, enhancement of sales would be enabled. Setting the higher ranked brand, consumption would be promoted.

7 Conclusion

It is often observed that consumers select the upper class brand when they buy the next time. Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix become an upper triangle matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand is selected from the lower brand in jumping way, corresponding part in an upper triangle matrix would be 0. A questionnaire investigation for automobile purchasing case was executed and the above structure was confirmed.

Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. In Takeyasu et al. (2007) [6], matrix structure was analyzed for the case brand selection was executed for upper class. In this paper, the method of building the ranking table utilizing correspondence analysis was newly proposed. It becomes clear that the ranking table utilizing correspondence analysis is effective with the demonstration of numerical example.

If the transition matrix is identified, a S-step forecasting can be executed. Generalized forecasting matrix components' equations were introduced. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Such research as questionnaire investigation of consumers' activities in brand wine / whisky purchasing cases should be executed in the near future to verify obtained results.

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