

Optimization in Allocating Goods to Shop Shelves for Cup Noodles

Koumei Suzuki¹, Yuki Higuchi² and Kazuhiro Takeyasu³

Abstract

How to allocate goods in shop shelves makes great influence to sales amount. Searching best fit allocation of goods to shelves is a kind of combinatorial problem. This becomes a problem of integer programming and utilizing genetic algorithm may be an effective method. Reviewing past researches, there are few researches made on this. Formerly, we have presented a papers concerning optimization in allocating goods to shop shelves utilizing genetic algorithm. In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examine the problem that allows goods to be

¹ Tokoha University.

² Setsunan University

³ Tokoha University.

allocated in multiple shelves and introduce the concept of sales profits and sales probabilities. Expansion of shelf is executed. Optimization in allocating goods to shop shelves is investigated. An application to the convenience store with POS sales data of cup noodles is executed. Utilizing genetic algorithm, optimum solution is pursued and verified by a numerical example. Various patterns of problems must be examined hereafter.

Mathematics Subject Classification: 90C27

Keywords: display; genetic algorithm; optimization; shelf

1 Introduction

Displaying method in the shop makes influence to sales amount, therefore various ideas are devised. What kind of items should be placed where in the shop, how to guide customers to what aisle in the shop are the big issues to be discussed. Searching best fit allocation of goods to shelves is also an important issue to be solved. In this paper, we seek how to optimize in allocating goods to shop shelves.

As for allocating good to shop shelves, following items are well known (Nagashima, 2005).

Shelf height is classified as follows.

- Shelf of 135cm height: Customers can see the whole space of the shop.

Specialty stores often use this type.

- Shelf of 150cm height: Female customers may feel pressure to the shelf height.

This height may be the upper limit to look over the shop.

- Shelf of 180cm height: It becomes hard to look over the shop. Therefore it should not be used for island display (display at the center or inside the shop).

Next, we show the following three functions of shelf for display.

1. Exhibition of goods function
2. Stock function
3. Display function

Effective range for exhibition is generally said to be 45cm-150cm. The range of 75cm-135cm is called golden zone especially. For the lower part under 45cm, goods are stocked as well as displaying.

Reviewing past papers, there are many papers concerning lay out problem. As for the problem of the distribution of equipment, we can see B. Korte *et al.* (2005), M. Gen *et al.* (1997) for the general research book. There are many researches made on this. Yamada *et al.* (2004) handles the lay out problem considering the aisle structure and intra-department material flow. Y. Wu *et al.* (2002) and Yamada *et al.* (2004) handle this problem considering aisle structure. Ito *et al.* (2006) considers multi-floor facility problem.

Although there are many researches on corresponding theme as stated above, we can hardly find researches on the problem of optimization in allocating goods to shop shelves.

Formerly, we have presented a paper concerning optimization in allocating

goods to shop shelves utilizing genetic algorithm (Takeyasu *et al.*,2008). In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examine the problem that allows goods to be allocated in multiple shelves and introduce the concept of sales profits and sales probabilities. Expansion of shelf is executed. Optimization in allocating goods to shop shelves is investigated. An application to the convenience store with POS sales data of cup noodles is executed. Utilizing genetic algorithm, optimum solution is pursued and verified by a numerical example.

The rest of the paper is organized as follows. Problem description is stated in section 2. Genetic Algorithm is developed in section 3. Numerical example is exhibited in section 4 which is followed by the remarks of section 5. Section 6 is a summary.

2 Problem Description

Shelf model is constructed as Figure 1. There are five shelf positions. Shelf position 1 is mainly to put big and heavy goods including stock function. Shelf position 3, 4 at the height of the range 75cm to 135cm are the space of golden zone. Thus, we can use shelves properly by assuming these shelves. In numerical example, we examine using these five shelves. First of all, we make problem description in the case there is only one shelf (case 1). Then we expand to the case

there are multiple shelves (case 2).

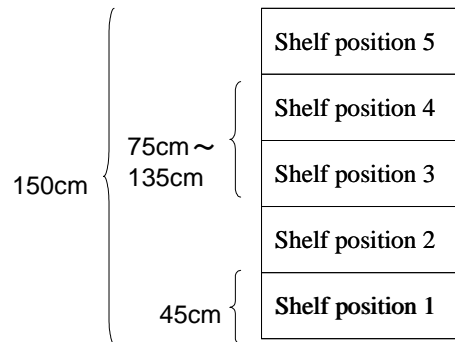


Figure 1: Shelf Model

(1)Case 1: The case that there is only one shelf

Although there are few cases that there is only one shelf, it makes the foundation for multiple shelves case. Therefore we pick it up as a fundamental one.

Suppose shelf position k is from 1 to L (Figure 2).

$k = L$
\vdots
$k = 3$
$k = 2$
$k = 1$

Figure 2: Shelf Position

Suppose there are N amount of goods ($i = 1, \dots, N$). Set sales profit of goods i as H^i . Table 1 shows the sales probabilities when each goods is placed at each shelf position. The values in this table are written for example.

Suppose goods are sold in the period from t_1 to t_n . In addition, a new goods i is replenished when goods i is sold out.

Set the accumulated sales probability of goods i in time zone t , shelf j , and shelf position k in the table as $HK_{t,j,k}^i$.

Then, the sales probability $K_{t_1/t_n}^{i,j,k}$ of goods i in the period will be described as follows.

$$K_{t_1/t_n}^{i,j,k} = \sum_{t=1}^n HK_{t,j,k}^i$$

This can take the value more than 1. For example, the value 2 means that 2 amount of goods were sold during the period.

Set Benefit in the sales period from t_1 to t_n as $P_{t_1/t_n}^{i,j,k}$ ($i = 1, \dots, N$) ($j = 1, \dots, m$) ($k = 1, \dots, L$) when goods i is placed at shelf j and shelf position k .

Where Benefit means:

$$\text{Benefit} = \text{SalesProbability} \times \text{SalesProfit}$$

Therefore, this equation is represented as follows.

$$P_{t_1/t_n}^{i,j,k} = K_{t_1/t_n}^{i,j,k} \cdot H^i \quad (1)$$

where $j=1$ because one shelf case is considered here.

Set $x_{i,k}$ as:

$$x_{i,k} = 1 : \text{Goods } i \text{ is placed at shelf}$$

$$x_{i,k} = 0 : \text{Else}$$

Suppose only one goods can be placed at one shelf position and also suppose that goods is allowed to allocate in multiple shelf positions. Then constraints are described as follows.

$$x_{i,k} = 1,0 \quad (i = 1, \dots, N) \quad (k = 1, \dots, L) \quad (2)$$

$$\sum_{i=1}^N x_{i,k} = 1 \quad (k = 1, \dots, L) \quad (3)$$

Under these constraints,

$$\text{Maximize } J = \sum_{k=1}^L \sum_{i=1}^N P_{t_1/t_n}^{i,j,k} x_{i,k} \quad (4)$$

(2) Case 2: The case that there are m shelves

Suppose there are m shelves (Figure 3). Set Benefit as $P_{t_1/t_n}^{i,j,k}$ ($i = 1, \dots, N$), ($j = 1, \dots, m$), ($k = 1, \dots, L_j$) where goods i is placed at shelf position k of shelf j . The sales period is the same with above stated (1).

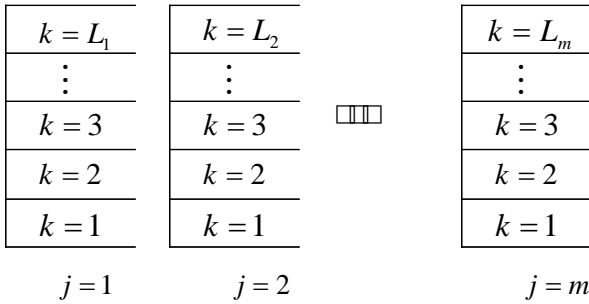


Figure 3: Shelf Position under multiple shelves

Set $x_{i,j,k}$ as:

$x_{i,j,k} = 1$: Goods is placed at shelf position k of shelf j

$x_{i,j,k} = 0$: Else

Suppose only one goods can be placed at one shelf position and also suppose that goods is allowed to allocate in multiple shelf positions. Then constraints are described as follows. The sales period is the same with before.

$$x_{i,j,k} = 1, 0 \quad (i = 1, \dots, N) \quad (j = 1, \dots, m) \quad (k = 1, \dots, L_j) \quad (5)$$

$$\sum_{i=1}^N x_{i,j,k} = 1 \quad (j = 1, \dots, m) \quad (k = 1, \dots, L_j) \quad (6)$$

Under these constraints,

$$\text{Maximize } J = \sum_{i=1}^N \sum_{j=1}^m \sum_{k=1}^{L_j} P_{t_1/t_n}^{i,j,k} x_{i,j,k} \quad (7)$$

3 Algorithm

We can make problem description as stated above, although these are somewhat under restricted cases. As far as only these are considered as they are, there is little difference between these and the conventional optimization problems. However, as soon as the number of involved shelves becomes larger, the number of variables dramatically grows greater, to which the application of Genetic Algorithm solution and Neural Network solutions may be appropriate. There are various means to solve this problem. When that variable takes the value of 0 or 1, the application of genetic algorithm would be a good method. As is well known, the calculation volume reaches numerous or even infinite amounts in these problems when the number of variables increases. It is reported that GA is effective for these problems (Gen et al. (1995), Lin et al. (2005), Zhang et al. (2005)).

A. The Variables

Suppose the number of goods, shelf position, and shelf are 20, 2, 9 respectively. In this paper, shelf is expanded from 2 to 9. Then the number of variables becomes two hundred.

$$x_{i,j,k} = 1,0 \quad (i = 1, \dots, 20) \quad (j = 1, \dots, 9) \quad (k = 1, 2)$$

Therefore, set chromosome as follows.

$$\begin{aligned}
 X = & (x_{1,1,1}, x_{2,1,1}, x_{3,1,1}, \dots, x_{20,1,1}, \\
 & x_{1,1,2}, x_{2,1,2}, x_{3,1,2}, \dots, x_{20,1,2}, \\
 & \vdots \\
 & x_{1,2,1}, x_{2,2,1}, x_{3,2,1}, \dots, x_{20,2,1}, \\
 & x_{1,2,2}, x_{2,2,2}, x_{3,2,2}, \dots, x_{20,2,2} \\
 & \vdots \\
 & x_{1,9,1}, x_{2,9,1}, x_{3,9,1}, \dots, x_{20,9,1}, \\
 & x_{1,9,2}, x_{2,9,2}, x_{3,9,2}, \dots, x_{20,9,2},)
 \end{aligned} \tag{8}$$

B. Initialize population

Initialization of population is executed. The number of initial population is M . Here set $M = 100$. Set gene at random and choose individual which satisfies constraints.

C. Selection

In this paper, we take elitism while selecting. Choose P individuals in the order which take maximum score of objective function.

Here, set $P = 20$

D. Crossover

Here, we take uniform crossover.

Set crossover rate as:

$$P_c = 0.7 \quad (9)$$

E. Mutation

Set mutation rate as:

$$P_m = 0.01 \quad (10)$$

Algorithm of GA is exhibited at Table 2.

Table 2: Algorithm of multi-step tournament selection method

Step 1 : Set maximum No. as g_{\max} , population size as P , crossover rate as p_c ,
mutation rate as p_m .

Step 2 : Set $t=1$ for generation No. and generate initial solution matrix

$$x_p(t) = (x_{i,j,k}^p) \quad (p=1, \dots, M).$$

Step 3 : Calculate Objective function $J(x_p(t))$ for all solution matrix $x_p(t)$
($p=1, \dots, P$) in generation t .

Step 4 : Set $t=t+1$ until $t > g_{\max}$.

Step 5 : Crossover

Generate new individual by crossover utilizing the method of above stated
 D .

Step 6 : Mutation

Reproduce by mutation utilizing the method of above stated E .

Step 7 : Calculate objective function for reproduction of generation t .

Step 8 : Selection

Next generation is selected by elitism.

Go to Step 4.

Introducing the variable y_s such that:

$$y_s = i \quad (11)$$

Where

$$s = k + (j - 1) \cdot 2 \quad (12)$$

When

$$x_{i,j,k} = 1$$

then (8) is expressed as:

$$Y = (y_1, y_2, \dots, y_{18}) \quad (13)$$

4 Numerical Example

Now, we execute numerical example using POS sales data. Numerical example is executed in “Case 2” of 2 (2). Suppose the sales period is 5 days for Monday through Friday. Table 3 shows the unit sales profit H^i of each goods.

Supposing a general daytime retail store, we set opening time to be 9 through 18 o'clock. Table 4 shows the sales probabilities of lot i as an example.

Table 5 shows the sales probability by shelf for each shelf position. Table 6 shows the value in which Table 4 and Table 5 are multiplied. Table 7 shows the

benefit Table in which accumulated probability of Table 6 and Sales Profit of Table 3 are multiplied.

Table 3: Unit Sales Price and Sales Profit of each goods

Lot i	Sales Price	H^i
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Table 4: Sales Probability of Lot i (Time Zone)

Day of the Week	Time Zone(t)	Sales Probability	Day of the Week	Time Zone(t)	Sales Probability
(Mon.)	9-10		(Thu.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	
(Tue.)	9-10		(Fri.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	

(Wed.)	9-10		(Sat.)	9-10	
	10-11			10-11	
	11-12			11-12	
	12-13			12-13	
	13-14			13-14	
	14-15			14-15	
	15-16			15-16	
	16-17			16-17	
	17-18			17-18	

In Table 5, shelf $j=2$ is located near the entrance therefore the table value reflects this condition.

Table 5: Sales Probability of Lot i (Shelf Position)

Shelf $j=1$		Shelf $j=2$		Shelf $j=3$		Shelf $j=4$		Shelf $j=5$		Shelf $j=6$		Shelf $j=7$		Shelf $j=8$		Shelf $j=9$	
Shelf Position		Shelf Position		Shelf Position		Shelf Position		Shelf Position		Shelf Position		Shelf Position		Shelf Position		Shelf Position	
$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$
1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.2	1.2	1.2	1.2	1.1	1.1	1.0	1.0	0.9	0.9

Table 6: Sales Probability of Lot i

Day of the Week	Time Zone(t)	Sales Probability											
		Shelf $j=1$		Shelf $j=2$		Shelf $j=3$...	Shelf $j=8$		Shelf $j=9$		
		$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	
(Mon.)	9-10											
	10-11											
	11-12							...					
	12-13											
	13-14											
	14-15											
	15-16											
	16-17											
	17-18											
(Tue.)	9-10											
	10-11											
	11-12											
	12-13											
	13-14											
	14-15											
	15-16											
	16-17											

	17-18										
(Sat.)	9-10										
	10-11										
	11-12										
	12-13										
	13-14										
	14-15										
	15-16										
	16-17										
	17-18										

Table 7 shows the benefit when each goods is placed at each shelf position of each shelf.

Table 7: Benefit Table

<i>i</i> Lot	Shelf 1		Shelf 2		Shelf 3		Shelf 4	
	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
1	500	800	510	810	510	810	630	1270
2	540	924	520	900	450	850	450	850
3	820	390	820	390	820	390	912	500
4	400	841	400	841	400	841	419	859
5	836	401	820	390	820	390	820	390
6	410	840	410	840	420	860	410	840
7	820	390	835	400	820	390	820	390
8	670	290	670	290	670	290	670	290
9	820	390	820	390	913	501	820	390
10	500	900	500	900	400	800	400	800
11	500	900	539	923	450	850	450	850
12	560	900	560	900	400	850	400	850
13	670	290	670	290	670	290	670	290
14	800	380	800	380	800	380	800	380
15	821	395	821	395	822	396	822	396
16	370	840	370	840	370	840	400	900
17	500	800	520	800	520	800	520	800
18	680	295	680	295	680	295	680	295

19	790	310	790	310	790	310	790	310
20	390	900	390	900	380	830	460	990

Shelf 5		Shelf 6		Shelf 7		Shelf 8		Shelf 9	
k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2	k = 1	k = 2
550	1010	540	1010	550	1100	560	1100	560	1010
450	850	450	850	450	850	450	850	450	850
800	360	800	360	670	300	670	300	670	300
400	841	400	841	400	841	400	841	400	841
800	360	800	360	670	300	670	300	670	300
410	840	410	840	410	840	410	840	410	840
800	380	800	380	670	310	670	310	670	310
670	290	670	290	680	300	680	300	689	301
790	350	790	350	660	280	660	280	660	280
500	1000	500	1000	500	1000	614	1272	500	1000
450	850	450	850	450	850	450	850	450	850
630	1140	550	1000	550	1000	550	1000	550	1000
670	290	670	290	688	300	690	302	688	300
815	403	800	380	680	300	680	300	680	300
801	401	801	400	681	295	681	296	911	500
400	900	629	1139	460	990	460	990	460	990
610	1100	615	1100	615	1273	600	1200	600	1100
680	295	680	295	691	303	685	300	685	300
790	310	814	402	680	290	680	290	680	290
460	990	460	990	460	990	460	990	605	1191

Experimental results are as follows. The expression Eq. (8) is complicated. Therefore we use expression by Eq. (13). A sample set of initial population is exhibited in Table 6.

Table 6: A Sample Set of Initial Population

$$\begin{aligned}
 Y_1 &= (1 \ 8 \ 1 \ 1 \ 4 \ 1 \ 1 \ 1 \ 6 \ 7 \ 1 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1) \\
 Y_2 &= (1 \ 1 \ 9 \ 2 \ 7 \ 1 \ 1 \ 1 \ 8 \ 2 \ 1 \ 1 \ 4 \ 4 \ 2 \ 1 \ 9 \ 1) \\
 Y_3 &= (1 \ 4 \ 1 \ 1 \ 9 \ 2 \ 1 \ 1 \ 7 \ 1 \ 8 \ 1 \ 1 \ 1 \ 1 \ 2 \ 1 \ 1) \\
 &\vdots \\
 Y_{98} &= (2 \ 1 \ 4 \ 1 \ 1 \ 1 \ 5 \ 1 \ 4 \ 4 \ 1 \ 1 \ 1 \ 1 \ 1 \ 7 \ 1 \ 1) \\
 Y_{99} &= (2 \ 2 \ 5 \ 1 \ 6 \ 1 \ 1 \ 7 \ 9 \ 7 \ 1 \ 7 \ 1 \ 5 \ 1 \ 1 \ 1 \ 5) \\
 Y_{100} &= (8 \ 7 \ 7 \ 8 \ 1 \ 7 \ 1 \ 2 \ 8 \ 1 \ 6 \ 1 \ 2 \ 4 \ 5 \ 1 \ 1 \ 2)
 \end{aligned}$$

Convergence process is exhibited in Figure 4.

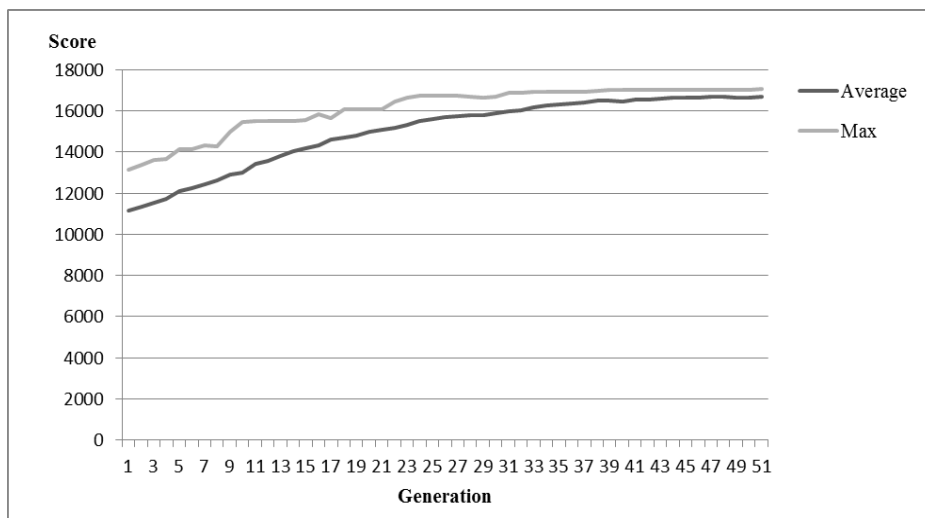


Figure 4: Convergence Process of Case 2

The problem is simple, so combination of genotype for crossover saturates in the 125th generation. Genotype in which objective function becomes maximum is as follows.

$$Y = (5, 2, 7, 11, 9, 6, 3, 1, 14, 12, 19, 16, 18, 17, 13, 10, 15, 20)$$

This coincides with the result of optimal solution by the calculation of all considerable cases, therefore it coincides with a theoretical optimal solution. We take up simple problem and we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

5 Remarks

As there are few papers made on this theme, we constructed prototype version before (Takeyasu et al., 2008). In this paper, we examined the problem that allowed goods to be allocated in multiple shelves and introduced the concept of sales profits and sales probabilities. An application to the shop with POS sales data was executed. We can see that genetic algorithm is effective for this problem.

In practice, following themes occur.

1. Sales probabilities should be arranged correctly.
2. There are various types of shelves corresponding to goods characteristics (For example, cold storage goods).
3. Furthermore, genotype must be devised in construction when there are huge number of goods and shelves.

For these issues, expanded version of the paper will be built hereafter consecutively. As for 1, constraints are relaxed than those of this paper. As for 2, expansion is easy to make. As for 3, constructing genotype from the shelf side would bear much more simple expression.

6 Conclusion

How to allocate goods in shop shelves makes great influence to sales amount. Searching best fit allocation of goods to shelves is a kind of combinatorial problem. This becomes a problem of integer programming and utilizing genetic algorithm may be an effective method. Reviewing past researches, there were few researches made on this. Formerly, we had presented papers concerning optimization in allocating goods to shop shelves utilizing genetic algorithm. In those papers, the problem that goods were not allowed to allocate in multiple shelves and the problem that goods were allowed to allocate in multiple shelves were pursued. In this paper, we examined the problem that allowed goods to be allocated in multiple shelves and introduced the concept of sales profits and sales probabilities. Expansion of shelf was executed. Optimization in allocating goods to shop shelves was investigated. An application to the convenience store with POS sales data of cup noodles was executed. Utilizing genetic algorithm, optimum solution was pursued and verified by a numerical example. Various patterns of problems should be examined hereafter.

References

- [1] Nagashima, Y., *Shop Arrangement and Displaying (in Japanese)*, Nihon-Jitsugyo-Press, 2005.
- [2] Atsumi, S., *Shop Lay Out (in Japanese)*. Jitsumu-Kyoiku-Press, 2008.
- [3] Kohno, H., *Making Charming Shop (in Japanese)*. Paru-Press, 2006.
- [4] Korte, B., Vygeu, J. (2005) *Combinatorial Optimization*. Springer, 2005.
- [5] Gen, M., Cheng, R., *Genetic Algorithms & Engineering Design*. JOHN WILEY & SONS,INC, 1997.
- [6] Michalewicz, Z., *Genetic Algorithms + Data Structures = Evolution Programs*. Springer, 1999.
- [7] Wu, Y., Appleton, E., The optimization of block layout and aisle structure by a genetic algorithm, *Computers & Industrial Engineering*, **41**, (2002), 371-382.
- [8] Yamada, T., Irohara, T. and Fujikawa, H., Design methodology for the Facility Layout Problem to Consider the Aisle Structure and Intra-Department Material Flow. *JIMA*, **55**(3), (2004), 111-120.
- [9] Yamada, T. and Irohara, T., Detailed Layout Design Methodology using Mixed Integer Programming and Simulated Annealing, *JIMA*, **57**(1), (2006), 39-50.
- [10] Ito, T. and Irohara, T., Multi-floor Facility Layout Technique Including Determination of Detailed *I/O* Location, *JIMA*, **57**(5), (2006), 395-403.
- [11] Arial. M., Masui, Y., Optimization of maritime container-transportation network through the use of genetic algorithm, *Japan Ship-Marine Engineering Society*, **4**, (2006),55-61.

- [12] Gen, M., Ida, K., Li, Y., Solving multi objective solid transportation problem by genetic algorithm, *JIMA*, **46**(5), (1995), 445-454.
- [13] Toyama, H., Ida, K., Teramatsu, C., A proposal of a genetic algorithm for fixed-charge transportation problem and related numerical experiments, *JIMA*, **57**(3), (2006), 227-230.
- [14] Takeyasu, K., Higuchi, Y., Optimization in Allocating Goods to Shop Shelves, *The Journal of Economic Studies*, Osaka Prefecture University, **54**(3), (2008), 55-64.