Journal of Computations & Modelling, vol.5, no.2, 2015, 49-58
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2015

A Suggested Approach in Mixed Zero-One Fuzzy Goal Programming

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Abstract

In this paper, a new approach for the mixed zero-one fuzzy goal programs is presented. This approach allows the decision-maker to set alternative fuzzy goal constraints to his/her original ones. The alternative fuzzy goal constraints are considered, instead of the original ones, if certain situation conditions are not satisfied. In addition to the linearization technique for the mixed zero-one fuzzy goal programs, which has been given by Chang [C.T. Chang, Binary fuzzy goal programming, European Journal of Operational Research, 180 (1), (2007), 29–37], another linearization technique for the binary goal programs is presented. The suggested approach of using alternative fuzzy goal constraints by simultaneously utilizing the two linearization techniques is illustrated by a numerical example.

Mathematics Subject Classification: 90C70

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Article Info: *Received* : January 4, 2015. *Revised* : February 3, 2015.
Published online : April 5, 2015.

Keywords: Fuzzy goal programming; Zero-one fuzzy goal programming; Mixed zero-one fuzzy goal programming

1 Introduction

The basic principles of decision making in fuzzy environments, which was set by Bellman and Zadeh [2] have been used as building blocks of fuzzy linear programming [15]. Fuzzy programming for solving multi-objective linear programming problems was first proposed by Zimmermann [14]. The relation between fuzzy programming and goal programming has been stated by Mohamed [9]. Since early 1980s, many attempts have been made in providing different concepts of fuzzy goal programming. The initial fuzzy goal programming model and its solution procedure were presented by Narasimhan [10], followed by Hannan [6] and Ignizio [7]. Tiwari et al. [11] proposed a concept of maximizing the sum of the weighted membership functions of the fuzzy goals. This concept has been improved by Yaghoobi et al. [13]. In fuzzy goal programming, models can consider goals of different importance and with preemptive priorities [5, 12]. However, providing crisp preemptive goal priorities is not an easy task. Uncertainty could be inherent in relative importance relations among the goals. In order to overcome this drawback, Aköz and Petrovic [1] have proposed a fuzzy goal programming model, where goal importance levels were defined and represented by fuzzy relations. In addition, Chanas and Kuchta [3] have provided a survey of various fuzzy goal programming models to represent a satisfaction degree of the decision-maker with respect to his/her preference structure.

Few attempts have been made in the area of binary fuzzy goal programming. One of the main contributions in this area is the approach, which has been presented by Chang [4]. According to his approach, any fuzzy goal may be removed from the model subject to the function of environment/resource constraint. This approach has been applied in the case of straight and U-shaped assembly line balancing [8]. In this paper, Chang’s approach is generalized by incorporating alternative fuzzy goals. In real cases, if certain situation conditions are not verified, it is more practical to have alternatives to the original fuzzy goals rather than just removing them from the model.

The paper is organized as follows: In the next section, the proposed approach is presented. The utilized linearization methods are given in Section 3. Section 4 is devoted for the implementation of the suggested approach by a numerical example. Finally, conclusions are provided in the last section.

2 Mixed Zero-One Fuzzy Goal Programs with Alternative Fuzzy Goals

In this section, the suggested approach of having alternative fuzzy goals is presented. Assume that the original mixed zero-one fuzzy goal constraints are given as

$$f_i(X, Y) \underset{\sim}{>} b_i, \; i = 1, 2, \dots, m_1, \tag{1}$$

$$f_i(X, Y) \underset{\sim}{<} b_i, \; i = m_1+1, m_1+2, \dots, m, \tag{2}$$

where $f_i(X, Y)$ is the ith linear function of binary (0-1) variables $X = (x_1, x_2, \dots, x_{n_1})$ and non-negative variables $Y = (y_1, y_2, \dots, y_{n_2})$. Also, $\underset{\sim}{>}$ and $\underset{\sim}{<}$ indicate approximately greater than or equal to and approximately less than or equal to, respectively, while b_i is the aspiration level of the ith fuzzy goal constraint. Let u_i and v_i that are determined by the decision-maker be, respectively, the lower and upper limits of the ith fuzzy goal constraint. Then, the ith membership function for the fuzzy goal constraint (1) is given by

$$\mu_{1i} = \begin{cases} 1 & \text{if} \quad f_i(X,Y) \geq b_i \,, \\ \dfrac{f_i(X,Y) - u_i}{b_i - u_i} & \text{if} \quad u_i < f_i(X,Y) < b_i \,, \\ 0 & \text{if} \quad f_i(X,Y) \leq u_i, \end{cases} \tag{3}$$

while for the fuzzy goal constraint (2), it is given by

$$\mu_{2i} = \begin{cases} 1 & \text{if} \quad f_i(X,Y) \leq b_i \,, \\ \dfrac{v_i - f_i(X,Y)}{v_i - b_i} & \text{if} \quad b_i < f_i(X,Y) < v_i \,, \\ 0 & \text{if} \quad f_i(X,Y) \geq v_i. \end{cases} \tag{4}$$

On the other hand, let $R_j(X)$ be the jth situation function, $j = 1, 2,\ldots, J$, where $R_j(X)$ is a Boolean function which indicates whether a certain condition is satisfied ($R_j(X) = 1$) or not ($R_j(X) = 0$). Then, it is assumed that $R_j(X) = \prod_{t \in T_j} x_t$, where T_j is the index set of binary variables that belong to the jth situation condition. Therefore, the decision-maker can set an alternative fuzzy goal constraint to the original one if his/her situation condition is not satisfied. This approach allows the decision-maker to set alternative fuzzy goal constraints instead of just getting rid of the original ones. Hence, let G_j be a disjoint index set of the fuzzy goal constraints that depend on the jth situation condition, $G = \bigcup_j G_j$, where G is the index set of all conditional fuzzy goal constraints.

Besides, let I_p be the index set of fuzzy goal constraints having priority level p, $p = 1, 2,\ldots, P$. Note that both the original fuzzy goal constraint and its alternative should have the same priority level.

Accordingly, the crisp goal programming model, including the alternative goal constraints, is presented as

$$\text{Lexicographically minimize } \Big\{ \sum_{i \in I_p} d_i + \sum_{\substack{i \in I_p \\ i \in G}} d_i^{\text{o}} \; : p = 1, 2,\ldots, P \Big\} \tag{5}$$

subject to:

$$L_i \, f_i\,(X, Y) \; - \; L_i \, u_i + d_i \geq 1, \;\; i = 1, 2,\ldots, m_1; \; i \notin G, \tag{6}$$

$$K_i\, v_i \,-\, K_i\, f_i\,(X, Y) + d_i \,\geq 1,\;\; i = m_1{+}1, m_1{+}2,\ldots, m;\;\; i \notin G\,, \tag{7}$$

$$r_j = R_j\,(X),\;\; j = 1, 2,\ldots, J, \tag{8}$$

$$L_i\, f_i\,(X, Y)\, r_j \,-\, L_i\, u_i\, r_j + d_i \geq r_j,\;\; i \in G_j;\;\; i = 1, 2,\ldots, m_1;\;\; j = 1, 2,\ldots, J, \tag{9}$$

$$K_i\, v_i\, r_j - K_i\, f_i\,(X, Y)\, r_j + d_i \,\geq r_j, i \in G_j; i = m_1{+}1, m_1{+}2,\ldots, m;\; j = 1, 2,\ldots, J, \tag{10}$$

$$L_i^{\mathrm{o}}\, f_i^{\mathrm{o}}(X, Y)\,(1 - r_j) \,-\, L_i^{\mathrm{o}}\, u_i^{\mathrm{o}}\,(1 - r_j) + d_i^{\mathrm{o}} \geq (1 - r_j),$$
$$i \in G_j;\;\; i = 1, 2,\ldots, m_1;\;\; j = 1, 2,\ldots, J, \tag{11}$$

$$K_i^{\mathrm{o}}\, v_i^{\mathrm{o}}\,(1 - r_j) \,- K_i^{\mathrm{o}} f_i^{\mathrm{o}}(X, Y)\,(1 - r_j) + d_i^{\mathrm{o}} \geq (1 - r_j),$$
$$i \in G_j;\;\; i = m_1{+}1, m_1{+}2,\ldots, m;\;\; j = 1, 2,\ldots, J, \tag{12}$$

$$\Phi_s(X, Y) \,\leq\, 0,\;\; s = 1, 2,\ldots, S, \tag{13}$$

$$0 \,\leq\, d_i \,\leq 1,\;\; i = 1, 2,\ldots, m, \tag{14}$$

$$0 \,\leq\, d_i^{\mathrm{o}} \leq 1,\;\; i \in G, \tag{15}$$

$$X, r_j \in \{0, 1\},\;\; j = 1, 2,\ldots, J, \tag{16}$$

$$Y \,\geq 0, \tag{17}$$

where $L_i = 1 / (b_i - u_i)$ and $K_i = 1 / (v_i - b_i)$. The under-achievement of the ith fuzzy goal constraint is represented by the deviational variable d_i. The superscript “o” indicates the alternative. For instance, $f_i^{\mathrm{o}}(X, Y)$ is the alternative to $f_i(X, Y)$. Any of the situation constraints (8) may be presented in the form of inequality, depending on the situation condition that is required by the decision-maker. The goal constraints (11) and (12) are the alternatives to (9) and (10), respectively. Constraint set (13) represents the set of linear system constraints, while constraints (14) and (15) ensure the non-negativity of both the deviational variables and the membership functions.

3 The Linearization Approaches

It is obvious that the model (5)-(17) takes the form of a non-linear mixed zero-one goal program. Therefore, this model is linearized by two approaches. The first is utilized for the product of binary variables, while the second (Chang’s approach) is utilized when a non-negative variable is multiplied by a binary variable.

3.1 The first linearization approach

For the binary variables x_t, t = 1, 2,…, T, let $\alpha = \prod_{t=1}^{T} x_t$ where α is a binary variable, i.e., $\alpha \in \{0, 1\}$. Thus, $\alpha = 1$ if all the binary variables are equal to one, and $\alpha = 0$ otherwise. This relation can be linearized by the following inequality:

$$1 - T + \sum_{t=1}^{T} x_t \leq \alpha \leq \frac{\sum_{t=1}^{T} x_t}{T}. \tag{18}$$

3.2 The second linearization approach (Chang's approach)

For any binary variable x and non-negative variable y, let $\beta = xy$ where $\beta = y$ if $x = 1$, and $\beta = 0$ if $x = 0$. Then, this relation can be linearized as follows:

$$\begin{aligned} &(x-1)W + y \leq \beta \leq (1-x)W + y, \\ &0 \leq \beta \leq xW, \end{aligned} \tag{19}$$

where W is a large positive number.

It is worth noting that the second linearization approach can replace the first one. However, it is recommended to use the first one to linearize the product of binary variables since it requires less number of constraints, as shown by the following proposition.

Proposition 3.1 The product of T binary variables, $\prod_{t=1}^{T} x_t$ can be linearized by one two-sided constraint according to the first linearization approach, while it requires $2(T-1)$ two-sided constraints according to the second linearization approach.

4 Computational Study

In this section, a numerical illustration for the suggested approach is given. Therefore, assume the following mixed zero-one fuzzy goal program:

$$(g_1)\ \ 50x_1 + 30x_2 + 40x_3 + 4y_1 + 6y_2 \underset{\sim}{>} 60,$$

$$(g_2)\ \ 20x_1 + 40x_2 + 10x_3 \underset{\sim}{<} 40,$$

$$(g_3)\ \ 10y_1 + 6y_2 \underset{\sim}{<} 30,$$

$$x_1 + x_2 + x_3 \geq 1,$$

$$3y_1 + 2y_2 \geq 11,$$

$$x_1, x_2, x_3 \in \{0, 1\},$$

$$y_1, y_2 \geq 0.$$

The tolerance limits for the three fuzzy goals are (50, 45, 35) respectively. The decision-maker states that if either x_1 and/or x_3 is equal to zero, then the following fuzzy goal constraints are respectively used:

$$(g_1^{\mathrm{o}})\ \ 40x_1 + 35x_2 + 40x_3 + 5y_1 + 5y_2 \underset{\sim}{>} 65,$$

$$(g_2^{\mathrm{o}})\ \ 20x_1 + 30x_2 + 20x_3 \underset{\sim}{<} 50,$$

$$(g_3^{\mathrm{o}})\ \ 7y_1 + 8y_2 \underset{\sim}{<} 25,$$

in addition to the same system constraints, and with the tolerance limits (60, 55, 35) respectively. Accordingly, one situation constraint is considered ($r_1 = x_1x_3$), where $G_1 = \{1, 2, 3\}$. Let $r = r_1$; hence if $r = 1$, then g_1, g_2, and g_3 are used, while if $r = 0$, then g_1^{o}, g_2^{o}, and g_3^{o} are used. Moreover, two priority levels are assumed. The first priority level is assigned to the first goal, while the second priority level is assigned to the second and the third goals. Therefore, by utilizing the two linearization approaches, the linearized crisp mixed zero-one goal program can be presented as follows:

Lexicographically minimize $\{ d_1 + d_1^o, d_2 + d_2^o + d_3 + d_3^o \}$

subject to:

$5\alpha_1 + 3\alpha_2 + 4\alpha_3 + 0.4\beta_1 + 0.6\beta_2 - 6r + d_1 \geq 0,$

$-4\alpha_1 - 8\alpha_2 - 2\alpha_3 + 8r + d_2 \geq 0,$

$-2\beta_1 - 1.2\beta_2 + 6r + d_3 \geq 0,$

$8x_1 + 7x_2 + 8x_3 + y_1 + y_2 - 8\alpha_1 - 7\alpha_2 - 8\alpha_3 - \beta_1 - \beta_2 + 13r - 13 + d_1^o \geq 0,$

$-4x_1 - 6x_2 - 4x_3 + 4\alpha_1 + 6\alpha_2 + 4\alpha_3 - 10r + 10 + d_2^o \geq 0,$

$-0.7y_1 - 0.8y_2 + 0.7\beta_1 + 0.8\beta_2 - 2.5r + 2.5 + d_3^o \geq 0,$

$x_1 + x_3 - 1 \leq r \leq (x_1 + x_3) / 2,$

$x_t + r - 1 \leq \alpha_t \leq (x_t + r) / 2, \; t = 1, 2, 3,$

$(r-1)W + y_q \leq \beta_q \leq (1-r)W + y_q, \; q = 1, 2,$

$0 \leq \beta_q \leq rW, \; q = 1, 2,$

$x_1 + x_2 + x_3 \geq 1,$

$3y_1 + 2y_2 \geq 11,$

$0 \leq d_1, d_2, d_3, d_1^o, d_2^o, d_3^o \leq 1,$

$r, x_1, x_2, x_3, \alpha_1, \alpha_2, \alpha_3 \in \{0, 1\},$

$y_1, y_2 \geq 0,$

where the value of W is presumed to be 10000. The General Algebraic Modeling System (GAMS win32 23.8.2) package is utilized in solving this model using the sequential method for the two priorities. The optimal solution is $x_1 = 1$, $x_2 = 1$, $x_3 = 0$, (or $x_1 = 0$, $x_2 = 1$, $x_3 = 1$), $y_1 = 3.667$, and $y_2 = 0$. This means that $r = 0$ and the alternative goals are considered in the model instead of the original ones. Besides, all the deviational variables are zero except $d_3^o = 0.067$. Hence, only the third alternative goal, which takes the second priority level, has not been achieved. Furthermore, when the system constraint ($3y_1 + 2y_2 \geq 11$) is relaxed by setting the right-hand side to be 10.5 instead of 11, all the deviational variables are equal to zero. Thus, the three alternative goals have been completely achieved.

On the other hand, solving the original model without utilizing the suggested approach, i.e., with no situation constraint and no alternative goals, gives the following optimal solution: $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, (or any one of x_1, x_2, x_3 equals one while the other two equal zero), $y_1 = 0$, $y_2 = 5.5$, $d_1 = 0$, $d_2 = 0$, and $d_3 = 0.6$. It is obvious that this solution is worse than the one we get when the suggested approach is utilized. In many cases, the solution according to the suggested approach is at least as good as the solution if the suggested approach is not exploited.

5 Conclusions

The fuzzy mixed zero-one goal program has a wide area of applications. For instance, the fuzzy facility location problem is a common one. Practically, the decision-maker may have some situation conditions where one of two fuzzy goal constraints should be considered based on the fulfillment of a specific situation condition. The suggested methodology deals with this case. In addition to the Chang's linearization technique, another linearization technique can be utilized to reduce the number of constraints in the linearized model. Furthermore, it has been shown that the suggested approach may improve the solution. Finally, other than the lexicographic minimization which has been used in this study, the proposed methodology can be implemented for other criteria such as weighted min-max and weighted additive approach.

References

[1] O. Aköz and D. Petrovic, A Fuzzy goal programming method with imprecise goal hierarchy, *European Journal of Operational Research*, **181** (3), (2007), 1427–1433.

[2] R.E. Bellman and L.A. Zadeh, Decision-making in a fuzzy environment, *Management Science*, **17** (4), (1970), B141–B164.

[3] S. Chanas and D. Kuchta, Fuzzy goal programming – one notion, many meanings, *Control and Cybernetics*, **31** (4), (2002), 871–890.

[4] C.T. Chang, Binary fuzzy goal programming, *European Journal of Operational Research*, **180** (1), (2007), 29–37.

[5] L.H. Chen and F.C. Tsai, Fuzzy goal programming with different importance and priorities, *European Journal of Operational Research*, **133**, (2001), 548–556.

[6] E.L. Hannan, On fuzzy goal programming, *Decision Sciences*, **12**, (1981), 522–531.

[7] J.P. Ignizio, On the (re)discovery of fuzzy goal programming, *Decision Sciences*, **13**, (1982), 331–336.

[8] Y. Kara, T. Paksoy and C.T. Chang, Binary fuzzy goal programming approach to single model straight and U-shaped assembly line balancing, *European Journal of Operational Research*, **195** (2), (2009), 335–347.

[9] R.H. Mohamed, The relationship between goal programming and fuzzy programming, *Fuzzy Sets and Systems*, **89**, (1997), 215–222.

[10] R. Narasimhan, Goal programming in a fuzzy environment, *Decision Sciences*, **11**, (1980), 325–336.

[11] R.N. Tiwari, S. Dharmar and J.R. Rao, Fuzzy goal programming – An additive model, *Fuzzy Sets and Systems*, **24** (1), (1987), 27–34.

[12] H.F. Wang and C-C. Fu, A generalization of fuzzy goal programming with preemptive structure, *Computers & Operations Research*, **24** (9), (1997), 819–828.

[13] M.A. Yaghoobi, D.F. Jones and M. Tamiz, Weighted additive models for solving fuzzy goal programming problems, *Asia-Pacific Journal of Operational Research*, **25**, (2008), 715–733.

[14] H.J. Zimmermann, Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, **1**, (1978), 45–55.

[15] H.J. Zimmermann, *Fuzzy Set Theory and Its Applications*, Kluwer Academic Publishers, USA, 2001.