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Improving Forecasting Accuracy by the Utilization of Genetic Algorithm –An Application to the Data of Frozen Cakes –

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Abstract

In industries, making a correct forecasting is inevitable. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following

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method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original production data of two kinds of cakes. Genetic Algorithm is utilized to search optimal weights for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Then forecasting is executed on these data. In particular, a new method for calculating forecasting error is also proposed.

Mathematics Subject Classification: 62J10; 62M10; 91B70

Keywords: forecasting, genetic algorithm, exponential smoothing method, trend

1 Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM), [1]-[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it doesn’t pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2^{nd} order non-linear function and 3^{rd} order non-linear function is executed to the original production data of two kinds of cakes. Genetic Algorithm is utilized to search optimal weights for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of the previously proposed method. Variance of forecasting error of this newly proposed method is assumed to be less than those of the previously proposed method. Another method for calculating forecasting error (FAR) is also newly proposed in this paper. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4.

Forecasting is executed in section 5, and estimation accuracy is examined.

2 Description of ESM Using ARMA Model

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \quad (1)$$

$$= \alpha x_t + (1-\alpha)\hat{x}_t \quad (2)$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t: realized value at t

α: smoothing constant $(0 < \alpha < 1)$

(2) is re-stated as

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \quad (3)$$

By the way, we consider the following (1,1)order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \quad (4)$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \quad (5)$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process

$x(t)\ t = 1,2,\cdots,N,\cdots$

$\{e_t\}$:Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the ralation that

$$E\left[e_t \middle| e_{t-1}, e_{t-2}, \cdots\right] = 0$$

we get the following equation from (4).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{6}$$

Operating this scheme on $t+1$, we finally get

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \end{aligned} \tag{7}$$

If we set $1 - \beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1. Comparing with (4) and (5), we obtain

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (7),

$$\alpha = 1 - \beta$$

Therefore, we get

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{8}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (5) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{9}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{10}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known.

$$\left.\begin{array}{l} \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \tag{11}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (4) (5) (8) (11), we get

$$\left.\begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array}\right\} \tag{12}$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{13}$$

The following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{14}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{15}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{16}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (14), using the next relation,

$$(1-b_1)^2 \ge 0$$
$$(1+b_1)^2 \ge 0$$

(16) always holds.

As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$\left.\begin{aligned} b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \tag{17}$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way. Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating 0th and 1st order autocorrelation function.

3 Trend Removal Method

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function

We set

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function. (a_2, b_2, c_2) and (a_3, b_3, c_3, d_3) are also parameters for a 2nd and a 3rd order non-linear functions which are estimated by using least square method.

[3] The combination of linear and non-linear function.

We set

$$\begin{aligned} y = {} & \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) \\ & + \alpha_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \end{aligned} \tag{21}$$

$$0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le \alpha_3 \le 1, \alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{22}$$

as the combination linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the original data by (21). The optimal weighting parameters $\alpha_1, \alpha_2, \alpha_3$ are determined by utilizing GA. GA method is precisely described in section 6.

4 Monthly Ratio

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\}\ (i = 1, \cdots, L)\ (j = 1, \cdots, 12)$$

Where, $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a shipping data of i-th year, j-th month. Then, monthly ratio $\tilde{x}_j (j = 1, \cdots, 12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12} x_{ij}} \tag{23}$$

Monthly trend is removed by dividing the data by (23). Numerical examples both of monthly trend removal case and non-removal case are discussed in 7.

5 Forecasting Accuracy

Forecasting accuracy is measured by calculating the variance of the forecasting error. Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{24}$$

Where, forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{26}$$

And, another method for calculating forecasting error is shown as follows (Forecasting Accuracy Ratio: FAR).

$$FAR = \left\{ 1 - \frac{\sum_{i=1}^{N} |\varepsilon_i|}{\sum_{i=1}^{N} x_i} \right\} \cdot 100 \qquad (27)$$

6 Searching Optimal Weights utilizing GA

6.1 Definition of the problem

We search $\alpha_1, \alpha_2, \alpha_3$ of (21) which minimizes (24) by utilizing GA. By (22), we only have to determine α_1 and α_2. ${\sigma_\varepsilon}^2$ ((24)) is a function of α_1 and α_2, therefore we express them as ${\sigma_\varepsilon}^2(\alpha_1, \alpha_2)$. Now, we pursue the following:

$$\begin{aligned} &\text{Minimize: } \sigma_\varepsilon^2(\alpha_1, \alpha_2) \\ &\text{subject to: } 0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, \alpha_1 + \alpha_2 \le 1 \end{aligned} \qquad (28)$$

We do not necessarily have to utilize GA for this problem which has small member of variables. Considering the possibility that variables increase when we use logistics curve etc. in the near future, we want to ascertain the effectiveness of GA.

6.2 The structure of the gene

Gene is expressed by the binary system using {0,1} bit. Domain of variable is [0,1] from (22). We suppose that variables take down to the second decimal place. As the length of domain of variable is 1-0=1, seven bits are required to express variables. The binary bit strings <bit6, ～,bit0> is decoded to the [0,1] domain real number by the following procedure.[14]

Procedure 1:Convert the binary number to the binary-coded decimal.

$$\begin{aligned}&\left(\langle bit_6, bit_5, bit_4, bit_3, bit_2, bit_1, bit_0 \rangle\right)_2\\ &= \left(\sum_{i=0}^{6} bit_i 2^i\right)_{10} \qquad (29)\\ &= X'\end{aligned}$$

Procedure 2: Convert the binary-coded decimal to the real number.

The real number

= (Left hand starting point of the domain) (30)

+ X'((Right hand ending point of the domain)/(2^7-1))

The decimal number, the binary number and the corresponding real number in the case of 7 bits are expressed in Table 1.

Table 1: Corresponding table of the decimal number, the binary number and the real number

The decimal number	The binary number							The Corresponding real number
	Position of the bit							
	6	5	4	3	2	1	0	
0	0	0	0	0	0	0	0	0.00
1	0	0	0	0	0	0	1	0.01
2	0	0	0	0	0	1	0	0.02
3	0	0	0	0	0	1	1	0.02
4	0	0	0	0	1	0	0	0.03
5	0	0	0	0	1	0	1	0.04
6	0	0	0	0	1	1	0	0.05
7	0	0	0	0	1	1	1	0.06

8	0	0	0	1	0	0	0	0.06
…								…
126	1	1	1	1	1	1	0	0.99
127	1	1	1	1	1	1	1	1.00

1 variable is expressed by 7 bits, therefore 2 variables needs 14 bits. The gene structure is exhibited in Table 2.

Table 2: The gene structure

α_1							α_2						
Position of the bit													
13	12	11	10	9	8	7	6	5	4	3	2	1	0
0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1

6.3 The flow of algorithm

The flow of algorithm is exhibited in Figure 1.

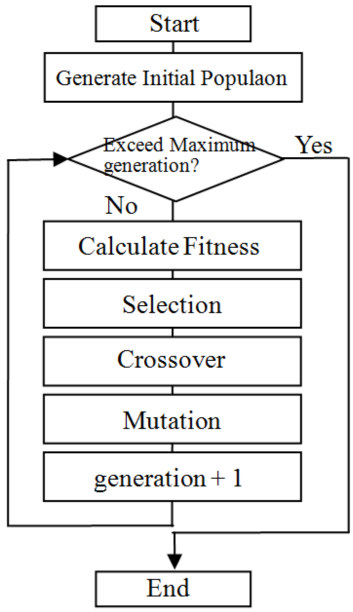

Figure 1: The flow of algorithm

A. Initial Population

Generate M initial population. Here, $M = 100$. Generate each individual so as to satisfy (22).

B. Calculation of Fitness

First of all, calculate forecasting value. There are 36 monthly data for each case. We use 24 data (1st to 24th) and remove trend by the method stated in section 3. Then we calculate monthly ratio by the method stated in section 4. After removing monthly trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (24). Calculation of fitness is exhibited in Figure 2.

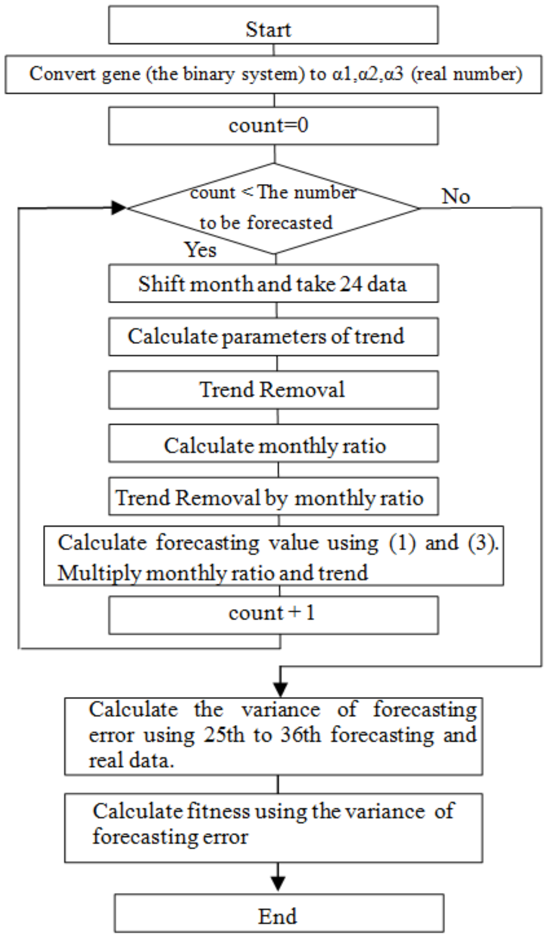

Figure 2: The flow of calculation of fitness

Scaling [15] is executed such that fitness becomes large when the variance of forecasting error becomes small. Fitness is defined as follows.

$$f(\alpha_1, \alpha_2) = U - \sigma_\varepsilon^{\ 2}(\alpha_1, \alpha_2) \tag{31}$$

Where U is the maximum of $\sigma_\varepsilon^{\ 2}(\alpha_1, \alpha_2)$ during the past W generation. Here, W is set to be 5.

C. Selection

Selection is executed by the combination of the general elitist selection and the tournament selection. Elitism is executed until the number of new elites reaches the predetermined number. After that, tournament selection is executed and selected.

D. Crossover

Crossover is executed by the uniform crossover. Crossover rate is set as follows.

$$P_c = 0.7 \tag{32}$$

E. Mutation

Mutation rate is set as follows.

$$P_m = 0.05 \tag{33}$$

Mutation is executed to each bit at the probability P_m, therefore all mutated bits in the population M becomes $P_m \times M \times 14$.

7 Numerical Example

7.1 Application to the original production data of frozen cakes

The original production data of frozen cakes for 2 cases from January 2008 to December 2010 are analyzed. Furthermore, GA results are compared with the calculation results of all considerable cases in order to confirm the effectiveness of GA approach. First of all, graphical charts of these time series data are exhibited in Figure 3, 4.

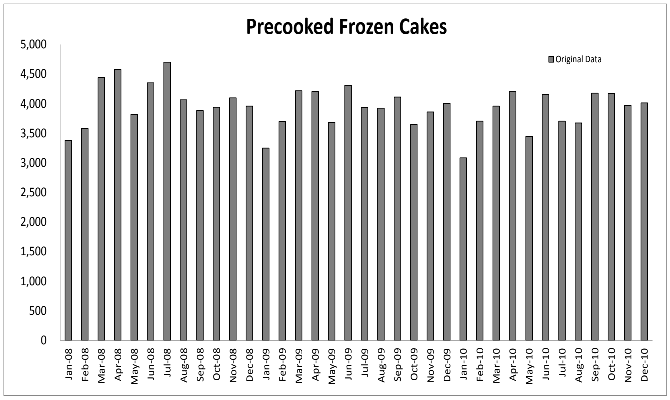

Figure 3: Data of Precooked Frozen Cakes

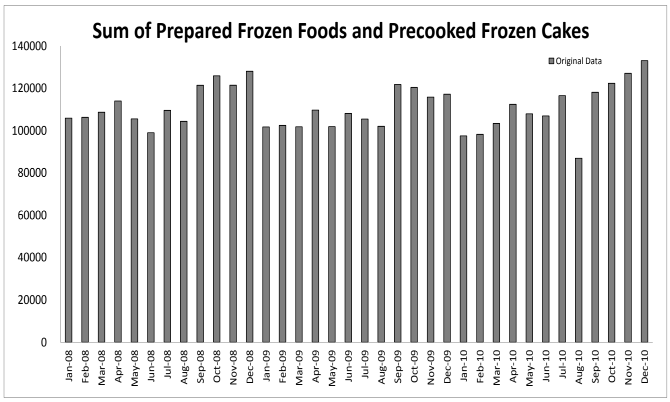

Figure 4: Data of Sum of Prepared Frozen Foods and Precooked Frozen Cakes

7.2 Execution Results

GA execution condition is exhibited in Table 3.

We made 10 times repetition and the maximum, average, minimum of the variance of forecasting error and the average of convergence generation are exhibited in Table 4 and 5.

Table 3: The gene structure

GA Execution Condition	
Population	100
Maximum Generation	50
Crossover rate	0.7
Mutation ratio	0.05
Scaling window size	5
The number of elites to retain	2
Tournament size	2

Table 4: GA execution results (Monthly ratio is not used)

	The variance of forecasting error		
	Maximum	Average	Minimum
Precooked Frozen Cakes	216,363.0375	216,363.0375	216,363.0375
Prepared Frozen Foods and Precooked Frozen Cakes	207,150,450	207,150,450	207,150,450

	Average of Convergence generation	FAR (to the minimum Variance)
Precooked Frozen Cakes	16.3	90.4%
Prepared Frozen Foods and Precooked Frozen Cakes	12.8	94.0%

Table 5: GA execution results (Monthly ratio is used)

	The variance of forecasting error		
	Maximum	Average	Minimum
Precooked Frozen Cakes	70,053.9185	70,053.9185	70,053.9185
Prepared Frozen Foods and Precooked Frozen Cakes	24,584,943.77	24,584,943.77	24,584,943.77

	Average of Convergence generation	FAR (to the minimum Variance)
Precooked Frozen Cakes	15.5	90.0%
Prepared Frozen Foods and Precooked Frozen Cakes	7.1	94.8%

The case monthly ratio is used is smaller than the case monthly ratio is not used concerning the variance of forecasting error in both cases. Seasonal trend can be observed in these data. Also, FAR found to be a sensitive good index.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

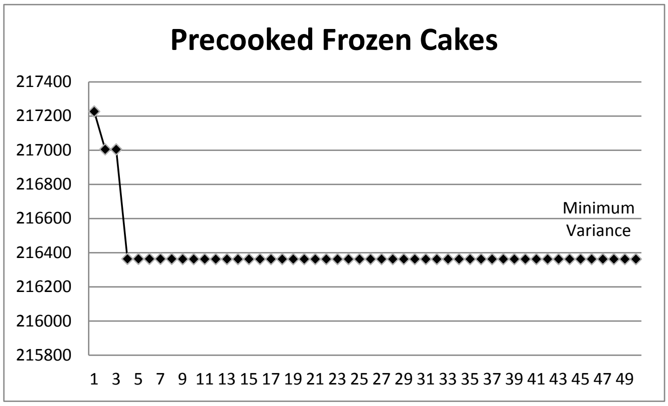

Figure 5: Convergence Process in the case of Precooked Frozen Cakes (Monthly ratio is not used)

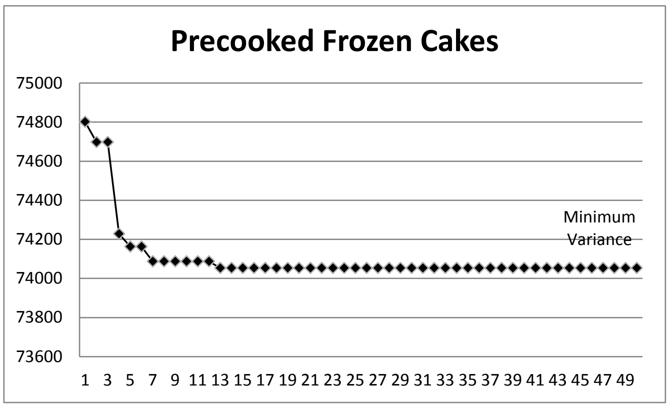

Figure 6: Convergence Process in the case of Precooked Frozen Cakes (Monthly ratio is used)

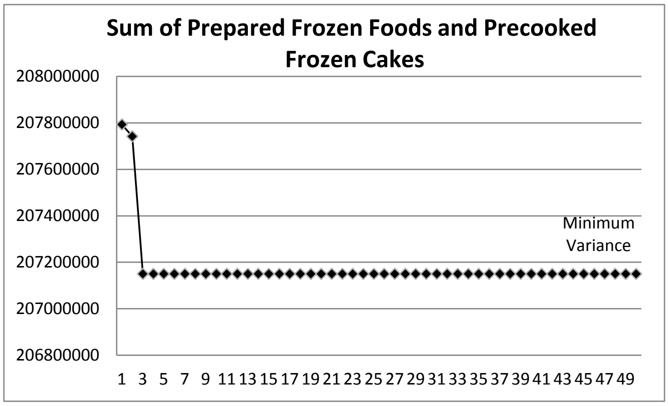

Figure 7: Convergence Process in the case of Prepared Frozen Foods and Precooked Frozen Cakes (Monthly ratio is not used)

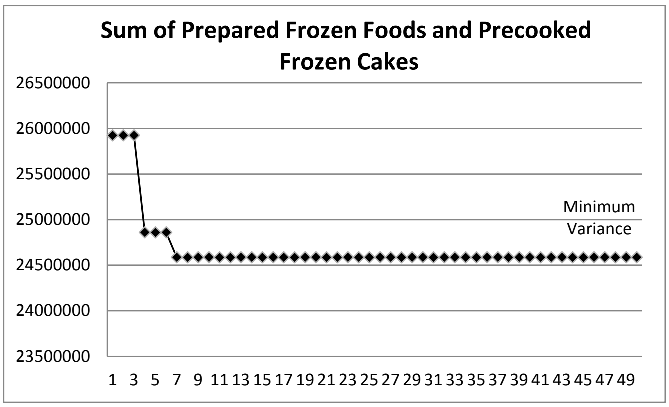

Figure 8: Convergence Process in the case of Prepared Frozen Foods and Precooked Frozen Cakes (Monthly ratio is used)

Next, optimal weights and their genes are exhibited in Table 6, 7.

Table 6: Optimal weights and their genes (Monthly ratio is not used)

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
Precooked Frozen Cakes	0.96	0.04	1	1	1	1	0	1	0	0	0	0	0	1	0	1
Prepared Frozen Foods and Precooked Frozen Cakes	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0

Table 7: Optimal weights and their genes (Monthly ratio is used)

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
Precooked Frozen Cakes	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Prepared Frozen Foods and Precooked Frozen Cakes	0.57	0.43	1	0	0	1	0	0	1	0	1	1	0	1	1	1

In the case monthly ratio is not used, the linear function model is best in both cases. In the case monthly ratio is used, the linear function model is best in both cases. Parameter estimation results for the trend of equation (21) using least square method are exhibited in Table 8 for the case of 1st to 24th data.

Table 8: Parameter estimation results for the trend of equation (21)

Data	a_1	b_1	a_2	b_2	c_2
Precooked Frozen Cakes	-5.448	4052.681	-0.991	19.326	3945.326
Prepared Frozen Foods and Precooked Frozen Cakes	227.109	107959.344	4.912	104.306	108491.493

Data	a_3	b_3	c_3	d_3
Precooked Frozen Cakes	0.436	-17.331	186.080	3562.974
Prepared Frozen Foods and Precooked Frozen Cakes	12.192	-452.296	4770.27	97792.816

Trend curves are exhibited in Figure 9, 10.

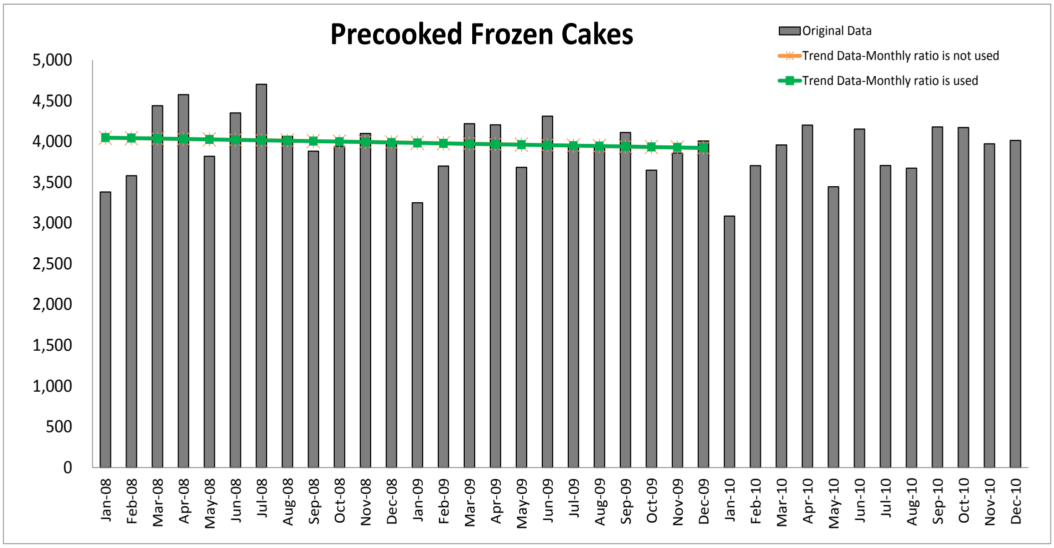

Figure 9: Trend of Precooked Frozen Cakes

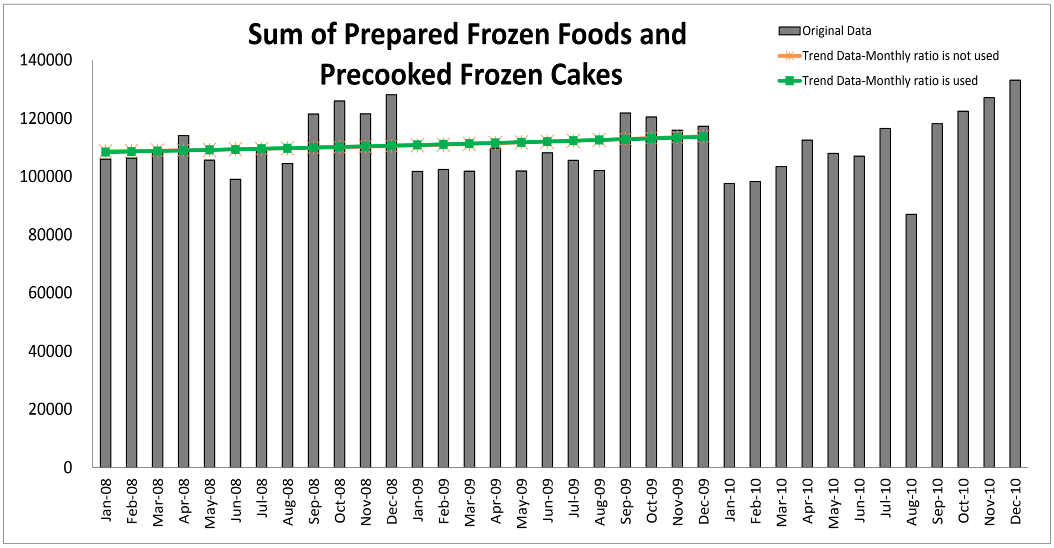

Figure 10: Trend of Prepared Frozen Foods and Precooked Frozen Cakes

Calculation results of Monthly ratio for 1st to 24th data are exhibited in Table 9.

Table 9: Parameter Estimation result of Monthly ratio

Month	1	2	3	4	5	6
Precooked Frozen Cakes	0.825	0.908	1.081	1.098	0.939	1.086
Prepared Frozen Foods and Precooked Frozen Cakes	0.948	0.951	0.957	1.016	0.940	0.935

Month	7	8	9	10	11	12
Precooked Frozen Cakes	1.084	1.004	1.007	0.957	1.004	1.007
Prepared Frozen Foods and Precooked Frozen Cakes	0.970	0.929	1.092	1.104	1.062	1.095

Estimation result of the smoothing constant of minimum variance for the 1st to 24th data are exhibited in Table 10, 11.

Table 10: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is not used)

Data	ρ_1	α
Precooked Frozen Cakes	-0.2037	0.7870
Prepared Frozen Foods and Precooked Frozen Cakes	-0.2956	0.6727

Table 11: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is used)

Data	ρ_1	α
Precooked Frozen Cakes	-0.0308	0.9692
Prepared Frozen Foods and Precooked Frozen Cakes	-0.4360	0.4145

Forecasting results are exhibited in Figure 11, 12.

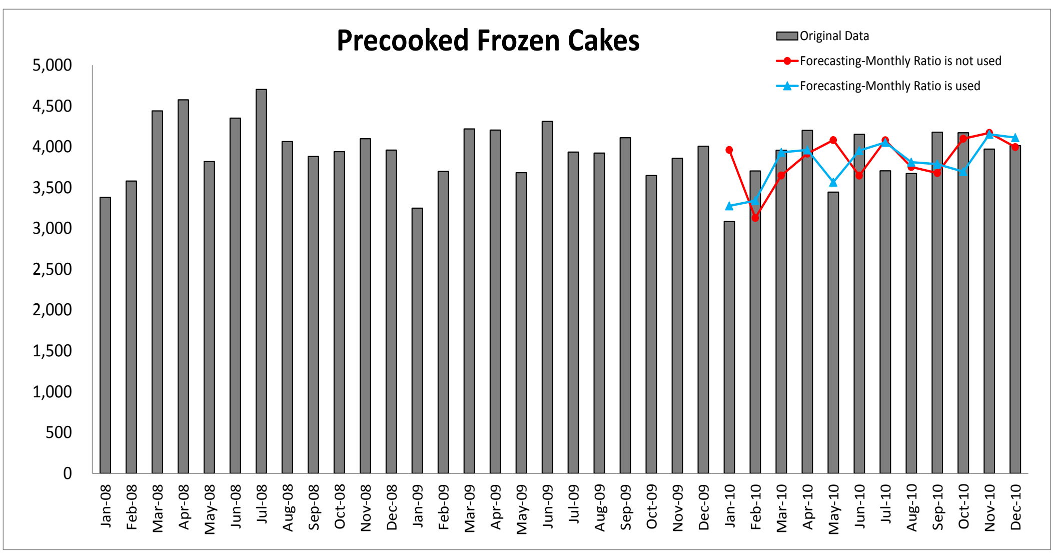

Figure 11: Forecasting Result of Precooked Frozen Cakes

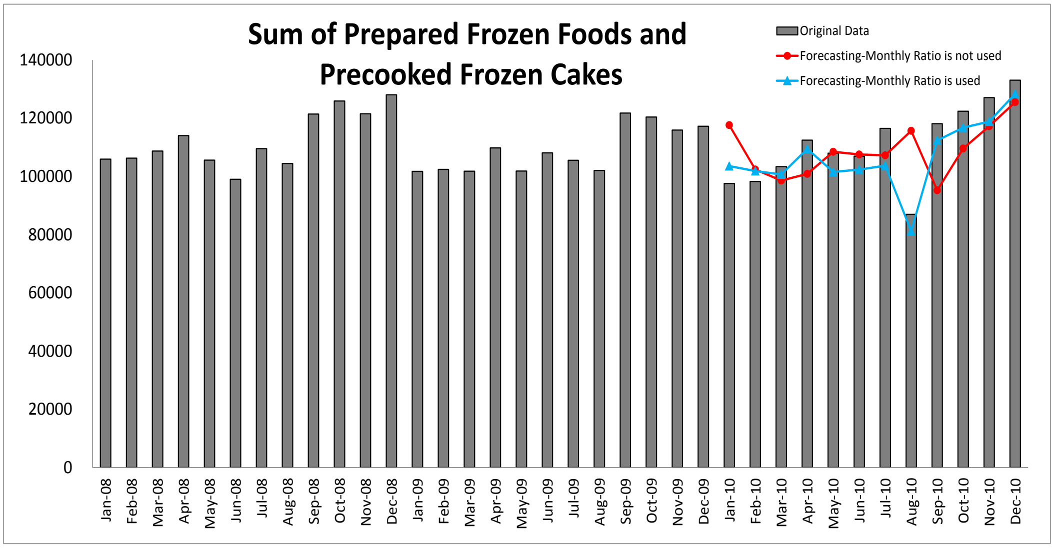

Figure 12: Forecasting Result of Prepared Frozen Foods and Precooked Frozen Cakes

7.3 Remarks

In both cases, that monthly ratio was used had a better forecasting accuracy. In the case monthly ratio is used, it can be said that the values of FAR are generally high. Precooked Frozen Cakes case had a good result in the linear function model, while Prepared Frozen Foods and Precooked Frozen Cakes case had a good result in the 1st+2nd order function model.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

8 Conclusion

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original production data of cakes. The combination of linear and non-linear function was also introduced in trend removal. Genetic Algorithm was utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting was executed on these data. Another method for calculating forecasting error (FAR) is also newly proposed in this paper, and it found to be a sensitive good index.

The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

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