# Free-Flight Trajectories of Small Caliber Projectiles and In-Bore Yaw Phenomenon Effects 

Elias E. Panagiotopoulos ${ }^{1}$ and Dimitrios N. Gkritzapis ${ }^{2}$


#### Abstract

This paper investigates the effects of in bore-yaw phenomenon on lateral throw-off and aerodynamic jump behavior for small caliber rotational symmetric projectiles launched horizontally at supersonic firing speeds and various altitudes from subsonic air vehicles. The ammunition used is the caliber . 50 API M8 bullet type firing from M2 machine automatic gun. The projectile is considered to be eccentrically engraved, tilted as it enters the rifling, and it is assumed that the tilt persists throughout its passage through the rifled barrel of the used weapon system. The modified linear 6-DOF flight simulation modeling is applied for the bullet free-flight trajectory predictions.


Keywords: In-Bore Yaw Effects, Lateral Throw-off, Aerodynamic Jump, API M8 bullet type, Modified Linear 6-DOF Trajectory Simulation, Supersonic Firing, Subsonic Air Vehicles, Tilt angle

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## 1 Introduction

The importance of the lateral throw-off and aerodynamic jump phenomena on the trajectories of spin-stabilized, flat-fire munitions is sufficient for the precision of firing sidewise from air vehicles. However, for modern high-velocity, these effects are often much larger than the drift, at moderate ranges.

The trajectory deflections caused by the above effects were first identified in 1909 by F. W. Mann [1], who called it as "X-Error". Lateral throw-off is the modern term for Mann’s "X-Error". Moreover, various authors have simulated the aerodynamic jump phenomenon caused by aerodynamic asymmetry and lateral force impulses [2].

The present study extends the work of [3], which examined the flat-fire launched disturbances due to aerodynamic jump phenomenon of the API M8 projectile firing at different angles relative to air vehicle's flight path motion. The examined bullet is assumed to be tilted as it enters the rifling, and it is assumed that the tilt persists throughout its passage through the rifled barrel. Under these assumptions, an engineering closed-form correlation is proposed for in-bore-yaw effects based on the bullet tilt angle, roll orientation angle at the muzzle and characteristic geometric distances of the examined ammunition.

A modified 6-DOF linear flight trajectory model depending on Costello’s theory [4] is applied for rapid predictions. The coupled pitching and yawing motion [5] is obtained for the first 100 meters of the whole atmospheric trajectory path.

## 2 Atmospheric Flight Trajectory Analysis

The projectile can be modelled as rigid body possessing six degrees-of-freedom (6-DOF) including three inertial position components (x, y, z) of the system mass center as well as the three Euler orientation angles $(\varphi, \theta, \psi)$.

Two mean coordinate systems are used for the computational approach of the atmospheric flight motion. The one is a plane fixed (inertial frame), at the ground surface. The other is a no-roll rotating coordinate system that is attached to, and moving with, the projectile's center of mass (no-roll-frame).

The modified linear theory equations of motion based on the analysis of [6] and must be numerically integrated to estimate rapid trajectories. Various simplifications were employed such as: (i) small yaw angle $\psi$, (ii) aerodynamic angles of attack $\alpha$ and sideslip $\beta$ remain small ( $<15^{\circ}$ ) for the main part of the trajectory, (iii) Magnus force components are small in comparison with the weight and aerodynamic force components, and so they are treated as negligible, and (iv) projectile is geometrically and aerodynamically symmetric about the station line.

The aerodynamic force and moment coefficients are projectile-specific functions of the Mach number and total angle of attack variations.

## 3 In-Bore Yaw Effects

The effect of in-bore yaw phenomenon [6] on lateral throw-off and aerodynamic jump is based on the assumption of a rotationally symmetric spin-stabilized projectile, both in configuration and mass distribution. The tilt angle (in-bore yaw angle) is denoted by the Greek letter $\mu$. Thus, if the center of gravity is either ahead of or behind this midpoint, there will also be a static unbalance, denoted by the Greek letter $\mu^{\prime}$. The relationship between $\mu^{\prime}$ and $\mu$ is given by the formula:

$$
\begin{equation*}
\frac{\mu^{\prime}}{D}=\left(L_{N}+\frac{1}{2} L_{C Y L}-C G_{N}\right) \tan \mu \tag{1}
\end{equation*}
$$

Taking the basic equations for the aerodynamic jump of a bullet due to in-bore yaw effects and the lateral throw-off phenomenon due to bullet tilt [6] we have the following expressions:

$$
\begin{align*}
& A_{J}=-i\left[\left(\frac{2 \pi}{\eta}\right)\left({K_{Y}}^{2}-K_{X}^{2}\right)\left(\frac{C_{L A}}{C_{M A}}\right) \tan \omega\right] e^{i \phi_{0}}  \tag{2}\\
& T_{L}=i\left[\left(\frac{2 \pi}{\eta}\right)\left(L_{N}+\frac{1}{2} L_{C Y L}-C G_{N}\right) \tan \omega\right] e^{i \phi_{0}} \tag{3}
\end{align*}
$$

## 4 Computational Results

The 6-DOF atmospheric trajectory with variable aerodynamic and Magnus coefficients of . 50 API M8 bullet motion is simulated while the wind effects are neglected. Horizontal fire from 100m height with almost $900 \mathrm{~m} / \mathrm{s}$ and in-bore clearance angle of 0.2 degrees are assumed firing right-hand side at two different angles (60 and 90 degrees) relative to aircraft's velocity of $230 \mathrm{~m} / \mathrm{s}$. The produced initial yaw (or sideslip) angles are approximately 11 degrees and 14 degrees, respectively, as shown in Figures 1a and 1b.


Figure 1: Coupled pitching and yawing motion for the . 50 API M8 bullet.


Figure 2: Deflections produced by lateral throw-off and aerodynamic jump.

The deflections produced by lateral throw-off and aerodynamic jump for firing sidewise at 100 m height with three different tilt angles ( $0.2,0.3$ and 0.4 degrees), due to flat sidewise firing at 20 degrees angle from an aircraft, are almost $114 \mathrm{~cm}, 170 \mathrm{~cm}$ and 227 cm , respectively, as shown in Figure 2a. The corresponding deflections firing at 90 degrees angle from an aircraft are approximately 123 cm, 185 cm and 247 cm, respectively, as depicted in Figure 2b.

## 5 Conclusion

The 6-DOF simulation flight dynamics model is applied for the prediction of the coupled pitching and yawing motion for spin-stabilized bullets firing sidewise from air vehicles. Variable aerodynamic coefficients based on Mach number variations and total angle of attack effects are included for the examined API M8 bullet trajectory simulations. An engineering closed-form correlation is proposed for the flat-fire disturbances due to in-bore yaw effects firing at different angles relative to air vehicle's flight path motion.

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[^0]:    ${ }^{1}$ Department of Mathematical and Engineering Sciences, Hellenic Military Academy, Vari, Attiki, 16673, Greece. E-mail: hpanxaxan@yahoo.gr
    ${ }^{2}$ Department of Mathematical and Engineering Sciences, Hellenic Military Academy, Vari, Attiki, 16673, Greece. E-mail: gritzap@yahoo.gr

