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Intermittent Demand Forecasting with the Estimation of Smoothing Constant of Minimum Variance Searching Optimal Parameters of Weight

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Abstract

In recent years, the needs for intermittent demand forecasting are increasing because of the constraints of strict Supply Chain Management. How to improve the forecasting accuracy is an important issue. There are many researches made on this. But there are rooms for improvement. In this paper, a new method for cumulative forecasting method is proposed. The data is cumulated and to this cumulated time series, the following method is applied to improve the forecasting accuracy. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, the new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing

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constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve the forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2^{nd} order non-linear function and 3^{rd} order non-linear function is executed to the production data of Emission CT apparatus and Superconducting magnetic resonance imaging system. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data.

The forecasting result is compared with those of the non-cumulative forecasting method. The new method shows that it is useful for the forecasting of intermittent demand data. The effectiveness of this method should be examined in various cases.

Mathematics Subject Classification: 62J10; 62M10; 91B70

Keywords: intermittent demand forecasting; minimum variance; exponential smoothing method; forecasting; trend

1. Introduction

In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are cases that intermittent demand forecasting is required. But the mere application of the past method does not bear good estimation of parameters and exquisite forecasting.

There are many researchers made on this.

Based upon the Croston's model (Croston 1972, Box et al.2008), Shenstone and Hyndma (2005) analyzed the intermittent demand forecasting.

Troung et al. (2011) applied Neural Network to intermittent demand forecasting. Ghobbar and Friend (1996) have made application to aircraft maintenance and inventory control.

Tanaka et al. (2012) has built sales forecasting model for book publishing, where they have devised cumulative forecasting method.

In this paper, we further develop this cumulative forecasting method in order to improve the forecasting accuracy for intermittent demand.

A new method for cumulative forecasting method is proposed.

The data is cumulated and to this cumulated time series, the following method is applied to improve the forecasting accuracy. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error[7]. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve the forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2^{nd} order non-linear function and 3^{rd} order non-linear function is executed to the data of Emission CT apparatus and Superconducting magnetic resonance imaging system. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data.

The forecasting result is compared with those of the non-cumulative forecasting method. The new method shows that it is useful for the forecasting of intermittent demand data. The effectiveness of this method should be examined in various cases.

The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2.Description of ESM using ARMA Model [7]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\begin{aligned}\hat{x}_{t+1} &= \hat{x}_t + \alpha\left(x_t - \hat{x}_t\right) \\ &= \alpha x_t + (1-\alpha)\hat{x}_t\end{aligned} \tag{1}$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t : realized value at t

α : smoothing constant $(0 < \alpha < 1)$

(1) is re-stated as

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \tag{2}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{3}$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{4}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$:Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (4) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E\left[e_t \middle| e_{t-1}, e_{t-2}, \cdots\right] = 0$$

we get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{5}$$

Operating this scheme on $t+1$, we finally get

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \end{aligned} \tag{6}$$

If we set $1-\beta=\alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1, [1, 3].

Comparing with (3) and (4), we obtain

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (6),

$$\alpha = 1 - \beta$$

Therefore, we get

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{7}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below. Let (4) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{8}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{9}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (8), (9), we get the following non-linear equations which are well known [3].

$$\left.\begin{array}{l} \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \tag{10}$$

For these equations, a recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (3) (4) (7) (10), we get

$$\left.\begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array}\right\} \tag{11}$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{12}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{13}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{14}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{15}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (13), using the next relation,

$$(1 - b_1)^2 \ge 0$$
$$(1 + b_1)^2 \ge 0$$

(15) always holds. As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$\left.\begin{aligned} b_1 &= \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \tag{16}$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

Here ρ_1 must satisfy

$$-\frac{1}{2} < \rho_1 < 0 \tag{17}$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3.Trend Removal Method[7]

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function

We set

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function

We set

$$y=\alpha_1\left(a_1x+b_1\right)+\alpha_2\left(a_2x^2+b_2x+c_2\right) \tag{21}$$

$$y=\beta_1\left(a_1x+b_1\right)+\beta_2\left(a_3x^3+b_3x^2+c_3x+d_3\right) \tag{22}$$

$$y=\gamma_1\left(a_1x+b_1\right)+\gamma_2\left(a_2x^2+b_2x+c_2\right)+\gamma_3\left(a_3x^3+b_3x^2+c_3x+d_3\right) \tag{23}$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, $\alpha_2=1-\alpha_1$, $\beta_2=1-\beta_1$, $\gamma_3=1-(\gamma_1+\gamma_2)$. Comparative discussion concerning (21), (22) and (23) are described in section 5.

4. Monthly Ratio [7]

For example, if there is the monthly data of L years as stated bellow:

$$\left\{x_{ij}\right\}\ \left(i=1,\cdots,L\right)\quad\left(j=1,\cdots,12\right)$$

Where, $x_{ij}\in R$ in which j means month and i means year and x_{ij} is a shipping data of i-th year, j-th month. Then, monthly ratio $\tilde{x}_j$, $\left(j=1,\cdots,12\right)$ is calculated as follows.

$$\tilde{x}_j=\frac{\frac{1}{L}\sum_{i=1}^{L}x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12}x_{ij}} \tag{24}$$

Monthly trend is removed by dividing the data by (24). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

5. Forecasting the Production Data

5.1 Analysis Procedure

Sum total data of production data of Emission CT apparatus and Superconducting magnetic resonance imaging system from January 2010 to December 2012 are analyzed. These data are obtained from the Annual Report of Statistical Investigation on Statistical-Survey-on-Trends-in-Pharmaceutical-Production by Ministry of Health, Labor and Welfare in Japan.

The original data and accumulated data are exhibited in Table 1 (Emission CT Apparatus) and Table 2 (Superconducting magnetic resonance imaging system).

Table 1: Original Data and Accumulated Data in Emission Apparatus

	Original Data	Accumulated Data
January /2010	5	5
February /2010	8	13
March /2010	13	26
April /2010	7	33
May /2010	0	33
June /2010	0	33
July /2010	0	33
August /2010	0	33
September /2010	12	45
October /2010	0	45
November /2010	0	45
December /2010	0	45
January /2011	0	45
February /2011	0	45

March /2011	15	60
April /2011	0	60
May /2011	0	60
June /2011	0	60
July /2011	5	65
August /2011	2	67
September /2011	21	88
October /2011	4	92
November /2011	3	95
December /2011	0	95
January /2011	6	101
February /2012	11	112
March /2012	22	134
April /2012	5	139
May /2012	5	144
June /2012	5	149
July /2012	0	149
August /2012	0	149
September /2012	0	149
October /2012	9	158
November /2012	8	166
December /2012	10	176

Table 2: Original Data and Accumulated Data in Superconducting magnetic resonance imaging system

	Original Data	Accumulated Data
January /2010	30	30
February /2010	39	69
March /2010	59	128
April /2010	17	145
May /2010	20	165

June /2010	24	189
July /2010	0	189
August /2010	31	220
September /2010	40	260
October /2010	29	289
November /2010	25	314
December /2010	27	341
January /2011	0	341
February /2011	0	341
March /2011	53	394
April /2011	67	461
May /2011	26	487
June /2011	21	508
July /2011	26	534
August /2011	43	577
September /2011	63	640
October /2011	27	667
November /2011	48	715
December /2011	98	813
January /2011	61	874
February /2012	54	928
March /2012	120	1,048
April /2012	50	1,098
May /2012	66	1,164
June /2012	60	1,224
July /2012	49	1,273
August /2012	48	1,321
September /2012	78	1,399
October /2012	44	1,443
November /2012	48	1,491
December /2012	86	1,577

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data (1 to 24) and remove trend by the method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{26}$$

Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{27}$$

5.2 Trend Removing

Trend is removed by dividing original data by,(21),(22),(23). The patterns of trend removal are exhibited in Table 3.

Table 3: The patterns of trend removal

Pattern1	α_1, α_2 are set 0.5 in the equation (21)
Pattern2	β_1, β_2 are set 0.5 in the equation (22)
Pattern3	α_1 is shifted by 0.01 increment in (21)
Pattern4	β_1 is shifted by 0.01 increment in (22)
Pattern5	γ_1 and γ_2 are shifted by 0.01 increment in (23)

In pattern1 and 2, the weight of α_1, α_2, β_1, β_2 are set 0.5 in the equation (21),(22). In pattern3, the weight of α_1 is shifted by 0.01 increment in (21) which satisfy the range $0 \le \alpha_1 \le 1.00$. In pattern4, the weight of β_1 is shifted in the same way which satisfy the range $0 \le \beta_1 \le 1.00$. In pattern5, the weight of γ_1 and γ_2 are shifted by 0.01 increment in (23) which satisfy the range $0 \le \gamma_1 \le 1.00$, $0 \le \gamma_2 \le 1.00$ .The best solution is selected which minimizes the variance of forecasting error.

5.3 Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 4.

5.4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (16). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (15). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval.

The intermittent demand data often include 0 data. If there are so many 0 data, there is a case we cannot calculate the theoretical solation of smoothing constant.

In that case, we add very tiny data which is not 0 but close to 0 that does not affect anything in calculating parameters (i.e. negligible small).

5.5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36^{th} data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (27).

As we have made accumulated data case and tiny data close to 0 added case, we have the following cases altogether.

1. Non Monthly Trend Removal

(1) Accumulated Data

(2) Non Accumulated Data

(2-1) Forecasting from the Accumulated data (Accumulated forecasting data at time n－Accumulated data (at time n-1))

A. Pattern 1

B. Pattern 2

C. Pattern 3

D. Pattern 4

E. Pattern 5

(2-2) Forecasting from the tiny data close to 0 added case

A. Pattern 1

B. Pattern 2

C. Pattern 3

D. Pattern 4

E. Pattern 5

2. Monthly Trend Removal

(1) Accumulated Data

(2) Non Accumulated Data

(2-1) Forecasting from the Accumulated data (Accumulated forecasting data at time n－Accumulated data (at time n-1))

A. Pattern 1

B. Pattern 2

C. Pattern 3

D. Pattern 4

E. Pattern 5

(2-2) Forecasting from the tiny data close to 0 added case

A. Pattern 1

B. Pattern 2

C. Pattern 3

D. Pattern 4

E. Pattern 5

We can make forecasting by reversely making the data from the forecasting accumulated data, i.e., that is shown at (2-1).

Now, we show them at Figure1 through 6.

Figure 1,2,3 show the Non-monthly Trend Removal Case in Emission CT Apparatus.

It includes all cases classified above.

Figure 1 shows the Accumulated Data Case in Non-Monthly Trend Removal.

Figure 2 shows the Forecasting from the Accumulated Data Case in Non-Monthly Trend Removal.

Figure 3 shows the Forecasting from the tiny data close to 0 added case in Non-Monthly Trend Removal.

Table 4,5 and 6 show the corresponding variance of forecasting error for each Figure 1,2 and 3.

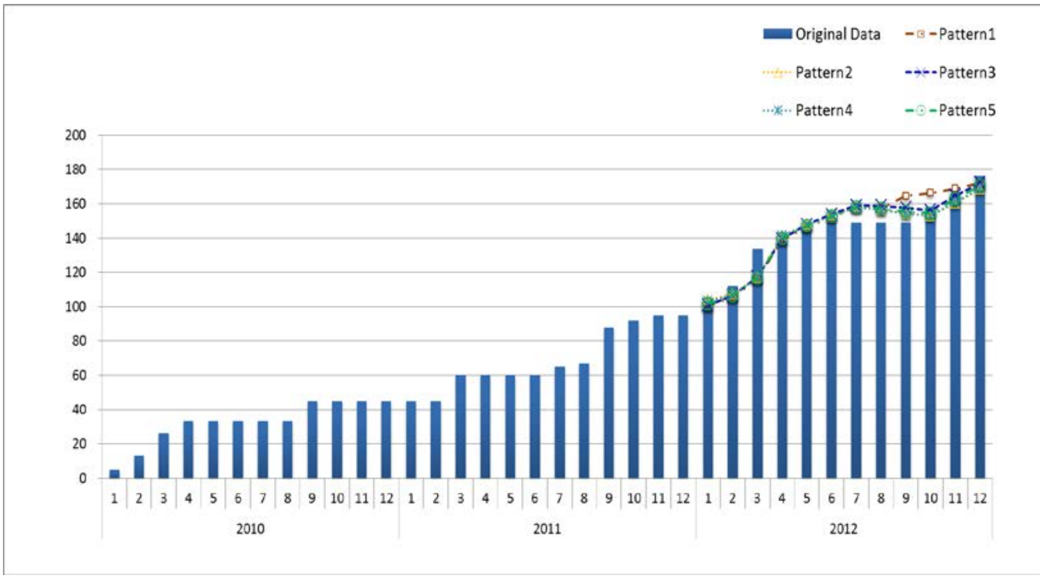

Figure 1: Forecasting from the Accumulated Data Case in Non-Monthly Trend Removal (1-(1))

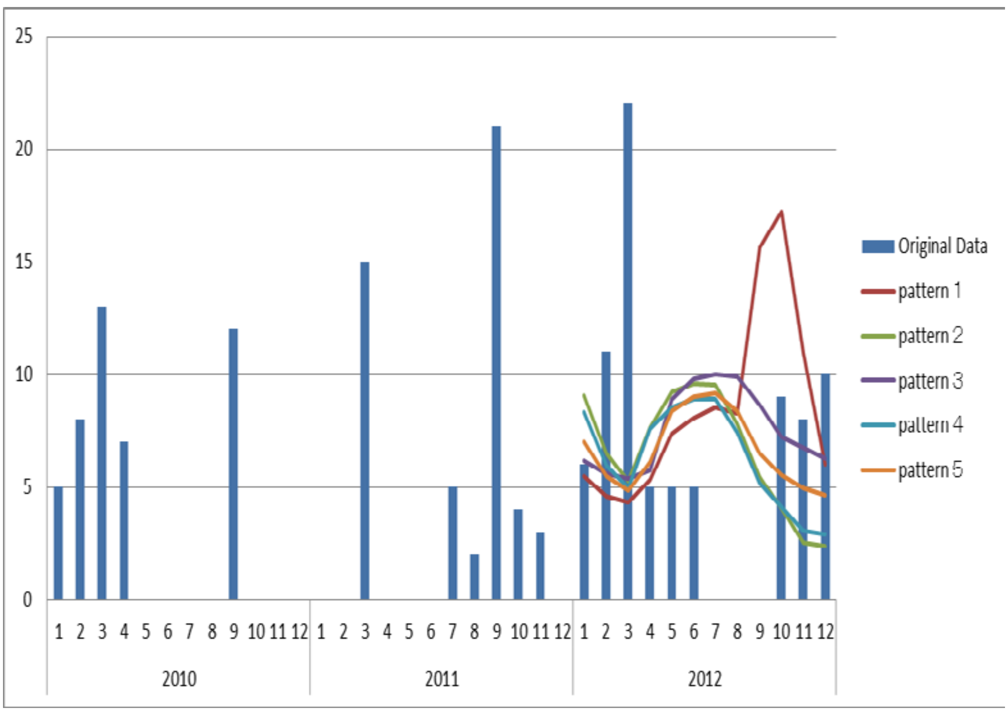

Figure 2: Forecasting from the Accumulated Data Case in Non-monthly Trend Removal (1-(2-1))

Table 4: Variance of Forecasting Error (1-(1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
792.4046691	634.1807914	717.4827357	642.8555513	681.0612363

Table 5: Variance of Forecasting Error (1-(2-1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
73.55953413	58.90308018	57.21871734	55.71935882	54.56690519

Table 6: Variance of Forecasting Error (1-(2-2))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
50.1848028	60.27175071	49.22124794	54.43312781	49.22124794

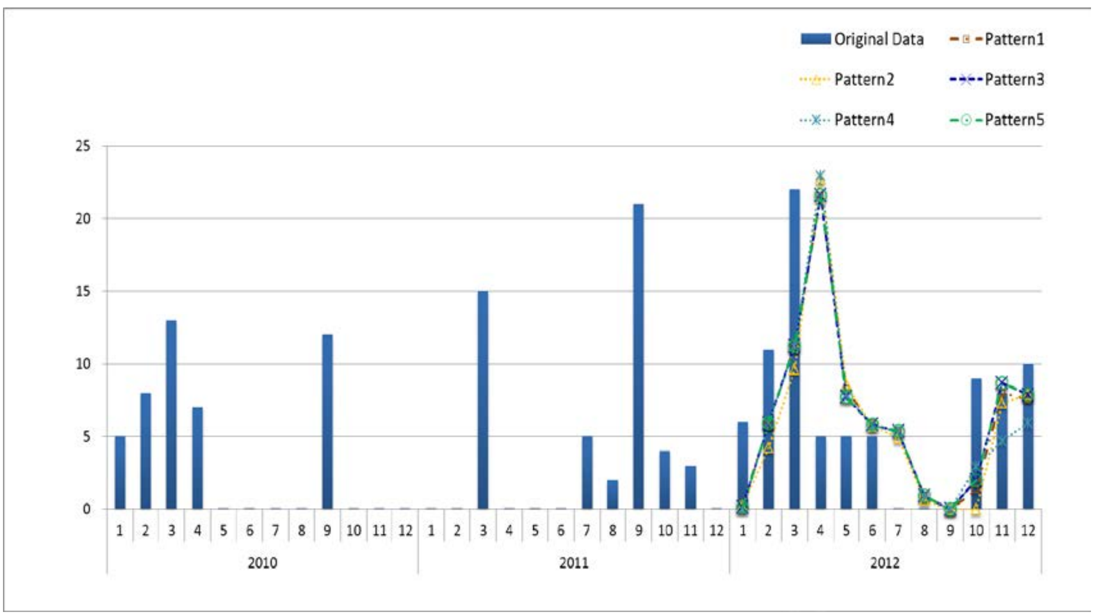

Figure 3: Forecasting from the Tiny Data close to 0 Added case in Non-Monthly Trend Removal (1-(2-2))

Next, we see the Monthly Trend Removal case.

Figure 4,5,6 show the Monthly Trend Removal Case in Emission CT Apparatus. It includes all cases classified above.

Figure 4 shows the Accumulated Data Case in Monthly Trend Removal.

Figure 5 shows the Forecasting from the Accumulated Data Case in Monthly Trend Removal.

Figure 6 shows the Forecasting from the tiny data close to 0 added case in Monthly Trend Removal.

Table 7,8 and 9 show the corresponding variance of forecasting error for each Figure 4,5 and 6.

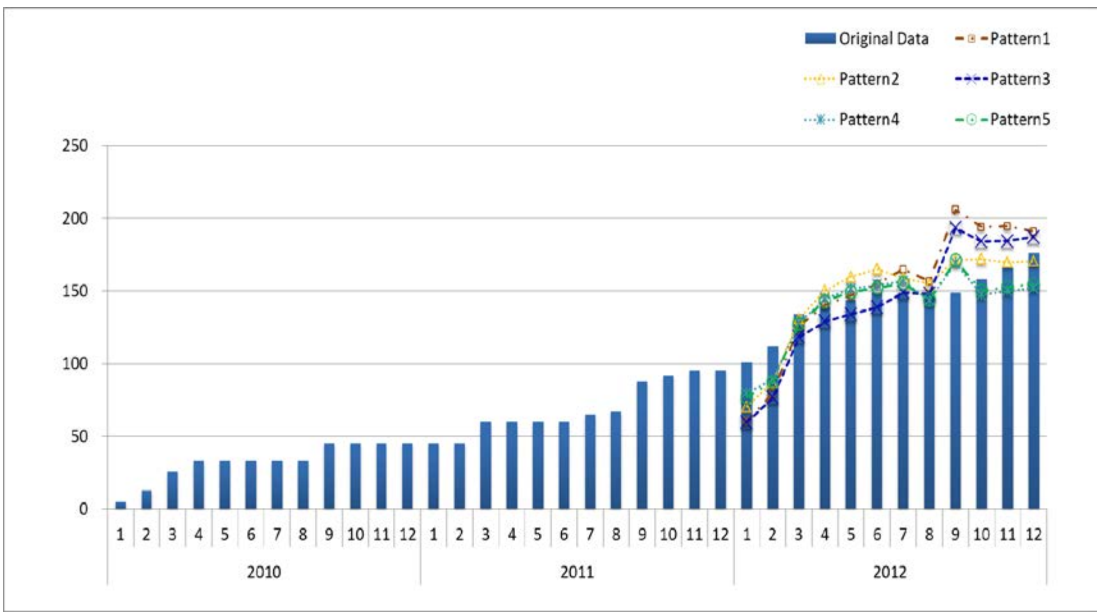

Figure 4: Accumulated Data case in Monthly Trend Removal (2-(1))

Table 7: Variance of Forecasting Error (2-(1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
2250.109446	1311.219764	1996.003907	897.3741051	958.4366137

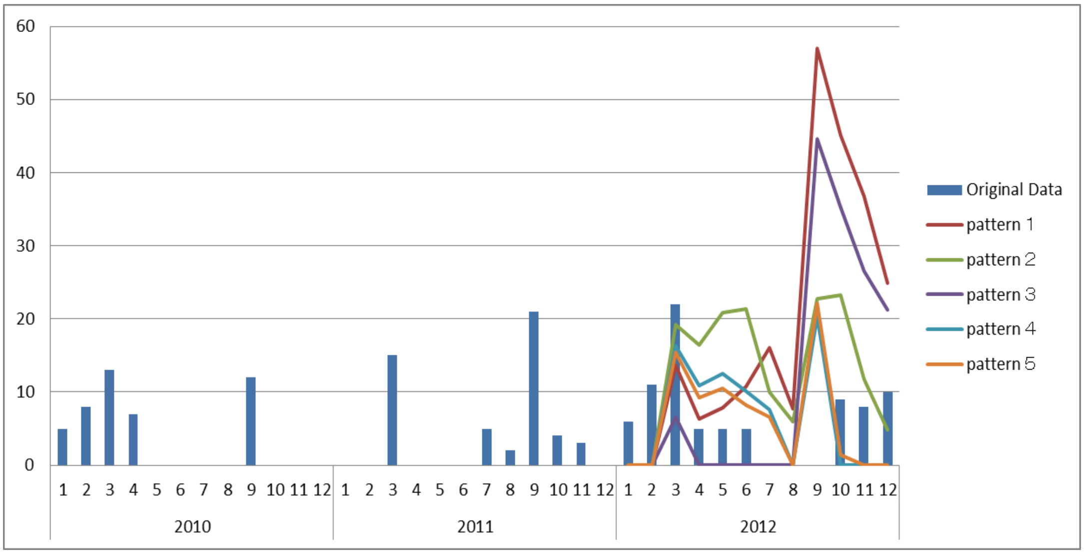

Figure 5: Forecasting from the accumulated Data case in Monthly Trend Removal (2-(2-1))

Table 8: Variance of Forecasting Error (2-(2-1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
403.281541	112.5876432	307.6240696	92.78690653	91.74218891

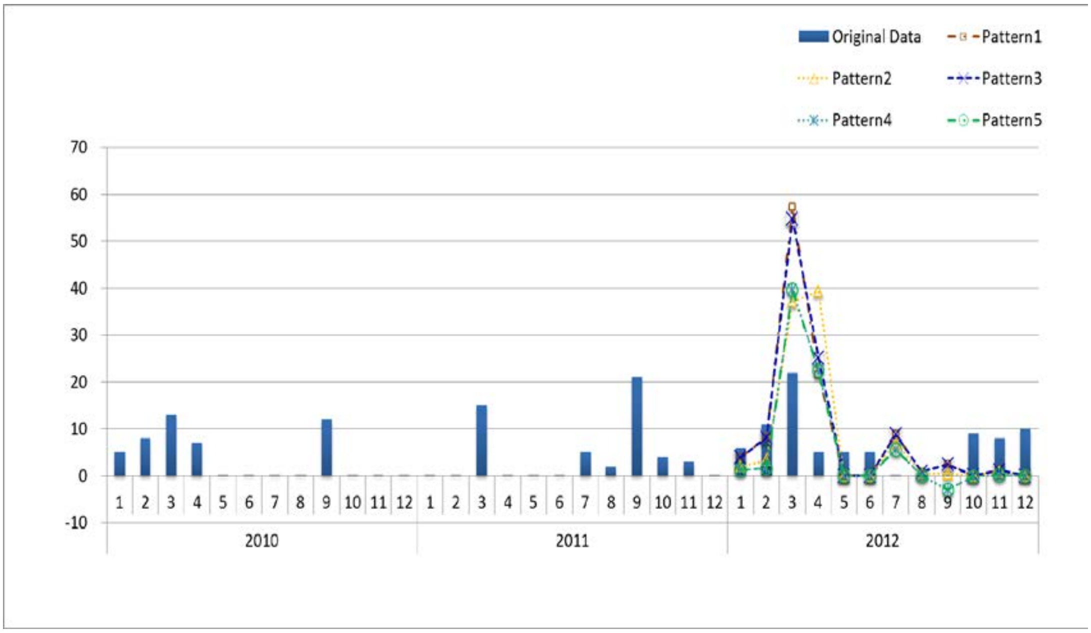

Figure 6: Forecasting from the Tiny Data close to 0 Added case in Monthly Trend Removal (2-(2-2))

Table 9: Variance of Forecasting Error (2-(2-2))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
167.9697542	164.7261374	163.6956109	95.11237194	95.11237194

Table 10 shows the summary for Emission CT by the Variance of forecasting error.

Table 10: Summary for Emission CT

		Monthly Trend Removal			Non Monthly Trend Removal		
Name	Emission CT	Accumulated Data	Forecasting Value — Accumulated Value	Tiny data close to 0 added case	Accumulated Data	Forecasting Value — Accumulated Value	Tiny data close to 0 added case
Minimum variance of Forecasting Error		897.3741051	91.74218891	95.11237194	634.1807914	54.56690519	49.22124794

Now, we proceed to the case of Superconducting magnetic resonance imaging system. Figure 7,8,9 show the Non-monthly Trend Removal Case in Superconducting magnetic resonance imaging system. It includes all cases classified above.

Figure 7 shows the Accumulated Data Case in Non-Monthly Trend Removal.

Figure 8 shows the Forecasting from the Accumulated Data Case in

Non-Monthly Trend Removal.

Figure 9 shows the Forecasting from the tiny data close to 0 added case in Non-Monthly Trend Removal.

Table 11,12 and 13 show the corresponding variance of forecasting error for each Figure 7,8 and 9.

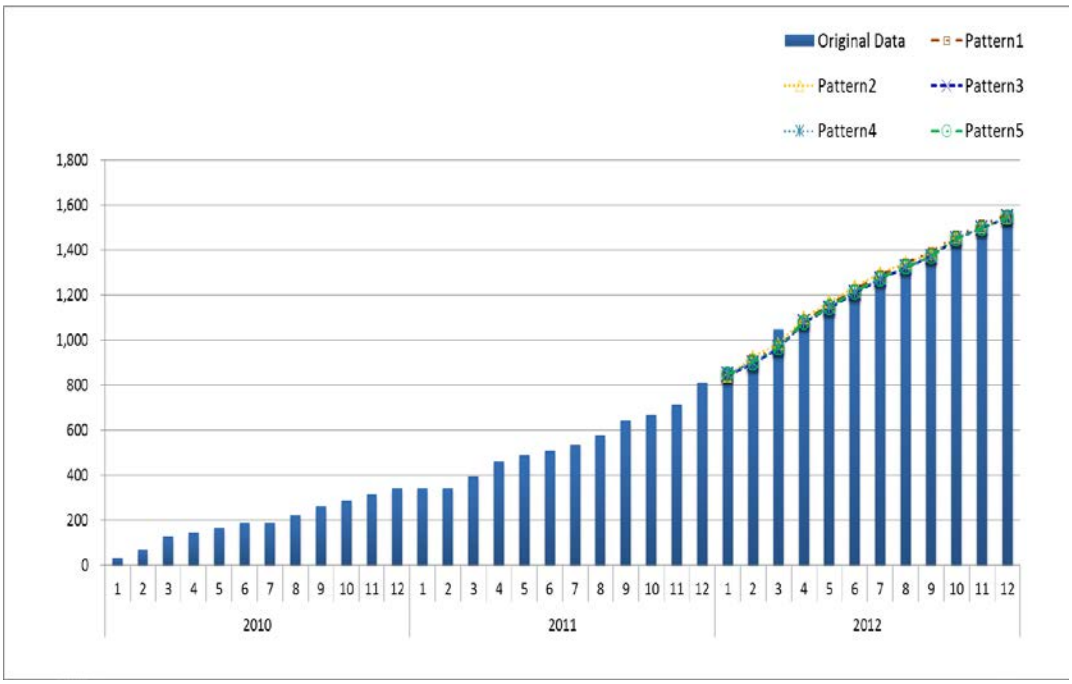

Figure 7: Accumulated Data case in Non-Monthly Trend Removal (1-(1))

Table 11: Variance of Forecasting Error (1-(1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
59410.71561	55835.78684	57675.7122	57038.31357	57038.31357

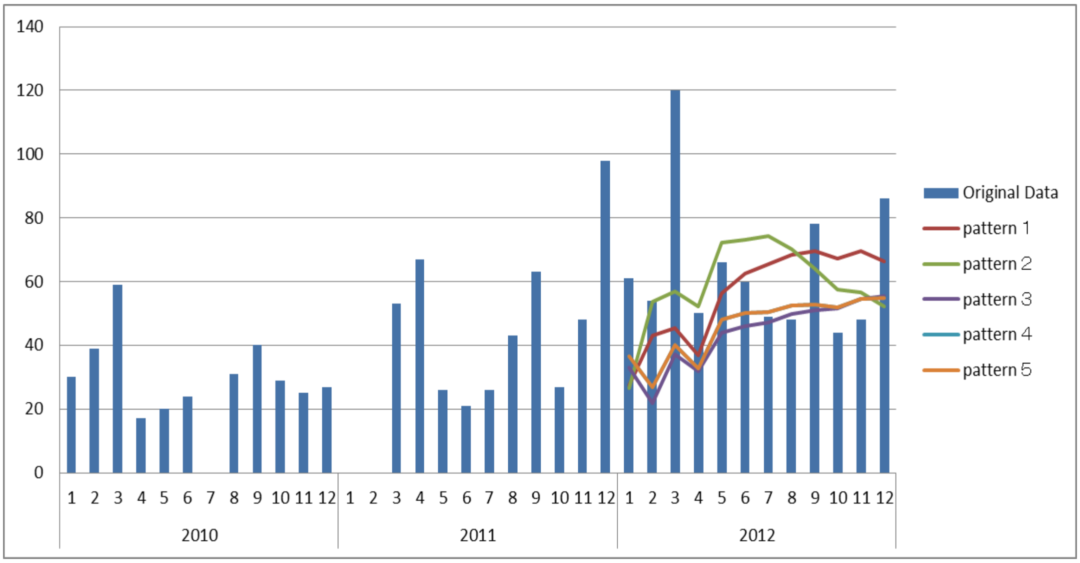

Figure 8: Forecasting from the accumulated Data case in Non-Monthly Trend Removal (1-(2-1))

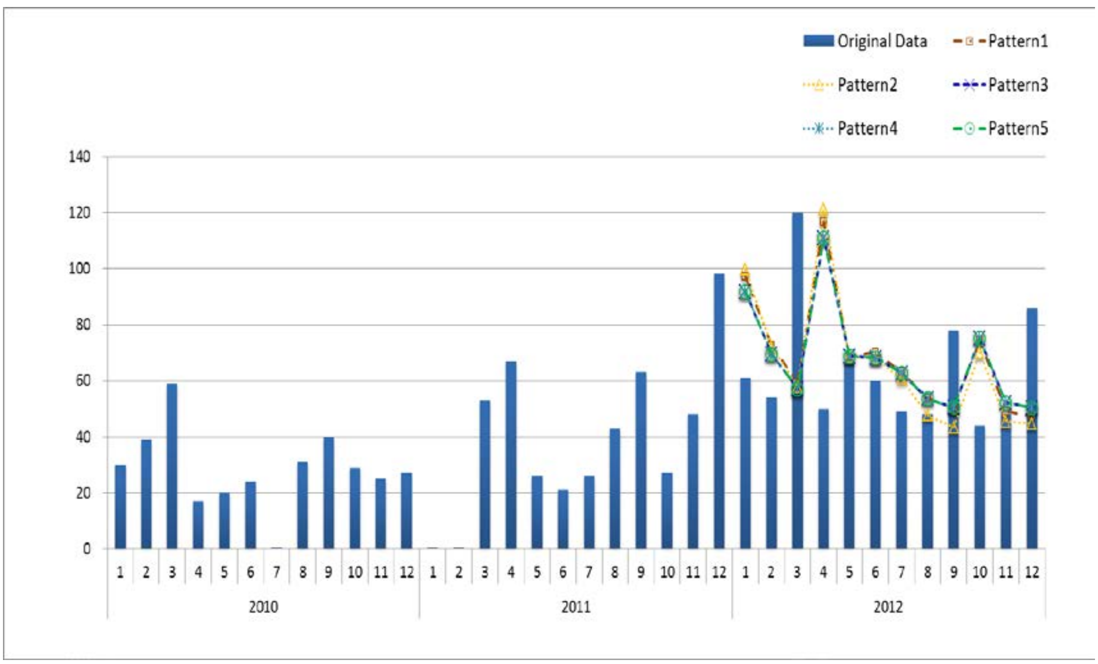

Figure 9: Forecasting from the Tiny Data close to 0 Added case in Non-Monthly Trend Removal (1-(2-2))

Table 12: Variance of Forecasting Error (1-(2-1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
784.796321	714.7271657	602.8689399	581.5812903	581.5812903

Table 13: Variance of Forecasting Error (1-(2-2))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
1186.363925	1315.05714	1088.423457	1088.423457	1088.423457

Next, we see the Monthly Trend Removal case.

Figure 10,11,12 show the Monthly Trend Removal Case in Superconducting magnetic resonance imaging system. It includes all cases classified above.

Figure 10 shows the Accumulated Data Case in Monthly Trend Removal.

Figure 11 shows the Forecasting from the Accumulated Data Case in Monthly Trend Removal.

Figure 12 shows the Forecasting from the tiny data close to 0 added case in Monthly Trend Removal.

Table 14,15 and 16 show the corresponding variance of forecasting error for each Figure 10,11 and 12.

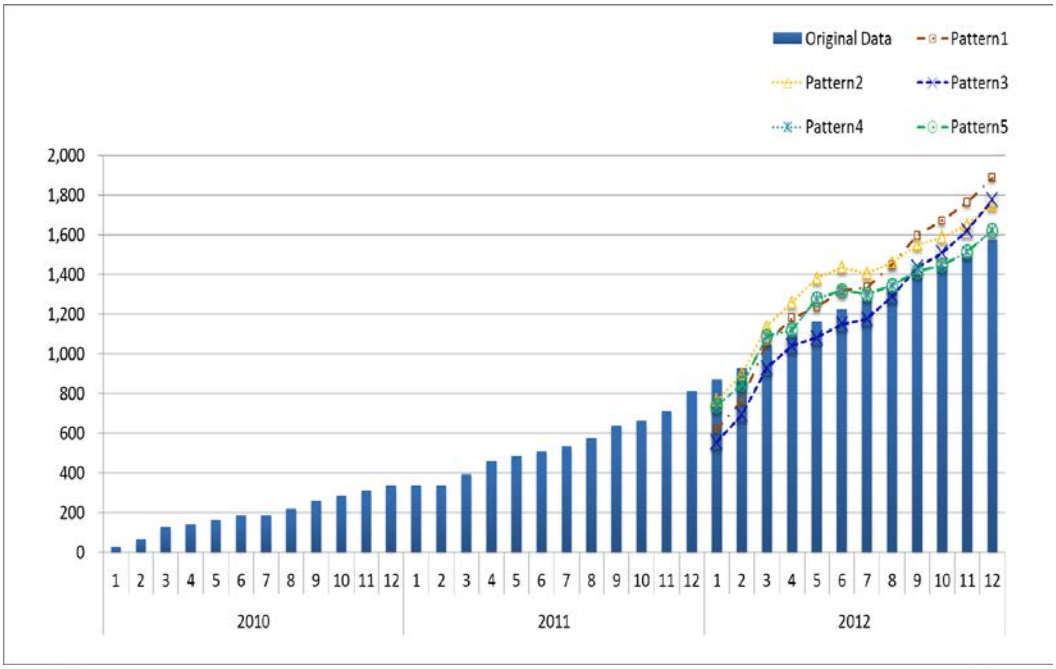

Figure 10: Accumulated Data case in Monthly Trend Removal (2-(1))

Table 14: Variance of Forecasting Error (2-(1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
149269.5167	89602.78443	133512.5389	69657.63343	69657.63343

Table 15: Variance of Forecasting Error (2-(2-1))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
14570.16293	7539.870327	8862.512621	2553.250314	2553.250314

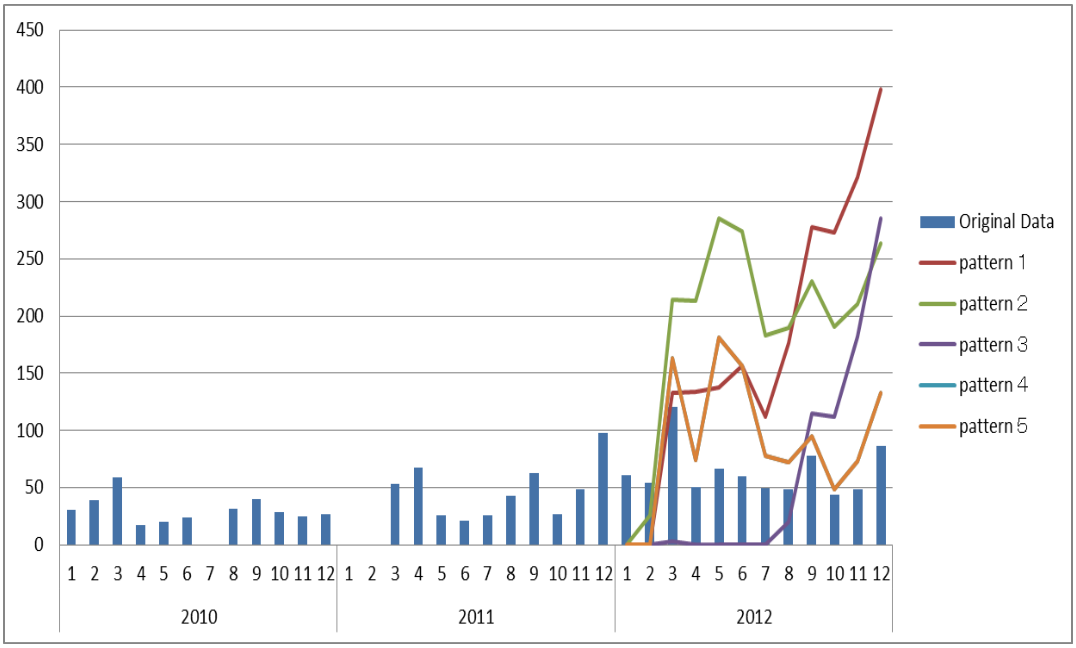

Figure 11: Forecasting from the accumulated Data case in Monthly Trend Removal (2-(2-1))

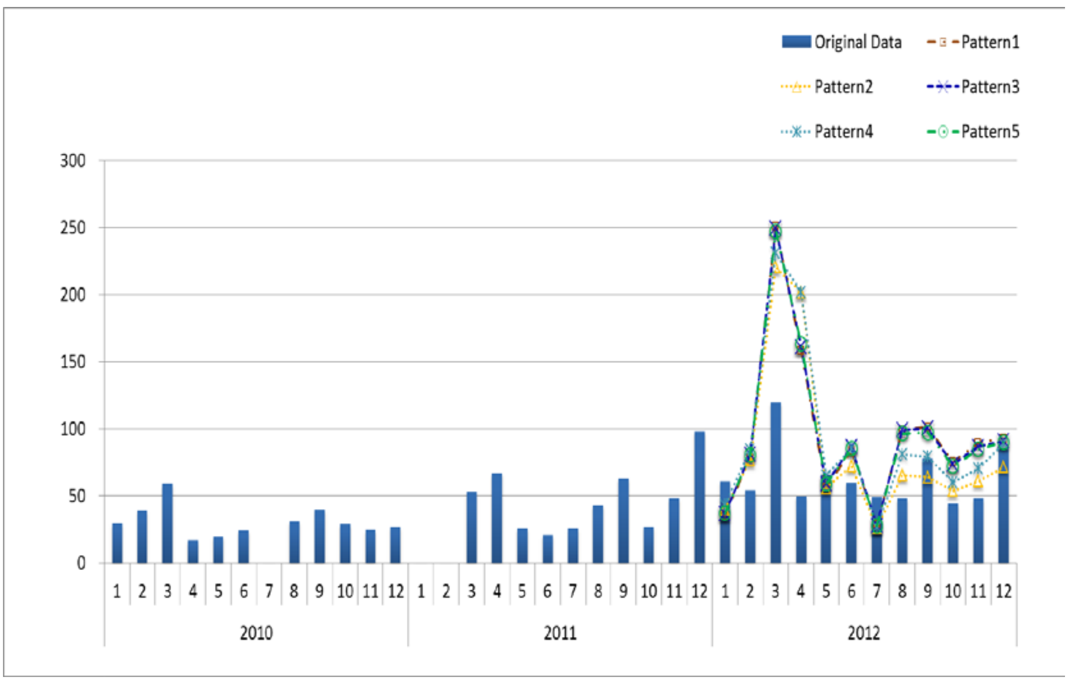

Figure 12: Forecasting from the Tiny Data close to 0 Added case in Monthly Trend Removal (2-(2-2))

Table 16: Variance of Forecasting Error (2-(2-2))

Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
2249.136971	2799.898976	2230.562172	2652.012791	2227.747506

Table 17 shows the summary for Superconducting magnetic resonance imaging system by the Variance of forecasting error.

Table 17: Summary for Superconducting magnetic resonance imaging system

		Monthly Trend Removal			Non Monthly Trend Removal		
Name	Superconducting magnetic resonance imaging system	Accumulated Data	Forecasting Value — Accumulated Value	Tiny data close to 0 added case	Accumulated Data	Forecasting Value — Accumulated Value	Tiny data close to 0 added case
Minimum variance of Forecasting Error		69657.63343	2553.250314	2227.747506	55835.78684	581.5812903	1088.423457

5.6 Remarks

In all cases, that non monthly trend removal case was better than monthly trend removal case. It may be because intermittent demand time series does not have regular cycle of demand. In the non-monthly trend removal for Emission CT Apparatus, 1-(2-2), i. e. , forecasting from the tiny data close to 0 added case was better than those of 1-(2-1), i. e. , forecasting from the accumulated data case. On the other hand, in the non-monthly trend removal for Superconducting magnetic resonance imaging system, 1-(2-1), i. e. , forecasting from the accumulated data case was better than those of 1-(2-2) , i. e. , forecasting from the tiny data close to 0 added case. Accumulated data case had a good result in forecasting accuracy.

6 Conclusion

The needs for intermittent demand forecasting are increasing. In this paper, a new method for cumulative forecasting method was proposed. The data was cumulated and to this cumulated time series, the following method was applied to improve the forecasting accuracy. Focusing that the equation of exponential smoothing method (ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way. Furthermore, combining the trend removing method with this method, we aimed to improve the forecasting accuracy. An approach to this method was executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function was executed to the production data of Emission CT apparatus and Superconducting magnetic resonance imaging system. The weights for these functions were set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights were searched. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting was executed on these data.

The forecasting result was compared with those of the non-cumulative forecasting method. The new method shows that it is useful for the forecasting of intermittent demand data. The effectiveness of this method should be examined in various cases.

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