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Procedure of Assessing the Electrical Transients with a View to Relative Extrema Localization

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Abstract

The assessment of transients in electrical systems requires numerical solving of systems of ordinary differential equations with the main purpose of localizing the relative extrema of state variables and of characteristic quantities in closely correlation with the selected state variables. Searching for relative extrema is traditionally performed immediately after the numerical integration is accomplished by sequentially processing a large file containing the amount of data recorded at all steps of numerical integration. Considering that, within the available software environments, the state variables time-related derivatives are usually discarded during numerical integration, finding approximations for the relative maxima and minima of each characteristic quantity could become a complex problem related to mathematical optimization. In this context, the present paper advances a simple procedure for real-time localizing of relative extrema of state variables and of the various characteristic quantities correlated with state

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variables by employing a high-order explicit linear multistep method. Computer experiments will be carried out by considering the more elaborate case of a synchronous generator subjected to sudden three-phase short circuit fault.

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1 Introduction

The accurate assessment of transient phenomena occurring in electrical systems requires developing of comprehensive state-space models, which are described by means of systems of ordinary differential equations [1-8]. Having in view that, in the majority of cases, the set of characteristic quantities, describing in an exhaustive manner an electrical transient, is not identical to the set of state-space variables, the computations can be complex, thus requiring the system analyst to operate with different software environments in order to solve a specific problem. Since there is no commercial software environment designed to depict the transient behaviour of each electrical system component, it remains the task of software engineers to develop dedicated environments with a view to in silico experimentation. Given that, in many cases, it is necessary to assess a large number of contingency cases, the computational capabilities of the developed software environment are of utmost significance.

The development of software environments for assessing transients in electrical systems has to be in accordance with the user requirements, which are customarily related to the computing rate and the visual representation of the evolution curves of characteristic quantities [9-14]. Within the available environments, a fourth- or a fifth-order integrator is commonly employed to carry out the numerical solving of the systems of differential equations, which describe the state-space models, whilst the visualization of the time-related evolution curves is practically the result of accessing a large file that stores the data received at all steps of numerical integration. It has to be emphasized that in the case in which the characteristic quantities, necessary to depict the system transient behaviour, entirely differ from the selected state-space variables, the user intervention could result in loss of performance due to the large volume of computations requested both during the numerical integration and immediately after the numerical integration process is completed.

To assess the transient phenomena in electrical systems, the analyst is especially interested in the relative extrema of certain characteristic quantities (currents, voltages) [15]. Hence, the system analyst requests access to the large amount of data recorded during numerical integration with the purpose of searching for relative maxima and minima of each characteristic quantity. Having this in view, the present investigation suggests a simple and effective procedure for localizing the relative extrema of state variables as well as of characteristic quantities that are closely correlated with the state variables. The procedure benefits from an eight-order explicit method, which is straightforwardly designed in the present paper starting from the generalized Adams-Bashforth predictor formula. It is common knowledge that unlike Runge-Kutta methods, at any given step of integration, the explicit multistep methods do not call on intermediate points in order to update the state variables. Instead of taking auxiliary points to increase accuracy, the explicit multistep methods use the values of state variables derivatives received at some previous steps during numerical integration. Hence, having in view that the information required for a mutistep method to make a step forward and update the state variables is precisely the one needed also for localizing the relative extrema, it follows that the employment of an explicit multistep method is entirely justified here. Furthermore, the computational capabilities can be improved both by increasing the size of the integration time step, which is allowable because of the high-order of the integrator, and by recording only the significant data received during numerical integration. Considering the more elaborate case of a salient-pole synchronous generator, it will be shown that the proposed procedure allows the localization of relative extrema of the characteristic quantities just in the course of integration, without further processing.

2 Design of the Numerical Integration Method

The relatively large number of state variables needed to comprehensively depict the behaviour of an electrical system during transients as well as the occurrence of time intervals over which the selected state variables have very fast variations require the use of a highly accurate numerical integration method for solving the systems of ordinary differential equations that describe the various state-space models. Valuable numerical results can be obtained by employing high-order multistep methods, which are recognized for the effectiveness acquired through the use of the values of state variables derivatives received at some previous steps of integration in the place of adding intermediate points at which the state variables derivatives are to be computed [16-19]. In the present paper, with the purpose of in silico experimentation, an eight-order linear multistep method is applied, with the start-up being achieved by employing the original fourth-order Runge-Kutta method to execute the first seven steps of integration. In what follows right away, we present the development of the multistep method based on the generalized Adams-Bashforth explicit formula. We consider the initial value problem:

$$\frac{dy}{dt} = g(t, y); \quad y(t_0) = y_0 \,. \tag{1}$$

Suppose that we have determined approximations $\{ y_s, s \in [1, n] \}$ for the solution of (1) at instants $\{ t_s = t_0 + s \cdot h, s \in [1, n] \}$. Next, we have to find an approximation for $y(t_{n+1})$ where $t_{n+1} = t_n + h$. We will employ the symbol g_s to designate the expression $g(t_s, y_s)$. We consider $p(t)$ as being the interpolation polynomial in Lagrange form associated with data points (t_s, g_s) where $s \in [n-7, n]$. We have:

$$p(t) = \sum_{s=n-7}^{n} \left(\prod_{\substack{l \in [n-7, n] \\ l \neq s}} \frac{t - t_l}{t_s - t_l} \right) \cdot g_s \tag{2}$$

and the Adams-Bashforth formula [18, 19]:

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} p(t)\, dt . \tag{3}$$

Relation (3) enables the computation of y_{n+1} that is the approximation of $y(t_{n+1})$, having at hand the values of function g at data points (t_s, y_s) with index $s \in [n-7, n]$. Based on relationships (2) and (3), we receive the expanded formula:

$$y_{n+1} = y_n + \sum_{s=n-7}^{n} \left[\int_{t_n}^{t_{n+1}} \left(\prod_{\substack{l \in [n-7, n] \\ l \neq s}} \frac{t - t_l}{t_s - t_l} \right) dt \right] \cdot g_s$$

equivalent to

$$\begin{gathered} y_{n+1} = y_n + h \sum_{s=n-7}^{n} B_s g_s \, , \\ y_s \approx y(t_s); \quad g_s = g(t_s, y_s); \quad h = t_{n+1} - t_n \end{gathered} \tag{4}$$

where we have to identify the coefficients:

$$B_s = \frac{1}{h} \int_{t_n}^{t_{n+1}} \left(\prod_{\substack{l \in [n-7,\, n] \\ l \neq s}} \frac{t - t_l}{t_s - t_l} \right) dt$$

for $s \in [n-7,\, n]$. For this purpose, we perform the following change of variable:

$$\tau = \frac{t - t_n}{h}\,, \tag{5}$$

which leads to

$$t = t_n + h\tau\,, \quad dt = h\,d\tau\,, \quad t - t_l = h \cdot (\tau + n - l)\,, \quad t_s - t_l = h \cdot (s - l)$$

and, eventually

$$B_s = \int_0^1 \left(\prod_{\substack{l \in [n-7,\, n] \\ l \neq s}} \frac{\tau + n - l}{s - l} \right) d\tau; \quad s \in [n-7,\, n].$$

Hence, by means of the change of variable defined by (5), the coefficients in the explicit formula (4) obtain a more convenient form:

$$B_s = \frac{(-1)^{n-s}}{[(s-n+7)!][(n-s)!]} \int_0^1 \frac{\tau \ldots (\tau + 7)}{\tau + n - s} \mathrm{d}\tau\,; \quad s \in [n-7,\, n].$$

Performing the calculations, one receives:

$$B_{n-7} = -36799/120960\,; \quad B_{n-6} = 295767/120960\,;$$

$$B_{n-5} = -1041723/120960\,; \quad B_{n-4} = 2102243/120960\,;$$

$$B_{n-3} = -2664477/120960\,; \quad B_{n-2} = 2183877/120960\,;$$

$$B_{n-1} = -1152169/120960\,; \quad B_n = 434241/120960\,. \tag{6}$$

Since the eight coefficients (6) have constant values, they can be employed within (4) to carry out the integration for any initial value problem, regardless the size of time step h.

Having in view (1), formula (4) emphasizes the state variable derivatives:

$$y_{n+1} = y_n + h \sum_{s=n-7}^{n} B_s \left(\frac{dy}{dt} \right)_s ; \quad \left(\frac{dy}{dt} \right)_s \approx \frac{dy}{dt}(t_s), \; s \in [n-7, n].$$

Thus, at each step of numerical integration, in order to compute the state variable value at the current point, the integrator has to evaluate and store the state variable time-related derivative corresponding to the previous point, taking into account that the other seven state variable derivatives are already stored as elements within a reserved vector. In other words, to compute y_{n+1}, the integrator evaluates:

$$\left(\frac{dy}{dt} \right)_n = g(t_n, y_n) \approx \frac{dy}{dt}(t_n). \tag{7}$$

It has to be emphasized that computing (7) at each step of numerical integration appears here to be of crucial interest in the process of localizing the relative extrema of state variable and of any other characteristic quantity in correlation with the state variable y.

3 The System under Consideration

We consider the more complex case of a three-phase salient-pole synchronous generator endowed with damping cage. As it is advanced in literature, the generalized *d*-*q* (orthogonal) axis mathematical model of synchronous generators is described by means of two distinctive sets of structural equations. These are the voltage equations, put forward as a set of ordinary differential equations, and the flux equations, depicted by a set of algebraic correlations between winding flux linkages and winding currents [20-22].

To facilitate the employment of mathematical concepts and language as well as to allow the generalization of conclusions, the synchronous generators representation is performed by adopting a per unit (p.u.) dimensionless system [21, 22]. The following base quantities will be considered:

- for voltages, denoted by symbol u, we have: $u_{base} = \sqrt{2} U_{rated}$;

- for currents, denoted by symbol i, we have: $i_{base} = \sqrt{2}\, I_{rated}$;

- for flux linkages, denoted by symbol ψ, we have: $\psi_{base} = \sqrt{2} U_{rated} / \omega_{rated}$;

- for resistances, denoted by symbol R, we have: $R_{base} = Z_{rated} = U_{rated} / I_{rated}$;

- for inductances, denoted by symbol L, we have: $L_{base} = Z_{rated} / \omega_{rated}$;

- for angular velocity, denoted by symbol ω, we have: $\omega_{base} = \omega_{rated}$;

- for time variable we have: $t_{base} = 1 / \omega_{rated}$.

The voltage equations of synchronous generators traditionally provide the time-related derivatives of winding flux linkages in terms of all winding currents and stator *d*-*q* axis flux linkages:

$$\frac{d\psi_d}{dt} = \omega\, \psi_q - R i_d - u_d \,, \tag{8}$$

$$\frac{d\psi_q}{dt} = -\omega\, \psi_d - R i_q - u_q \,, \tag{9}$$

$$\frac{d\psi_f}{dt} = -R_f\, i_f + u_f \,, \tag{10}$$

$$\frac{d\psi_D}{dt} = -R_D i_D \,, \tag{11}$$

$$\frac{d\psi_Q}{dt} = -R_Q i_Q \,. \tag{12}$$

The flux equations of synchronous generators are given by means of the following algebraic correlations:

$$\psi_d = L_\sigma i_d + L_{md} \cdot \left(i_d + i_f + i_D\right) = L_d\, i_d + L_{md} \cdot \left(i_f + i_D\right); \quad L_d = L_\sigma + L_{md} \,, \tag{13}$$

$$\psi_q = L_\sigma i_q + L_{mq} \cdot \left(i_q + i_Q\right) = L_q i_q + L_{mq} i_Q \,; \quad L_q = L_\sigma + L_{mq} \,, \tag{14}$$

$$\begin{aligned}\psi_f &= L_{f\sigma} i_f + L_{md}\cdot\left(i_d + i_f + i_D\right) = L_f i_f + L_{md}\cdot\left(i_d + i_D\right)\,; \\ L_f &= L_{f\sigma} + L_{md}\,,\end{aligned} \tag{15}$$

$$\begin{aligned}\psi_D &= L_{D\sigma} i_D + L_{md}\cdot\left(i_d + i_f + i_D\right) = L_D i_D + L_{md}\cdot\left(i_d + i_f\right)\,; \\ L_D &= L_{D\sigma} + L_{md}\,,\end{aligned} \tag{16}$$

$$\begin{aligned}\psi_Q &= L_{Q\sigma} i_Q + L_{mq}\cdot\left(i_q + i_Q\right) = L_Q i_Q + L_{mq} i_q\,; \\ L_Q &= L_{Q\sigma} + L_{mq}\,.\end{aligned} \tag{17}$$

Within (8)-(17), the subscripts are as follows: d - designates the “direct” axis components; q - designates the “quadrature” axis components; f - associated with field winding; D - associated with “direct” axis damper circuit; Q - associated with “quadrature” axis damper circuit; m - associated with magnetizing circuit; σ - denotes leakage inductances.

The role of flux equations (13)-(17) is to enable the selection of state variables (currents and/or flux linkages) in a manner suitable for the intended purpose. Since the values of stator *d-q* axis winding currents are of significant interest and, besides, taking into account that all parameters are considered here to be constant, in order to get the most from the numerical integration routine, we proceed to select all winding currents as state variables. Thus, the vector of state variables is

$$\mathbf{I} = \begin{bmatrix} i_d & i_q & i_f & i_D & i_Q \end{bmatrix}^T .$$

Having in view (13)-(17), we have now to process the set of voltage equations (8)-(12) in order to receive a state-space model, prepared for software implementation. Employing correlations (13)-(17) to replace the flux linkage variables within (8)-(12), one obtains the following differential-algebraic structure:

$$L_d \frac{di_d}{dt} + L_{md}\frac{di_f}{dt} + L_{md}\frac{di_D}{dt} = -R i_d + \omega\,\psi_q\left(i_q\,,\, i_Q\right) - u_d\,, \tag{18}$$

$$L_q \frac{di_q}{dt} + L_{mq}\frac{di_Q}{dt} = -\omega\,\psi_d\left(i_d\,,\; i_f\,,\; i_D\right) - R i_q - u_q\,, \tag{19}$$

$$L_{md}\frac{di_d}{dt}+L_f\frac{di_f}{dt}+L_{md}\frac{di_D}{dt}=-R_f i_f+u_f\,, \tag{20}$$

$$L_{md}\frac{di_d}{dt}+L_{md}\frac{di_f}{dt}+L_D\frac{di_D}{dt}=-R_D i_D\,, \tag{21}$$

$$L_{mq}\frac{di_q}{dt}+L_Q\frac{di_Q}{dt}=-R_Q i_Q\,. \tag{22}$$

With a view to transient assessment, structure (18)-(22) is routinely implemented without change within various software environments. In this way, the environments are forced to call an elimination procedure at the beginning of each step of numerical integration in order to provide the values of state currents time-related derivatives, which are requested by the integrator to update the state currents values. To increase the computational efficiency, we will symbolically manipulate structure (18)-(22) with the purpose of explicitly expressing the state currents derivatives in the most convenient form. Thus, the next step in the course of derivation here is represented by the processing of system (18)-(22) in order to receive an explicit (normal) form. We observe that equations (18), (20), (21) can be coupled to identify the set of expressions of the *d*-axis winding currents derivatives i.e.

$$DER_d=\left\{\frac{di_d}{dt},\quad \frac{di_f}{dt},\quad \frac{di_D}{dt}\right\}$$

whilst equations (19), (22) yield the set of expressions of the time-related derivatives of *q*-axis winding currents i.e.

$$DER_q=\left\{\frac{di_q}{dt},\quad \frac{di_Q}{dt}\right\}.$$

Accordingly, solving the set of equations (18), (20), (21) in relation to winding currents time-related derivatives, we receive:

$$\frac{dy_k}{dt}=c_{d1,k}i_d+c_{d2,k}i_q+c_{d3,k}i_f+c_{d4,k}i_D+c_{d5,k}i_Q+\alpha_{d1,k}u_d+\alpha_{d2,k}u_f\,; \qquad k\in\{1,2,3\};\quad y_1\equiv i_d\,,\, y_2\equiv i_f\,,\, y_3\equiv i_D\,. \tag{23}$$

The coefficients of currents and voltages interfering in (23) are the following:

$$c_{d1,k} = (-1)^k R\,\delta_{1k} \big/ \Delta_d \;, \quad c_{d2,k} = (-1)^{k+1} \omega L_q\, \delta_{1k} \big/ \Delta_d \;,$$

$$c_{d3,k} = (-1)^{k+1} R_f\, \delta_{2k} \big/ \Delta_d \;, \quad c_{d4,k} = (-1)^k R_D\, \delta_{3k} \big/ \Delta_d \;,$$

$$c_{d5,k} = (-1)^{k+1} \omega L_{mq}\, \delta_{1k} \big/ \Delta_d \;,$$

$$\alpha_{d1,k} = (-1)^k \delta_{1k} \big/ \Delta_d \;, \quad \alpha_{d2,k} = (-1)^k \delta_{2k} \big/ \Delta_d$$

wherein

$$\Delta_d = L_\sigma L_{f\sigma} L_{D\sigma} + \left(L_\sigma L_{f\sigma} + L_\sigma L_{D\sigma} + L_{f\sigma} L_{D\sigma} \right) L_{md} \;,$$

$$\delta_{11} = L_{f\sigma} L_{D\sigma} + \left(L_{f\sigma} + L_{D\sigma} \right) L_{md} \,,\; \delta_{12} = L_{D\sigma} L_{md} \,,\; \delta_{13} = -L_{f\sigma} L_{md} \,,$$

$$\delta_{21} \equiv \delta_{12} \,,\; \delta_{22} = L_\sigma L_{D\sigma} + \left(L_\sigma + L_{D\sigma} \right) L_{md} \,,\; \delta_{23} = L_\sigma L_{md} \,,$$

$$\delta_{31} \equiv \delta_{13} \,,\; \delta_{32} \equiv \delta_{23} \,,\; \delta_{33} = L_\sigma L_{f\sigma} + \left(L_\sigma + L_{f\sigma} \right) L_{md} \;.$$

With a view to optimal software implementation, the coefficients of state currents in (23) should be computed in the following manner:

$$aux \leftarrow R \,/\, \Delta_d \,;\; c_{d1,1} \leftarrow -\,\delta_{11} \cdot aux;\; c_{d1,2} \leftarrow \delta_{12} \cdot aux;\; c_{d1,3} \leftarrow -\,\delta_{13} \cdot aux;$$

$$aux \leftarrow \omega L_q \big/ \Delta_d \,;\; c_{d2,1} \leftarrow \delta_{11} \cdot aux;\; c_{d2,2} \leftarrow -\,\delta_{12} \cdot aux;\; c_{d2,3} \leftarrow \delta_{13} \cdot aux;$$

$$aux \leftarrow R_f \big/ \Delta_d \,;\; c_{d3,1} \leftarrow \delta_{21} \cdot aux;\; c_{d3,2} \leftarrow -\,\delta_{22} \cdot aux;\; c_{d3,3} \leftarrow \delta_{23} \cdot aux;$$

$$aux \leftarrow R_D \big/ \Delta_d \,;\; c_{d4,1} \leftarrow -\,\delta_{31} \cdot aux;\; c_{d4,2} \leftarrow \delta_{32} \cdot aux;\; c_{d4,3} \leftarrow -\,\delta_{33} \cdot aux;$$

$$aux \leftarrow \omega L_{mq} \big/ \Delta_d \,;\; c_{d5,1} \leftarrow \delta_{11} \cdot aux;\; c_{d5,2} \leftarrow -\,\delta_{12} \cdot aux;\; c_{d5,3} \leftarrow \delta_{13} \cdot aux \,.$$

Solving now the set of equations (19), (22) in relation to *q*-axis winding currents derivatives, one obtains:

$$\frac{di_q}{dt} = c_{q1,1} i_d + c_{q2,1} i_q + c_{q3,1} i_f + c_{q4,1} i_D + c_{q5,1} i_Q + \alpha_{q1} u_q \,, \tag{24}$$

$$\frac{di_Q}{dt} = c_{q1,2} i_d + c_{q2,2} i_q + c_{q3,2} i_f + c_{q4,2} i_D + c_{q5,2} i_Q + \alpha_{q2} u_q \,, \tag{25}$$

wherein the coefficients are:

$$c_{q1,1} = -\omega L_d L_Q \big/ \Delta_q \,,\; c_{q1,2} = \omega L_d L_{mq} \big/ \Delta_q \,,$$

$$c_{q2,1} = -R\,L_Q \big/ \Delta_q\,,\; c_{q2,2} = R\,L_{mq} \big/ \Delta_q\,,$$

$$c_{q3,1} = -\omega\,L_{md}\,L_Q \big/ \Delta_q\,,\; c_{q3,2} = \omega\,L_{md}\,L_{mq} \big/ \Delta_q\,,\; c_{q4,1} \equiv c_{q3,1}\,,\; c_{q4,2} \equiv c_{q3,2}\,,$$

$$c_{q5,1} = R_Q\,L_{mq} \big/ \Delta_q\,,\; c_{q5,2} = -R_Q\,L_q \big/ \Delta_q\,,$$

$$\alpha_{q1} = -L_Q \big/ \Delta_q\,,\; \alpha_{q2} = L_{mq} \big/ \Delta_q\,,$$

with

$$\Delta_q = L_\sigma\,L_{Q\sigma} + \left(L_\sigma + L_{Q\sigma}\right) L_{mq}\,.$$

Equations (23), (24) and (25) describe a state-space model in explicit (normal) form:

$$\frac{d}{dt}\mathbf{I} = \mathbf{C} \times \mathbf{I} + \mathbf{D} \times \mathbf{U} \tag{26}$$

where

$$\mathbf{I} = \begin{bmatrix} i_d & i_q & i_f & i_D & i_Q \end{bmatrix}^T ,$$

$$\mathbf{C} = \begin{bmatrix} c_{d1,1} & c_{q1,1} & c_{d1,2} & c_{d1,3} & c_{q1,2} \\ c_{d2,1} & c_{q2,1} & c_{d2,2} & c_{d2,3} & c_{q2,2} \\ c_{d3,1} & c_{q3,1} & c_{d3,2} & c_{d3,3} & c_{q3,2} \\ c_{d4,1} & c_{q4,1} & c_{d4,2} & c_{d4,3} & c_{q4,2} \\ c_{d5,1} & c_{q5,1} & c_{d5,2} & c_{d5,3} & c_{q5,2} \end{bmatrix}^T ,$$

$$\mathbf{D} = \begin{bmatrix} \alpha_{d1,1} & 0 & \alpha_{d1,2} & \alpha_{d1,3} & 0 \\ 0 & \alpha_{q1} & 0 & 0 & \alpha_{q2} \\ \alpha_{d2,1} & 0 & \alpha_{d2,2} & \alpha_{d2,3} & 0 \end{bmatrix}^T$$

represent the state currents vector and the matrices of coefficients whilst

$$\mathbf{U} = \begin{bmatrix} u_d & u_q & u_f \end{bmatrix}^T$$

is the voltages vector, selected here as input.

In the absence of the zero sequence component, a certain stator phase current results from the well-known converse Park-Gorev transform in terms of stator *d-q* axis winding currents [20, 21]:

$$i_{ph} = i_d \cos\left(\gamma_0 - \omega t\right) + i_q \sin\left(\gamma_0 - \omega t\right)$$

or, at synchronous angular velocity $(\omega = 1\,\text{p.u.})$

$$i_{ph} = i_d \cos(t-\gamma_0) - i_q \sin(t-\gamma_0) \tag{27}$$

wherein γ_0 is the initial value of the rotor lag angle.

From (27) we obtain the time-related derivative of stator phase current in terms of stator *d*-*q* axis winding currents, selected here as state variables, and their time-related derivatives:

$$\frac{d i_{ph}}{dt} = \left(\frac{d i_d}{dt} - i_q\right)\cos(t-\gamma_0) - \left(\frac{d i_q}{dt} + i_d\right)\sin(t-\gamma_0). \tag{28}$$

Extending formula (4) to solve system (26), we get the vector of state variables at step $(n+1)$ of numerical integration based on the values of state and input variables at the previous eight steps:

$$\mathbf{I}_{n+1} = \mathbf{I}_n + h\sum_{s=n-7}^{n} B_s\left(\mathbf{C}\times\mathbf{I}_s + \mathbf{D}\times\mathbf{U}_s\right) \tag{29}$$

where:

$$\begin{cases} \mathbf{I}_s = \left[i_{d,s} \quad i_{q,s} \quad i_{f,s} \quad i_{D,s} \quad i_{Q,s} \right]^T; \quad s \in [n-7, n], \\ i_{d,s} \approx i_d(t_s),\ i_{q,s} \approx i_q(t_s),\ i_{f,s} \approx i_f(t_s),\ i_{D,s} \approx i_D(t_s),\ i_{Q,s} \approx i_Q(t_s); \end{cases}$$

$$\mathbf{U}_s = \left[u_d(t_s) \quad u_q(t_s) \quad u_f(t_s) \right]^T; \quad s \in [n-7, n].$$

More specifically, in order to update the values of state currents at step $(n+1)$ of integration, it is necessary to evaluate the expressions of state currents time-related derivatives (26) at step n of integration, having in view that, at the other previous seven steps, the values of state currents derivatives are already stored as array elements.

4 Computer Experiments

The transient selected in order to emphasize the benefits of the suggested assessment procedure is represented by the sudden three-phase short-circuit fault at the terminals of a salient-pole synchronous generator with the following per unit (dimensionless) parameters [22]:

$$R = 0.0256\,,\quad R_f = 0.0032\,,\quad R_D = 0.088\,,\quad R_Q = 0.036\,,$$

$$L_\sigma = 0.088\,,\quad L_{f\sigma} = 0.258\,,\quad L_{D\sigma} = 0.33\,,\quad L_{Q\sigma} = 0.066\,,$$

$$L_{md} = 1.31\,,\quad L_{mq} = 0.705\,.$$

The selection of this transient for performing numerical experiments is justified by the fact that dynamic simulation has been extensively employed as the benchmark approach in order to evaluate the accuracy of the results received from various standardized semi-rigorous procedures designed for short circuit fault analysis in electrical systems [23-28]. In silico experimentation has been carried out for the widely accepted initial circumstance of generator no-load operation, with rated phase voltage. We have also assumed that the generator operates at synchronous velocity.

The outlined assumptions correspond to the following initial condition (in per unit):

$$i_{d,0} = 0\,,\quad i_{q,0} = 0\,,\quad i_{f,0} = 1/L_{md}\,,\quad i_{D,0} = 0\,,\quad i_{Q,0} = 0\,. \tag{30}$$

Besides, the restrictive conditions characteristic of three-phase short circuit are:

$$u_d(t) \equiv u_q(t) \equiv 0\,. \tag{31}$$

Hence, state-space model (26), wherein we have (31) and $u_f = R_f\, i_{f,0} = \text{const.}$, together with (30) provide here the initial value problem.

Figure 1 and Figure 2 indicate the evolution curves of stator *d-q* axis currents, along with their time-related derivatives, recorded in real-time i.e. during numerical integration. The evolution of stator phase current (27), which is of major practical interest, is presented in Figure 3 together with its time-related derivative, computed in real-time with (28). Both stator phase current curve and stator phase current derivative, wholly depicted in Figure 3, correspond to the initial lag angle of the rotor $\gamma_0 = \gamma_{0,\,lag} = -\,\pi/4$.

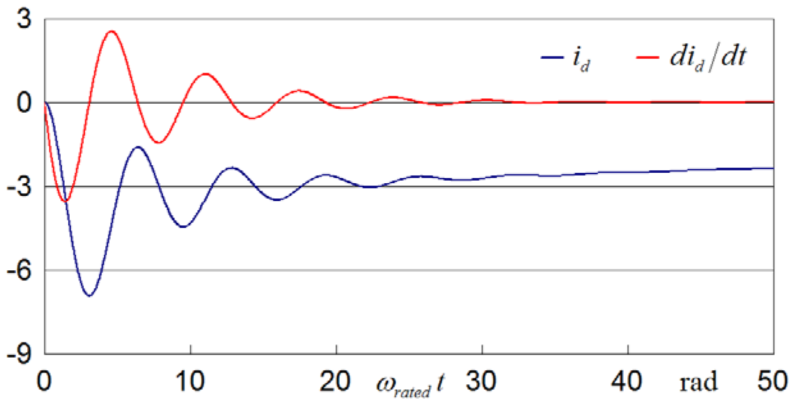

Figure 1: Evolution curves of stator *d*-axis current together with its time-related derivative, recorded during numerical integration

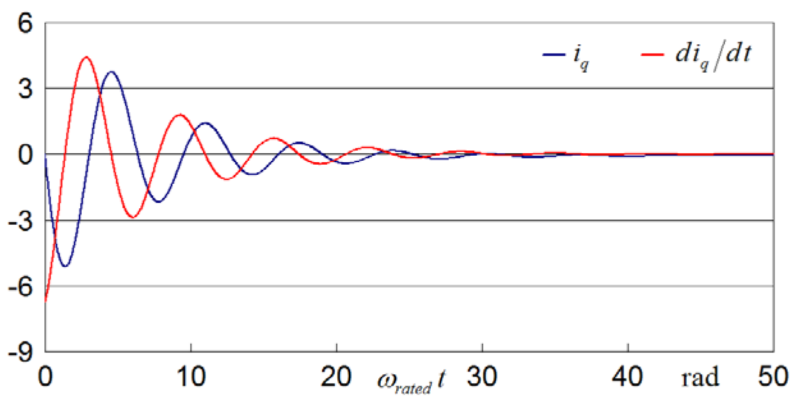

Figure 2: Evolution curves of stator *q*-axis current together with its time-related derivative, recorded during numerical integration

As aforementioned, the base quantity for time variable is $t_{base} = 1/\omega_{rated}$, where ω_{rated} represents the rated angular velocity. Having this in view, the integration has been carried out with a step size of 0.001 rad. over the interval [0 rad., 50 rad.]. It has to be emphasized that by adopting a 10-bytes extended data representation (1.9E-4932…1.1E+4932), no changes have been observed in the results at step size reduction below 0.001 rad. To develop the dedicated software environment, we have employed Free Pascal IDE [29] on Ubuntu OS.

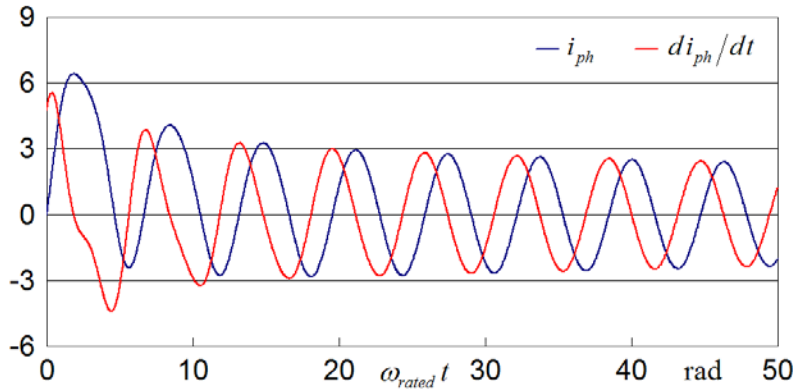

Figure 3: Evolution curves of stator phase current (27) together with its time-related derivative (28), recorded during numerical integration

The currents curves plotted in Figure 1, Figure 2 and Figure 3 highlight the traditional manner of assessing an electrical transient. More precisely, the evolution curves of currents in Figure 1 up to Figure 3 are the result of accessing the huge file that stores the data corresponding to all steps of numerical integration i.e. time variable values, currents values. Thus, although merely the relative maxima and minima of stator phase current (27), which is the main characteristic quantity of the process, are of practical interest to assess the selected short circuit scenario, the currents values have been traditionally recorded at each step of numerical integration. It has to be pointed out that in this case, the access time to the storage media is extremely high, having in view that data have been computed and recorded at all steps during the integration. This observation comes to be very significant in the situation in which a set of contingency cases is to be solved in online mode.

Since the main characteristic quantity here is the stator phase current, provided by (27), the next set of experiments will refer specifically to this quantity. To highlight the domains of interest in the evolution of stator phase current and to increase the computational capabilities of the dedicated software environment, we have proceeded to evaluate and record the selected stator phase current only if the following condition:

$$\left|\frac{d\,i_{ph}}{dt}\right| < \varepsilon$$

$$\Leftrightarrow \left| \left(\frac{d\,i_d}{dt} - i_q\right)\cos\left(t-\gamma_0\right) - \left(\frac{d\,i_q}{dt} + i_d\right)\sin\left(t-\gamma_0\right) \right| < \varepsilon \qquad (32)$$

has been met. More precisely, at each step of numerical integration, we have computed the absolute value of stator phase current derivative (28) to decide whether the value of stator phase current is within the area of practical interest, determined here by means of extremum ε in (32). The efficiency of this assessing procedure to localize the relative maxima and minima is suggestively revealed by the data of Figure 4, plotted for $\varepsilon \in \{3, 2.5, 2, 1.5, 1, 0.5\}$.

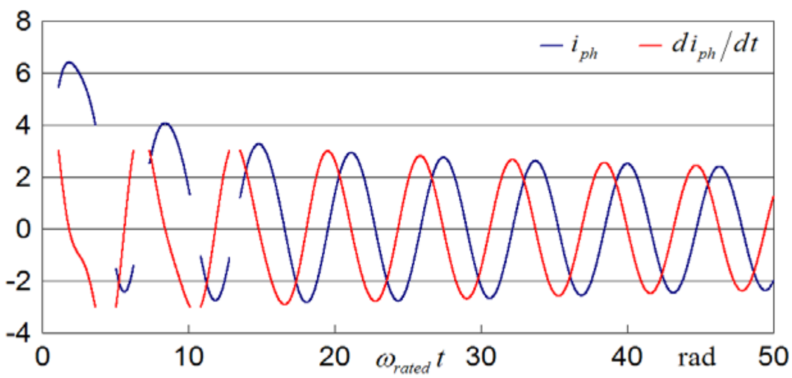

Figure 4(a): Values received for $\varepsilon = 3$ in criterion (32)

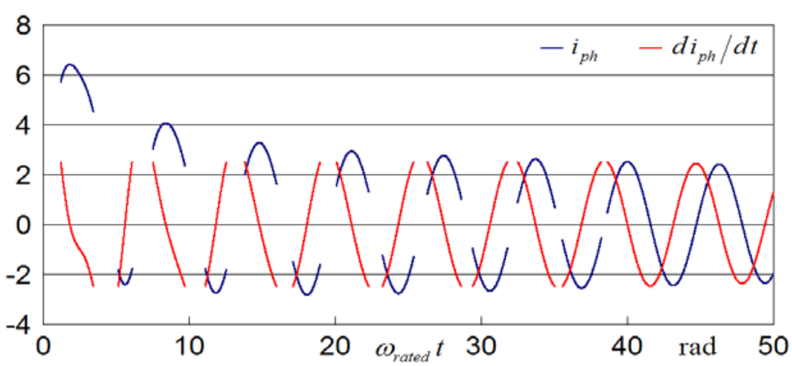

Figure 4(b): Values received for $\varepsilon = 2.5$ in criterion (32)

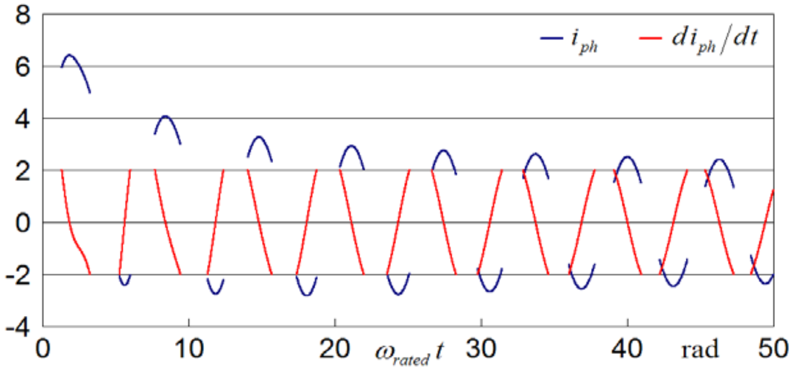

Figure 4(c): Values received for $\varepsilon = 2$ in criterion (32)

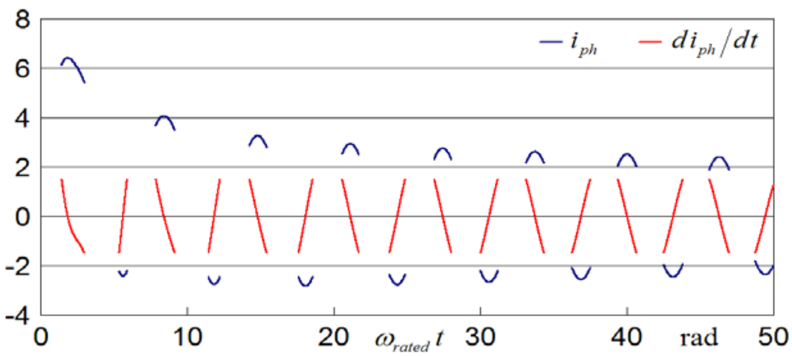

Figure 4(d): Values received for $\varepsilon = 1.5$ in criterion (32)

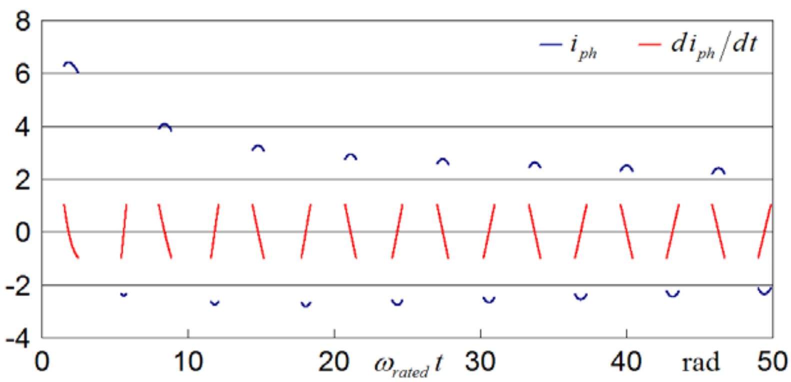

Figure 4(e): Values received for $\varepsilon = 1$ in criterion (32)

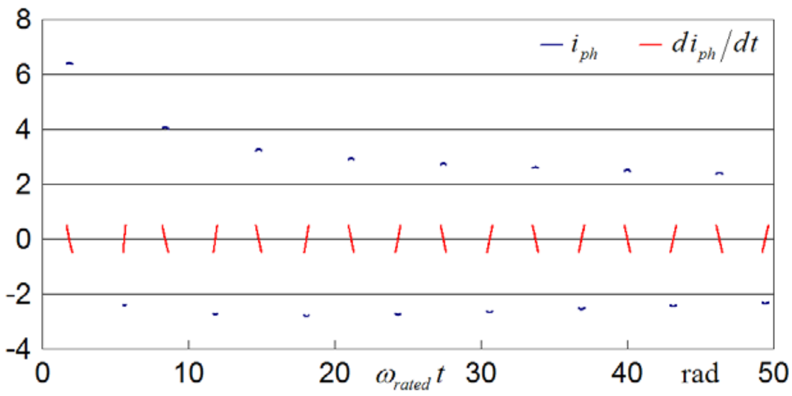

Figure 4(f): Values received for $\varepsilon = 0.5$ in criterion (32)

Figure 4: Values of selected stator phase current and its time-related derivative, received for $\varepsilon \in \{3, 2.5, 2, 1.5, 1, 0.5\}$ in criterion (32) of data recording: (a) $\varepsilon = 3$; (b) $\varepsilon = 2.5$; (c) $\varepsilon = 2$; (d) $\varepsilon = 1.5$; (e) $\varepsilon = 1$; (f) $\varepsilon = 0.5$

Applying of criterion (32) of data recording calls for the values of stator *d-q* axis currents and their time-related derivatives at each step of numerical integration. However, no further processing is needed since, in contrast to stator phase current, the stator *d-q* axis currents, depicted in Figure 1 and Figure 2, are precisely state variables and, implicitly, along with their time-related derivatives, they are to be updated at each step of numerical integration.

5 Conclusion

The present paper puts forward a new procedure of assessing the transients in electrical systems with a view to relative extrema localizing and computing rate increasing. The suggested procedure involves an eight-order explicit multistep method, which is straightforwardly designed in the paper based on the generalized Adams-Bashforth formula. The employment of an explicit multistep method is justified having in view that the information required in order to update the state variables is exactly the one needed also to localize the relative extrema of state variables and of any characteristic quantity correlated with the selected state variables. To facilitate the understanding, the complex case of a synchronous generator subjected to sudden three-phase short circuit fault is depicted in detail by plotting various novel and suggestive characteristic curves. To highlight the areas of interest, encompassing the relative maxima and minima, as well as to optimize the size of the file that stores the significant values received during numerical integration, we have advanced a criterion of data recording, which requires a test that involves the state variables and their time-related derivatives.

References

[1] T. Noda, K. Takenaka and T. Inoue, Numerical integration by the 2-stage diagonally implicit Runge-Kutta method for electromagnetic transient simulations, *IEEE Transactions on Power Delivery*, **24**(1), (January 2009), 390-399.

[2] J. Mahseredjian, S. Dennetiere, L. Dube, B. Khodabakhchian and L. Gerin-Lajoie, On a new approach for the simulation of transients in power systems, *Electric Power Systems Research*, **77**(11), (September 2007), 1514-1520.

[3] A.M. Gole, S. Filizadeh, R.W. Menzies and P.L. Wilson, Optimization-enabled electromagnetic transient simulation, *IEEE Transactions on Power Delivery*, **20**(1), (January 2005), 512-518.

[4] N. Watson and J. Arrillaga, *Power Systems Electromagnetic Transients Simulation*, The Institution of Electrical Engineers (IEE), London, UK, 2003.

[5] N. Bijl and L. Der Van Sluis, New approach to the calculation of electrical transients, Part I: Theory, *European Transactions on Electrical Power*, **8**(3), (May/June 1998), 175-179.

[6] N. Bijl and L. Der Van Sluis, New approach to the calculation of electrical transients, Part II: Applications, *European Transactions on Electrical Power*, **8**(3), (May/June 1998), 181-186.

[7] A. Greenwood, *Electrical Transients in Power Systems*, 2nd edition, John Wiley and Sons, New York, NY, USA, 1991.

[8] A. Semlyen and F. De Leon, Computation of electromagnetic transients using dual or multiple time steps, *IEEE Transactions on Power Systems*, **8**(3), (August 1993), 1274-1281.

[9] J. Mahseredjian and F. Alvarado, Creating an electromagnetic transients program in MATLAB: MatEMTP, *IEEE Transactions on Power Delivery*, **12**(1), (January 1997), 380-388.

[10] J.R. Marti and L.R. Linares, Real-time EMTP-based transients simulation, *IEEE Transactions on Power Systems*, **9**(3), (August 1994), 1309-1317.

[11] L.A. Dessaint, K. Al-Haddad, H. Le-Huy, G. Sybille and P. Brunelle, A power system simulation tool based on Simulink, *IEEE Transactions on Industrial Electronics*, **46**(6), (December 1999), 1252-1254.

[12] W. Long, D. Cotcher, D. Ruiu, P. Adam, S. Lee and R. Adapa, EMTP - a powerful tool for analyzing power system transients, *IEEE Computer Applications in Power*, **3**(3), (July 1990), 36-41.

[13] J.A. Martinez, Educational use of EMTP models for the study of rotating machine transients, *IEEE Transactions on Power Systems*, **8**(4), (November 1993), 1392-1399.

[14] J. Mahseredjian, G. Benmouyal, X. Lombard, M. Zouiti, B. Bressac and L. Gerin-Lajoie, A link between EMTP and MATLAB for user-defined modelling, *IEEE Transactions on Power Delivery*, **13**(2), (April 1998), 667-674.

[15] W.H. Enright, Software for ordinary and delay differential equations: Accurate discrete approximate solutions are not enough, *Applied Numerical Mathematics*, **56**(3-4), (March–April 2006), 459-471.

[16] T.E. Hull, W.H. Enright, B.M. Fellen and A.E. Sedgwick, Comparing numerical methods for ordinary differential equations, *SIAM Journal on Numerical Analysis*, **9**(4), (1972), 603-637.

[17] G. Soderlind and L. Wang, Evaluating numerical ODE/DAE methods, algorithms and software, *Journal of Computational and Applied Mathematics*, **185**(2), (January 2006), 244-260.

[18] J.C. Butcher, Numerical methods for ordinary differential equations in the 20th century, *Journal of Computational and Applied Mathematics*, **125**(1-2), (December 2000), 1-29.

[19] U.M. Ascher and L.R. Petzold, *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1998.

[20] A.A. Gorev, *Transient Processes of Synchronous Machine*, Nauka, Sankt-Petersburg, Russia, 1985.

[21] I. Boldea, *The Electric Generators Handbook: Synchronous Generators*, CRC Press, Boca Raton, FL, USA, 2006.

[22] L. Lupsa-Tataru, An extension of flux linkage state-space model of synchronous generators with a view to dynamic simulation, *WSEAS Transactions on Power Systems*, **1**(12), (December 2006), 2017-2022.

[23] J.C. Das, Study of generator source short-circuit currents with respect to interrupting the duty of generator circuit breakers, EMTP simulation, ANSI/IEEE and IEC methods, *International Journal of Emerging Electric Power Systems*, **9**(3), (April 2008), article 3.

[24] E. Haginomori and S. Ohtsuka, Sudden short circuiting of synchronous machines by EMTP, *Electrical Engineering in Japan*, **146**(1), (January 2004), 78-88.

[25] A. Berizzi, S. Massucco, A. Silvestri and D. Zaninelli, Short-circuit calculation: a comparison between IEC and ANSI standards using dynamic simulation as reference, *IEEE Transactions on Industry Applications*, **30**(4), (July/August 1994), 1099-1106.

[26] F. Castelli-Dezza, A. Silvestri and D. Zaninelli, The IEC 909 standard and dynamic simulation of short-circuit currents, *European Transactions on Electrical Power Engineering*, **4**(3), (May/June 1994), 213-221.

[27] A. Rodolakis, A comparison of North American (ANSI) and European (IEC) fault calculation guidelines, *IEEE Transactions on Industry Applications*, **29**(3), (May/June 1993), 515-521.

[28] O.E. Roennspiess and A.E. Efthymiadis, A comparison of static and dynamic short circuit analysis procedures, *IEEE Transactions on Industry Applications*, **26**(3), (May/June 1990), 463-475.

[29] M. Van Canneyt, *Run-Time Library (RTL): Reference Guide. Free Pascal version 2.6.0: Reference Guide for RTL Units*, Document version 2.6, December 2011, http://www.freepascal.org