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Brand Selection and its Matrix Structure
-Expansion to the Second Order Lag-

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Abstract

Focusing that consumers' are apt to buy superior brand when they are accustomed or bored to use current brand, new analysis method is introduced. Before buying data and after buying data is stated using liner model. When above stated events occur, transition matrix becomes upper triangular matrix. In this paper, equation using transition matrix is extended to the second order lag and the method is newly re-built. These are confirmed by numerical examples. S-step forecasting model is also introduced. This approach makes it possible to identify brand position in the market and it can be utilized for building useful and effective marketing plan.

Keywords: brand selection; matrix structure; brand position

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1 Introduction

It is often observed that consumers select upper class brand when they buy next time after they are bored to use current brand. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. If the top brand were selected from lower brand skipping intermediate brands, corresponding part in upper triangular matrix would be 0. These are verified in numerical examples with simple models.

If transition matrix is identified, s-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planners for products notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach makes it effective to execute marketing plan and/or establish new brand.

Quantitative analysis concerning brand selection has been executed by Yamanaka[5], Takahashi et al.[4]. Yamanaka[5] examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi et al.[4] made analysis by the Brand Selection Probability model using logistics distribution. In Takeyasu et al. (2007), matrix structure was analyzed for the case brand selection was executed for upper class. In this paper, equation using transition matrix is extended to the second order lag and the method is newly re-built. Such research is quite a new one.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. Extension of the model to the second order lag is made in section 3. Numerical calculation is executed in section 4. Section 5 is a summary.

2 Brand Selection and Its Matrix Structure

2.1 Upper shift of brand selection

Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand. Consumer's behavior of selecting brand might be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few.

Suppose that x is current buying variable, and x_b is previous buying variable. Shift to x is executed from x_b, y_b, or z_b. Therefore, x is stated in the following equation. a_{ij} represents transition probability from j-th to i-th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

and

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{1}$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \ \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \ \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{AX_b} \tag{2}$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times 3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix. To examine this, generating following data, which are all consisted by the data in which transition is made from lower brand to upper brand,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \cdots \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \cdots \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1, \qquad 2, \qquad \cdots \qquad N$$

parameter can be estimated using least square method. Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

where

$$\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1,2,\cdots,N$$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \to Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT} \right)^{-1} \tag{7}$$

In the data group which are all consisted by the data in which transition is made from lower brand to upper brand, estimated value $\hat{\mathbf{A}}$ should be upper triangular matrix. If following data which shift to lower brand are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{\mathbf{b}}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part triangle.

2.2 Sorting brand ranking by re-arranging row

In a general data, variables may not be in order as x, y, z. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in upper triangular matrix, and the small value parts are gathered in lower triangular matrix.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix}} \quad \xleftarrow{\text{Shifting row}} \quad \begin{pmatrix} z \\ x \\ y \end{pmatrix} \overset{\hat{\mathbf{A}}}{\begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix}} \tag{8}$$

2.3 Matrix structure under the case skipping intermediate class brand is skipped

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand. We suppose v, w, x, y, z brands (suppose they are laid from upper position to lower position as $v > w > x > y > z$). In the above case, selection shifts would be

$v \leftarrow z$

$v \leftarrow y$

Suppose they do not shift to y, x, w from z, to x, w from y, and to w from x, then Matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \tag{9}$$

3 Extension of The Model to The Second Order Lag

We extend Eq.(2) to the second order lag in this section. We have analyzed the automobile purchasing case (Takeyasu et al. (2007)). In that case, we could obtain the data (current buying data, former buying data, before former buying data). We have analyzed them by dividing the data (current buying data, former buying data) and (former buying data before former buying data), and put them to Eq.(5) to apply the model.

But this is a kind of a simplified method to apply to the model. If we have a further time lag model and we can utilized the data as it is, the estimation accuracy of parameter would be more accurate and the forecasting would be more precise. Therefore we introduce a new model which extends Eq.(2) to the second order lag model as follows.

$$\mathbf{X}_t = \mathbf{A}_1 \mathbf{X}_{t-1} + \mathbf{A}_2 \mathbf{X}_{t-2} \tag{10}$$

Where

$$\mathbf{X}_t = \begin{pmatrix} x_1^t \\ x_2^t \\ \vdots \\ x_p^t \end{pmatrix} \quad t = 1, 2 \cdots$$

$$\mathbf{A}_1 = \begin{pmatrix} a_{11}^{(1)}, & a_{12}^{(1)}, & \cdots, & a_{1p}^{(1)} \\ a_{21}^{(1)}, & a_{22}^{(1)}, & \cdots, & a_{2p}^{(1)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(1)}, & a_{p1}^{(1)}, & \ldots, & a_{pp}^{(1)} \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} a_{11}^{(2)}, & a_{12}^{(2)}, & \cdots, & a_{1p}^{(2)} \\ a_{21}^{(2)}, & a_{22}^{(2)}, & \cdots, & a_{2p}^{(2)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(2)}, & a_{p1}^{(2)}, & \ldots, & a_{pp}^{(2)} \end{pmatrix}$$

$$\mathbf{X}_t \in \mathbf{R}^p \left(t = 1, 2, \cdots\right) \quad \mathbf{A}_1 \in \mathbf{R}^{p \times p}, \mathbf{A}_2 \in \mathbf{R}^{p \times p}$$

In order to estimate $\mathbf{A_1}, \mathbf{A_2}$, we set the following equation in the same way as before.

$$\mathbf{X}_t^i = \mathbf{A}_1 \mathbf{X}_{t-1}^i + \mathbf{A}_2 \mathbf{X}_{t-2}^i + \boldsymbol{\varepsilon}_t^i \quad \left(t = 1, 2, \cdots, N\right) \tag{11}$$

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}_t^{iT} \boldsymbol{\varepsilon}_t^{i} \to Min \tag{12}$$

Eq.(11) is expressed as follows.

$$\mathbf{X}_t^i = \left(\mathbf{A}_1, \quad \mathbf{A}_2\right) \begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix} + \boldsymbol{\varepsilon}_t^i \tag{13}$$

$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right)$ which is an estimated value of $\left(\mathbf{A}_1, \quad \mathbf{A}_2\right)$ is obtained as follows in the same way as Eq.(7).

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\sum_{i=1}^{N} \mathbf{X}_t^i \left(\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT} \right) \right) \left(\sum_{i=1}^{N} \begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix} \left(\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT} \right) \right)^{-1} \tag{14}$$

This is re-written as :

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-1}^{iT}, \quad \sum_{i=1}^{N} \mathbf{X}_t^i \mathbf{X}_{t-2}^{iT}\right) \begin{pmatrix} \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-1}^i \mathbf{X}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-2}^i \mathbf{X}_{t-2}^{iT} \end{pmatrix}^{-1} \tag{15}$$

We set this as :

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\hat{\mathbf{B}}, \quad \hat{\mathbf{C}}\right) \begin{pmatrix} \hat{\mathbf{D}} & \hat{\mathbf{E}} \\ \hat{\mathbf{E}}^T & \hat{\mathbf{F}} \end{pmatrix}^{-1} \tag{16}$$

In the data group of upper shift brand, $\hat{\mathbf{E}}$ becomes an upper triangular matrix. While $\hat{\mathbf{D}}$ and $\hat{\mathbf{F}}$ are diagonal matrix in any case. This will be made clear in the numerical calculation later.

4 Forecasting

After transition matrix is estimated, we can make forecasting. We show some of them in the following equations.

$$\hat{\mathbf{X}}_{t+1} = \hat{\mathbf{A}}_1 \mathbf{X}_t + \hat{\mathbf{A}}_2 \mathbf{X}_{t-1} \tag{17}$$

$$\hat{\mathbf{X}}_{t+2} = \left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right)\mathbf{X}_t + \hat{\mathbf{A}}_1 \hat{\mathbf{A}}_2 \mathbf{X}_{t-1} \tag{18}$$

$$\hat{\mathbf{X}}_{t+3} = \left(\hat{\mathbf{A}}_1{}^3 + \hat{\mathbf{A}}_1 \hat{\mathbf{A}}_2 + \hat{\mathbf{A}}_2 \hat{\mathbf{A}}_1\right)\mathbf{X}_t + \left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right)\hat{\mathbf{A}}_2 \mathbf{X}_{t-1} \tag{19}$$

$$\begin{aligned} \hat{\mathbf{X}}_{t+4} = &\left(\hat{\mathbf{A}}_1{}^4 + \hat{\mathbf{A}}_1{}^2 \hat{\mathbf{A}}_2 + \hat{\mathbf{A}}_1 \hat{\mathbf{A}}_2 \hat{\mathbf{A}}_1 + \hat{\mathbf{A}}_2 \hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2{}^2\right)\mathbf{X}_t \\ &+ \left\{\hat{\mathbf{A}}_1\left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right) + \hat{\mathbf{A}}_2 \hat{\mathbf{A}}_1\right\}\hat{\mathbf{A}}_2 \mathbf{X}_{t-1} \end{aligned} \tag{20}$$

5 Numerical Example

In this section, we consider the case there is no shift to lower brand. We consider the case that brand selection shifts to the same class or upper classes. As above referenced, corresponding part of transition matrix must be an upper triangular matrix. Suppose following events occur. Here we set $p=3$ in Eq.(10).

< $\mathbf{X}_{t-2}$ to $\mathbf{X}_{t-1}$ >

① Shift from lower brand to middle brand : 5 events
② Shift from lower brand to lower brand : 3 events
③ Shift from lower brand to lower brand : 4 events
④ Shift from lower brand to upper brand : 3 events
⑤ Shift from lower brand to middle brand : 4 events
⑥ Shift from lower brand to lower brand : 3 events
⑦ Shift from middle brand to middle brand : 2 events
⑧ Shift from middle brand to middle brand : 3 events
⑨ Shift from upper brand to upper brand : 1 event
⑩ Shift from middle brand to upper brand : 2 events
⑪ -
⑫ -
⑬ -

< $\mathbf{X}_{t-1}$ to $\mathbf{X}_{t}$ >

① Shift from middle brand to upper brand : 5 events
② Shift from lower brand to upper brand : 3 events
③ Shift from lower brand to middle brand : 4 events
④ Shift from upper brand to upper brand : 3 events
⑤ Shift from middle brand to middle brand : 4 events
⑥ Shift from lower brand to lower brand : 3 events
⑦ Shift from middle brand to upper brand : 2 events

⑧ Shift from middle brand to middle brand : 3 events

⑨ Shift from upper brand to upper brand : 1 event

⑩ Shift from upper brand to upper brand : 2 events

⑪ Shift from lower brand to middle brand : 2 events

⑫ Shift from middle brand to middle brand : 1 event

⑬ Shift from middle brand to upper brand : 2 events

Vector $\mathbf{X}_t, \mathbf{X}_{t-1}, \mathbf{X}_{t-2}$ in these cases are expressed as follows.

① $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

② $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

③ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

④ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑤ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑥ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑦ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑧ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑨ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

⑩ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑪ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑫ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑬ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Substituting these to Eq.(14), we obtain the following equation.

$$\left(\hat{\mathbf{A}}_1,\quad \hat{\mathbf{A}}_2\right)=\begin{pmatrix} 6 & 9 & 3 & 1 & 4 & 11 \\ 0 & 8 & 6 & 0 & 3 & 8 \\ 0 & 0 & 3 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 6 & 0 & 0 & 1 & 2 & 3 \\ 0 & 17 & 0 & 0 & 5 & 9 \\ 0 & 0 & 12 & 0 & 0 & 10 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 5 & 0 & 0 & 7 & 0 \\ 3 & 9 & 10 & 0 & 0 & 22 \end{pmatrix}^{-1} \tag{21}$$

As we have seen before, we can confirm that

$\hat{\mathbf{E}}$ part in Eq.(17) is an upper triangular matrix and

$\hat{\mathbf{D}}, \hat{\mathbf{F}}$ part in Eq.(17) are diagonal matrices.

$\hat{\mathbf{E}}^T$ part is there by a lower triangular matrix.

We can find that if $\hat{\mathbf{E}}$ part becomes an upper triangular matrix, then the items compose upper shift or the same level shift. Calculation results of $\left(\hat{\mathbf{A}}_1,\quad \hat{\mathbf{A}}_2\right)$ become as follows.

$$\left(\hat{\mathbf{A}}_1,\quad \hat{\mathbf{A}}_2\right)=$$

$$\begin{pmatrix} 1.006 & 0.529 & 0.206 & -0.006 & -0.094 & 0.053 \\ 0.110 & 0.568 & 0.649 & -0.110 & -0.008 & -0.178 \\ -0.116 & -0.097 & 0.145 & 0.116 & 0.102 & 0.126 \end{pmatrix} \tag{22}$$

6 Conclusion

Consumers often buy higher grade brand products as they are accustomed or bored to use current brand products they have.

Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. In Takeyasu et al. (2007), matrix structure was analyzed for the case brand selection was executed for upper class. In this paper, equation using transition matrix was extended to the second order lag and the method was newly re-built. In the numerical example, matrix structure's hypothesis was verified. We can utilized the data as it is for the data in which time lag exist by this new model and estimation accuracy of parameter becomes more accurate and forecasting becomes more precise. Such research as questionnaire investigation of consumers activity in automobile purchasing should be executed in the near future to verify obtained results.

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