A Hybrid Method to Improve Forecasting Accuracy

–An Application to the Processed Cooked Rice–

Hiromasa Takeyasu¹, Daisuke Takeyasu² and Kazuhiro Takeyasu³

Abstract

In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method (ESM) is equivalent to (1,1) order ARMA model equation, new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the

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following method. Trend removing by the combination of linear and 2\textsuperscript{nd} order non-linear function and 3\textsuperscript{rd} order non-linear function is executed to the original production data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. Genetic Algorithm is utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

**Keywords**: minimum variance, exponential smoothing method, forecasting, trend, bread

1 Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM), [1]-[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under
supposed noise because he could not grasp observation noise. It can be said that it doesn’t pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn’t show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original production data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. Genetic Algorithm is utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.
2 Description of ESM using ARMA model

In ESM, forecasting at time \( t + 1 \) is stated in the following equation.

\[
\hat{x}_{t+1} = \hat{x}_t + \alpha (x_t - \hat{x}_t)
\]

(1)

\[
= \alpha x_t + (1 - \alpha) \hat{x}_t
\]

(2)

Here,
\[
\hat{x}_{t+1} : \text{forecasting at } t + 1
\]
\[x_t : \text{realized value at } t\]
\[\alpha : \text{smoothing constant } (0 < \alpha < 1)\]

(2) is re-stated as

\[
\hat{x}_{t+1} = \sum_{l=0}^{\alpha} \alpha (1 - \alpha)^l x_{t-l}
\]

(3)

By the way, we consider the following (1,1) order ARMA model.

\[
x_t - x_{t-1} = e_t - \beta e_{t-1}
\]

(4)

Generally, \((p,q)\) order ARMA model is stated as

\[
x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j}
\]

(5)

Here,
\[
\{x_t\} : \text{Sample process of Stationary Ergodic Gaussian Process } x(t), \ t = 1,2,\ldots,N
\]
\[
\{e_t\} : \text{Gaussian White Noise with 0 mean } \sigma_e^2 \text{ variance}
\]

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that

\[
E[e_t | e_{t-1}, e_{t-2}, \ldots] = 0
\]

we get the following equation from (4).

\[
\hat{x}_t = x_{t-1} - \beta e_{t-1}
\]

(6)
Operating this scheme on $t+1$, we finally get
\[ \hat{x}_{t+1} = \hat{x}_t + (1 - \beta) e_t = \hat{x}_t + (1 - \beta) (x_t - \hat{x}_t) \] (7)

If we set $1 - \beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is $-1$. Comparing with (4) and (5), we obtain
\[ a_t = -1 \]
\[ b_t = -\beta \]

From (1), (7),
\[ \alpha = 1 - \beta \]

Therefore, we get
\[ a_t = -1 \]
\[ b_t = -\beta = \alpha - 1 \] (8)

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (5) be
\[ \tilde{x}_t = x_t + \sum_{i=1}^{n} a_t x_{t-i} \] (9)
\[ \tilde{e}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \] (10)

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known.
\[ \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j}, & k \leq q \\ 0, & k \geq q + 1 \end{cases} \] (11)
\[ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \]
For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only $b_1$, so it can be solved in the following way.

From (4) (5) (8) (11), we get

$$
q = 1 \\
a_1 = -1 \\
b_1 = -\beta = \alpha - 1 \\
\tilde{r}_0 = (1 + b_1^2)\sigma^2_e \\
\tilde{r}_1 = b_1\sigma^2_e
$$

If we set

$$
\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} 
$$

the following equation is derived.

$$
\rho_k = \frac{b_1}{1 + b_1^2} 
$$

We can get $b_1$ as follows.

$$
b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} 
$$

In order to have real roots, $\rho_1$ must satisfy

$$
|\rho_1| \leq \frac{1}{2} 
$$

From invertibility condition, $b_1$ must satisfy

$$
|b_1| < 1
$$

From (14), using the next relation,

$$
(1 - b_1)^2 \geq 0 \\
(1 + b_1)^2 \geq 0
$$

(16) always holds.

As
\[ \alpha = b_1 + 1 \]

\[ b_1 \text{ is within the range of } -1 < b_1 < 0 \]

Finally we get

\[ b_1 = \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \]

\[ \alpha = \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \]  \hspace{1cm} (17)

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way. Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating 0th and 1st order autocorrelation function.

### 3 Trend removal method

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

\[ y = a_1x + b_1 \]  \hspace{1cm} (18)

as a linear function.

[2] Non-linear function

We set

\[ y = a_2x^2 + b_2x + c_2 \]  \hspace{1cm} (19)

\[ y = a_3x^3 + b_3x^2 + c_3x + d_3 \]  \hspace{1cm} (20)
as a 2nd and a 3rd order non-linear function. \((a_2, b_2, c_2)\) and \((a_3, b_3, c_3, d_3)\) are also parameters for a 2nd and a 3rd order non-linear functions which are estimated by using least square method.


We set

\[
y = a_1(a_xx + b_1) + a_2(a_xx^2 + b_2x + c_2) + a_3(a_xx^3 + b_3x^2 + c_3x + d_3)
\]  

(21)

\[
0 \leq a_1 \leq 1, 0 \leq a_2 \leq 1, 0 \leq a_3 \leq 1, a_1 + a_2 + a_3 = 1
\]  

(22)

as the combination linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the original data by (21). The optimal weighting parameter \(a_1, a_2, a_3\), are determined by utilizing GA. GA method is precisely described in section 6.

4 Monthly ratio

For example, if there is the monthly data of \(L\) years as stated bellow:

\[
\{x_{ij}\} \quad (i = 1, 2, \ldots, L) \quad (j = 1, 2, \ldots, 12)
\]

where, \(x_{ij} \in R\) in which \(j\) means month and \(i\) means year and \(x_{ij}\) is a production data of \(i\)-th year, \(j\)-th month. Then, monthly ratio \(\tilde{x}_j\) \((j = 1, 2, \ldots, 12)\) is calculated as follows.

\[
\tilde{x}_j = \frac{\frac{1}{L} \sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \frac{1}{12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij}}
\]  

(23)

Monthly trend is removed by dividing the data by (23). Numerical examples both of monthly trend removal case and non-removal case are discussed in 7.
5 Forecasting accuracy

Forecasting accuracy is measured by calculating the variance of the forecasting error. Variance of forecasting error is calculated by:

$$\sigma^2_\varepsilon = \frac{1}{N-1} \sum_{i=1}^{N} (\varepsilon_i - \bar{\varepsilon})^2$$  \hspace{1cm} (24)

Where, forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i$$  \hspace{1cm} (25)

$$\bar{\varepsilon} = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_i$$  \hspace{1cm} (26)

6 Searching optimal weights utilizing GA

6.1 Definition of the problem

We search $a_1, a_2, a_3$ of (21) which minimizes (24) by utilizing GA. By (22), we only have to determine $a_1$ and $a_2$. $\sigma^2_\varepsilon ((24))$ is a function of $a_1$ and $a_2$, therefore we express them as $\sigma^2_\varepsilon(a_1, a_2)$. Now, we pursue the following:

Minimize: $\sigma^2_\varepsilon(a_1, a_2)$  \hspace{1cm} (27)

subject to: $0 \leq a_1 \leq 1, \ 0 \leq a_2 \leq 1 \ a_1 + a_2 \leq 1$

We do not necessarily have to utilize GA for this problem which has small member of variables. Considering the possibility that variables increase when we use logistics curve etc in the near future, we want to ascertain the effectiveness of GA.
6.2 The structure of the gene

Gene is expressed by the binary system using \{0,1\} bit. Domain of variable is [0,1] from (22). We suppose that variables take down to the second decimal place. As the length of domain of variable is 1-0=1, seven bits are required to express variables. The binary bit strings \langle bit6, \sim, bit0 \rangle is decoded to the [0,1] domain real number by the following procedure [14].

Table 6-1: Corresponding table of the decimal number, the binary number and the real number

<table>
<thead>
<tr>
<th>The decimal number</th>
<th>The binary number</th>
<th>The corresponding real number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0 0 0 0</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 0 0 0 1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 0 1 0</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 0 1 0 1</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 0 1 0 0</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>0 0 0 0 1 0 1</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>0 0 0 0 1 1 0</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>0 0 0 0 1 1 1</td>
<td>0.06</td>
</tr>
<tr>
<td>8</td>
<td>0 0 0 1 0 0 0</td>
<td>0.06</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>126</td>
<td>1 1 1 1 1 1 0</td>
<td>0.99</td>
</tr>
<tr>
<td>127</td>
<td>1 1 1 1 1 1 1</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Procedure 1: Convert the binary number to the binary-coded decimal.

\[
\left( (\text{bit}_6, \text{bit}_5, \text{bit}_4, \text{bit}_3, \text{bit}_2, \text{bit}_1, \text{bit}_0) \right)_2 = \left( \sum_{i=0}^{6} \text{bit}_i 2^i \right)_{10} = X'
\]

Procedure 2: Convert the binary-coded decimal to the real number.

The real number

\[
= (\text{Left hand starting point of the domain}) + X'((\text{Right hand ending point of the domain})/(2^7 - 1))
\]

The decimal number, the binary number and the corresponding real number in the case of 7 bits are expressed in Table 6-1.

1 variable is expressed by 7 bits, therefore 2 variables needs 14 bits. The gene structure is exhibited in Table 6-2.

<table>
<thead>
<tr>
<th>Position of the bit</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>12</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>11</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>10</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>9</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>8</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>7</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>6</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>5</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>4</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>3</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>2</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>1</td>
<td>0-1</td>
<td>0-1</td>
</tr>
<tr>
<td>0</td>
<td>0-1</td>
<td>0-1</td>
</tr>
</tbody>
</table>

6.3 The flow of Algorithm

The flow of algorithm is exhibited in Figure 6-1.
A. Initial Population

Generate $M$ initial population. Here, $M = 100$. Generate each individual so as to satisfy (22).

B. Calculation of Fitness

First of all, calculate forecasting value. There are 36 monthly data for each case. We use 24 data (1st to 24th) and remove trend by the method stated in section 3. Then we calculate monthly ratio by the method stated in section 4. After removing monthly trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (24). Calculation of fitness is exhibited in Figure 6-2.
Scaling [15] is executed such that fitness becomes large when the variance of forecasting error becomes small. Fitness is defined as follows.

\[ f(\alpha_1, \alpha_2) = U - \sigma_e^2(\alpha_1, \alpha_2) \]  

(30)

where \( U \) is the maximum of \( \sigma_e^2(a_1, a_2) \) during the past \( W \) generation. Here, \( W \) is set to be 5.

C. Selection

Selection is executed by the combination of the general elitist selection and the tournament selection. Elitism is executed until the number of new elites reaches
the predetermined number. After that, tournament selection is executed and selected.

**D. Crossover**

Crossover is executed by the uniform crossover. Crossover rate is set as follows.

\[ P_c = 0.7 \]  

(31)

**E. Mutation**

Mutation rate is set as follows.

\[ P_m = 0.05 \]  

(32)

Mutation is executed to each bit at the probability \( P_m \), therefore all mutated bits in the population \( M \) becomes \( P_m \times M \times 14 \).

7 Numerical example

7.1 Application to the original production data of processed cooked rice

The original production data of processed cooked rice for 3 cases (Data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice: Annual Report of Statistical Research, Ministry of Agriculture, Forestry and Fisheries, Japan) from January 2008 to December 2010 are analyzed. Furthermore, GA results are compared with the calculation results of all considerable cases in order to confirm the effectiveness of GA approach. First of all, graphical charts of these time series data are exhibited in Figure 7-1 - 7-3.
Figure 7-1: Data of Processed Cooked Rice (Total)

Figure 7-2: Data of Retort Cooked Rice
7.2 Execution Results

GA execution condition is exhibited in Table 7-1.

<table>
<thead>
<tr>
<th>GA Execution Condition</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>100</td>
</tr>
<tr>
<td>Maximum Generation</td>
<td>50</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation ratio</td>
<td>0.05</td>
</tr>
<tr>
<td>Scaling window size</td>
<td>5</td>
</tr>
<tr>
<td>The number of elites to retain</td>
<td>2</td>
</tr>
<tr>
<td>Tournament size</td>
<td>2</td>
</tr>
</tbody>
</table>

We made 10 times repetition and the maximum, average, minimum of the variance of forecasting error and the average of convergence generation are exhibited in Table 7-2 and 7-3.

Table 7-2: GA execution results (Monthly ratio is not used)
Hiromasa Takeyasu, Daisuke Takeyasu and Kazuhiro Takeyasu

<table>
<thead>
<tr>
<th>Food</th>
<th>The variance of forecasting error</th>
<th>Average of convergence generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Average</td>
</tr>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>6254952.198</td>
<td>4773756.223</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>216829.0258</td>
<td>155165.7454</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>4989.709619</td>
<td>3404.034352</td>
</tr>
</tbody>
</table>

Table 7-3: GA execution results (Monthly ratio is used)

<table>
<thead>
<tr>
<th>Food</th>
<th>The variance of forecasting error</th>
<th>Average of convergence generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Average</td>
</tr>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>5279063.541</td>
<td>2765813.356</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>143425.3998</td>
<td>60059.74106</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>4464.665018</td>
<td>2147.230534</td>
</tr>
</tbody>
</table>

The case monthly ratio is used is smaller than the case monthly ratio is not used concerning the variance of forecasting error in every case.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.
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Figure 7-4: Convergence Process in the case of Processed Cooked Rice (Total) (Monthly ratio is not used)

Figure 7-5: Convergence Process in the case of Processed Cooked Rice (Total) (Monthly ratio is used)

Figure 7-6: Convergence Process in the case of Retort Cooked Rice (Monthly ratio is not used)
Figure 7-7: Convergence Process in the case of Retort Cooked Rice (Monthly ratio is used)

Figure 7-8: Convergence Process in the case of Dehydrated Cooked Rice (Monthly ratio is not used)

Figure 7-9: Convergence Process in the case of Dehydrated Cooked Rice (Monthly ratio is used)
Next, optimal weights and their genes are exhibited in Table 7-4,7-5.

Table 7-4: Optimal weights and their genes (Monthly ratio is not used)

<table>
<thead>
<tr>
<th>Data</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>position of the bit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>13 12 11 10 9 8 7 6 5 4 3 2 1 0</td>
</tr>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>1.00</td>
<td>0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>1.00</td>
<td>0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>0.50</td>
<td>0.5</td>
<td>1 0 0 0 0 0 0 0 0 0 1 1 1 1 1</td>
</tr>
</tbody>
</table>

Table 7-5: Optimal weights and their genes (Monthly ratio is used)

<table>
<thead>
<tr>
<th>Data</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>position of the bit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>13 12 11 10 9 8 7 6 5 4 3 2 1 0</td>
</tr>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>1.00</td>
<td>0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>1.00</td>
<td>0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>1.00</td>
<td>0</td>
<td>1 1 1 1 1 1 1 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

In the case monthly ratio is not used, the linear function model is best in two cases for Processed Cooked Rice (Total) and Retort Rice, while Dehydrated Rice has chosen linear and 2\textsuperscript{nd} order non-linear function as the best one. In the case monthly ratio is used, the linear function model is best in all cases. Parameter estimation results for the trend of equation (21) using least square method are exhibited in Table 7-6 for the case of 1st to 24th data.
Table 7-6: Parameter estimation results for the trend of equation (21)

<table>
<thead>
<tr>
<th>Data</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c_2$</th>
<th>$a_3$</th>
<th>$b_3$</th>
<th>$c_3$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>42.32</td>
<td>22999.63</td>
<td>-3.26</td>
<td>123.83</td>
<td>22646.4</td>
<td>1.66</td>
<td>-65.34</td>
<td>757.38</td>
<td>21193.73</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>19.04</td>
<td>1579.91</td>
<td>0.04</td>
<td>18.07</td>
<td>1584.12</td>
<td>0.69</td>
<td>-25.88</td>
<td>282.56</td>
<td>977.67</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>-3.36</td>
<td>489.34</td>
<td>0.21</td>
<td>-8.6</td>
<td>512.04</td>
<td>0.02</td>
<td>-0.67</td>
<td>0.36</td>
<td>491.5</td>
</tr>
</tbody>
</table>

Trend curves are exhibited in Figure 7-10 - 7-12.

Figure 7-10: Trend of Processed Cooked Rice (Total)

Figure 7-11: Trend of Retort Cooked Rice
Figure 7-12: Trend of Dehydrated Cooked Rice

Calculation results of Monthly ratio for 1st to 24th data are exhibited in Table 7-7.

Table 7-7: Parameter Estimation result of Monthly ratio

<table>
<thead>
<tr>
<th>Data</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processed Cooked Rice</td>
<td>0.89</td>
<td>0.92</td>
<td>1.05</td>
<td>1.08</td>
<td>0.95</td>
<td>0.98</td>
<td>1.05</td>
<td>1.01</td>
<td>1.02</td>
<td>1.03</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>0.84</td>
<td>0.89</td>
<td>0.88</td>
<td>0.93</td>
<td>0.95</td>
<td>0.95</td>
<td>1.08</td>
<td>0.98</td>
<td>1.04</td>
<td>1.15</td>
<td>1.11</td>
<td>1.20</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>1.05</td>
<td>1.07</td>
<td>1.19</td>
<td>0.92</td>
<td>0.80</td>
<td>0.64</td>
<td>0.90</td>
<td>1.00</td>
<td>1.20</td>
<td>1.11</td>
<td>1.01</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Estimation result of the smoothing constant of minimum variance for the 1st to 24th data are exhibited in Table 7-8, 7-9.

Table 7-8: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is not used)

<table>
<thead>
<tr>
<th>Data</th>
<th>ρ1</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processed Cooked Rice</td>
<td>-0.210408</td>
<td>0.779348</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>-0.382185</td>
<td>0.535275</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>-0.199611</td>
<td>0.791730</td>
</tr>
</tbody>
</table>
Table 7-9: Smoothing constant of Minimum Variance of equation (17)  
(Monthly ratio is used)

<table>
<thead>
<tr>
<th>Data</th>
<th>$\rho_1$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processed Cooked Rice (Total)</td>
<td>-0.192750</td>
<td>0.799501</td>
</tr>
<tr>
<td>Retort Cooked Rice</td>
<td>-0.465967</td>
<td>0.316076</td>
</tr>
<tr>
<td>Dehydrated Cooked Rice</td>
<td>-0.327017</td>
<td>0.627642</td>
</tr>
</tbody>
</table>

Forecasting results are exhibited in Table 7-13 - 7-15.

Figure 7-13: Forecasting Result of Processed Cooked Rice (Total)

Figure 7-14: Forecasting Result of Retort Cooked Rice
7.3 Remarks

In the case monthly ratio is not used, the linear function model is best in two cases for Processed Cooked Rice (Total) and Retort Rice, while Dehydrated Rice has chosen linear and 2nd order non-linear function as the best one. In the case monthly ratio is used, the linear function model is best in all cases.

The case monthly ratio is used is smaller than the case monthly ratio is not used concerning the variance of forecasting error in every case.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

8 Conclusion

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was
proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrary. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the stock market price data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. The combination of linear and non-linear function was also introduced in trend removal. Genetic Algorithm was utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

References


