Journal of Computations & Modelling, vol.3, no.2, 2013, 125-150
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2013

A Hybrid Method to Improve Forecasting Accuracy
–An Application to the Processed Cooked Rice–

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Abstract

In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the

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Article Info: *Received* : March 21, 2013. *Revised* : June 2, 2013
Published online : June 20, 2013

following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original production data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. Genetic Algorithm is utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

Keywords: minimum variance, exponential smoothing method, forecasting, trend, bread

1 Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM), [1]-[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2^{nd} order non-linear function and 3^{rd} order non-linear function is executed to the original production data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. Genetic Algorithm is utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2 Description of ESM using ARMA model

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \tag{1}$$

$$= \alpha x_t + (1-\alpha)\hat{x}_t \tag{2}$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t : realized value at t

α : smoothing constant $(0 < a < 1)$

(2) is re-stated as

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \tag{3}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{4}$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{5}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$, $t = 1, 2, \ldots, N$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

we get the following equation from (4).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{6}$$

Operating this scheme on $t+1$, we finally get

$$\hat{x}_{t+1} = \hat{x}_t + (1-\beta)e_t = \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \tag{7}$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1. Comparing with (4) and (5), we obtain

$$a_1 = -1$$
$$b_1 = -\beta$$

From (1), (7),

$$\alpha = 1-\beta$$

Therefore, we get

$$\begin{aligned} a_1 &= -1 \\ b_1 &= -\beta = \alpha - 1 \end{aligned} \tag{8}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (5) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{9}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{10}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known.

$$\tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j}, & k \le q \\ 0, & k \ge q+1 \end{cases} \tag{11}$$

$$\tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (4) (5) (8) (11), we get

$$\begin{aligned} &q = 1 \\ &a_1 = -1 \\ &b_1 = -\beta = \alpha - 1 \\ &\tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ &\tilde{r}_1 = b_1\sigma_e^2 \end{aligned} \tag{12}$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{13}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{14}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{15}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{16}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (14), using the next relation,

$$\begin{aligned} &(1 - b_1)^2 \ge 0 \\ &(1 + b_1)^2 \ge 0 \end{aligned}$$

(16) always holds.

As

$\alpha = b_1 + 1$

b_1 is within the range of

$-1 < b_1 < 0$

Finally we get

$$
\begin{aligned}
b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\
\alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1}
\end{aligned}
\tag{17}
$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way. Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating 0th and 1st order autocorrelation function.

3 Trend removal method

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function

We set

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function. (a_2, b_2, c_2) and (a_3, b_3, c_3, d_3) are also parameters for a 2nd and a 3rd order non-linear functions which are estimated by using least square method.

[3] The combination of linear and non-linear function.
We set

$$\begin{aligned} y &= \alpha_1(a_1x+b_1)+\alpha_2\left(a_2x^2+b_2x+c_2\right) \\ &\quad +\alpha_3\left(a_3x^3+b_3x^2+c_3x+d_3\right) \end{aligned} \tag{21}$$

$$0 \le \alpha_1 \le 1, 0 \le \alpha_2 \le 1, 0 \le \alpha_3 \le 1, \alpha_1+\alpha_2+\alpha_3=1 \tag{22}$$

as the combination linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the original data by (21). The optimal weighting parameter a_1, a_2, a_3, are determined by utilizing GA. GA method is precisely described in section 6.

4 Monthly ratio

For example, if there is the monthly data of L years as stated bellow:
$\{x_{ij}\}$ $(i=1,2,\ldots,L)$ $(j=1,2,\ldots,12)$

where, $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a production data of i -th year, j -th month. Then, monthly ratio $\tilde{x}_j$ $(j=1,2,\ldots,12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12} x_{ij}} \tag{23}$$

Monthly trend is removed by dividing the data by (23). Numerical examples both of monthly trend removal case and non-removal case are discussed in 7.

5 Forecasting accuracy

Forecasting accuracy is measured by calculating the variance of the forecasting error. Variance of forecasting error is calculated by:

$$\sigma_{\varepsilon}^{2} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{24}$$

Where, forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{26}$$

6 Searching optimal weights utilizing GA

6.1 Definition of the problem

We search a_1, a_2, a_3 of (21) which minimizes (24) by utilizing GA. By (22), we only have to determine a_1 and a_2, σ_e^2 ((24)) is a function of a_1 and a_2, therefore we express them as $\sigma_e^2(a_1, a_2)$. Now, we pursue the following:

Minimize: $\sigma_e^2(a_1, a_2)$

subject to: $0 \le a_1 \le 1,\ \ 0 \le a_2 \le 1 \quad a_1 + a_2 \le 1$ (27)

We do not necessarily have to utilize GA for this problem which has small member of variables. Considering the possibility that variables increase when we use logistics curve etc in the near future, we want to ascertain the effectiveness of GA.

6.2 The structure of the gene

Gene is expressed by the binary system using {0,1} bit. Domain of variable is [0,1] from (22). We suppose that variables take down to the second decimal place. As the length of domain of variable is 1-0=1, seven bits are required to express variables. The binary bit strings <bit6, ～,bit0> is decoded to the [0,1] domain real number by the following procedure [14].

Table 6-1: Corresponding table of the decimal number, the binary number and the real number

The decimal number	The binary number							The Corresponding real number
	Position of the bit							
	6	5	4	3	2	1	0	
0	0	0	0	0	0	0	0	0.00
1	0	0	0	0	0	0	1	0.01
2	0	0	0	0	0	1	0	0.02
3	0	0	0	0	0	1	1	0.02
4	0	0	0	0	1	0	0	0.03
5	0	0	0	0	1	0	1	0.04
6	0	0	0	0	1	1	0	0.05
7	0	0	0	0	1	1	1	0.06
8	0	0	0	1	0	0	0	0.06
…								…
126	1	1	1	1	1	1	0	0.99
127	1	1	1	1	1	1	1	1.00

Procedure 1: Convert the binary number to the binary-coded decimal.

$$\left(\langle bit_6, bit_5, bit_4, bit_3, bit_2, bit_1, bit_0 \rangle\right)_2 = \left(\sum_{i=0}^{6} bit_i 2^i\right)_{10} = X' \tag{28}$$

Procedure 2: Convert the binary-coded decimal to the real number.

The real number
= (Left hand starting point of the domain) (29)
+ X'((Right hand ending point of the domain)/($2^7 - 1$))

The decimal number, the binary number and the corresponding real number in the case of 7 bits are expressed in Table 6-1.

1 variable is expressed by 7 bits, therefore 2 variables needs 14 bits. The gene structure is exhibited in Table 6-2.

Table 6-2: The gene structure

α_1							α_2						
Position of the bit													
13	12	11	10	9	8	7	6	5	4	3	2	1	0
0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1	0-1

6.3 The flow of Algorithm

The flow of algorithm is exhibited in Figure 6-1.

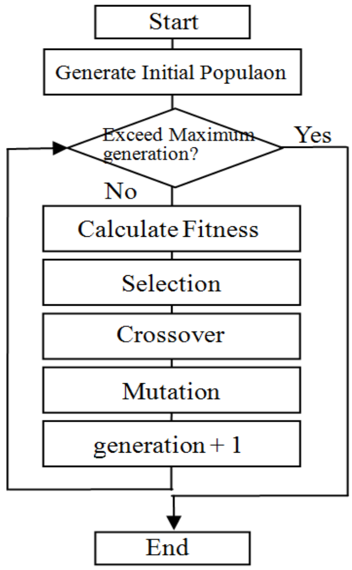

Figure 6-1: The flow of algorithm

A. Initial Population

Generate M initial population. Here, $M = 100$. Generate each individual so as to satisfy (22).

B. Calculation of Fitness

First of all, calculate forecasting value. There are 36 monthly data for each case. We use 24 data (1st to 24th) and remove trend by the method stated in section 3. Then we calculate monthly ratio by the method stated in section 4. After removing monthly trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (24). Calculation of fitness is exhibited in Figure 6-2.

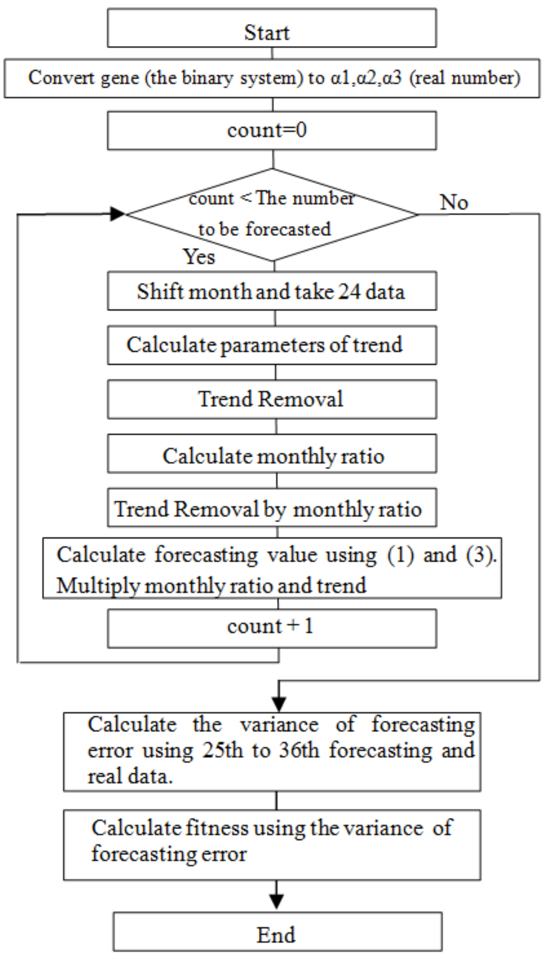

Figure 6-2: The flow of calculation of fitness

Scaling [15] is executed such that fitness becomes large when the variance of forecasting error becomes small. Fitness is defined as follows.

$$f(\alpha_1, \alpha_2) = U - \sigma_\varepsilon{}^2(\alpha_1, \alpha_2) \tag{30}$$

where U is the maximum of $\sigma_e^2(a_1, a_2)$ during the past W generation. Here, W is set to be 5.

C. Selection

Selection is executed by the combination of the general elitist selection and the tournament selection. Elitism is executed until the number of new elites reaches the predetermined number. After that, tournament selection is executed and selected.

D. Crossover

Crossover is executed by the uniform crossover. Crossover rate is set as follows.

$$P_e = 0.7 \tag{31}$$

E. Mutation

Mutation rate is set as follows.

$$P_m = 0.05 \tag{32}$$

Mutation is executed to each bit at the probability P_m, therefore all mutated bits in the population M becomes $P_m \times M \times 14$.

7 Numerical example

7.1 Application to the original production data of processed cooked rice

The original production data of processed cooked rice for 3 cases (Data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice: Annual Report of Statistical Research, Ministry of Agriculture, Forestry and Fisheries, Japan) from January 2008 to December 2010 are analyzed. Furthermore, GA results are compared with the calculation results of all considerable cases in order to confirm the effectiveness of GA approach. First of all, graphical charts of these time series data are exhibited in Figure 7-1 - 7-3.

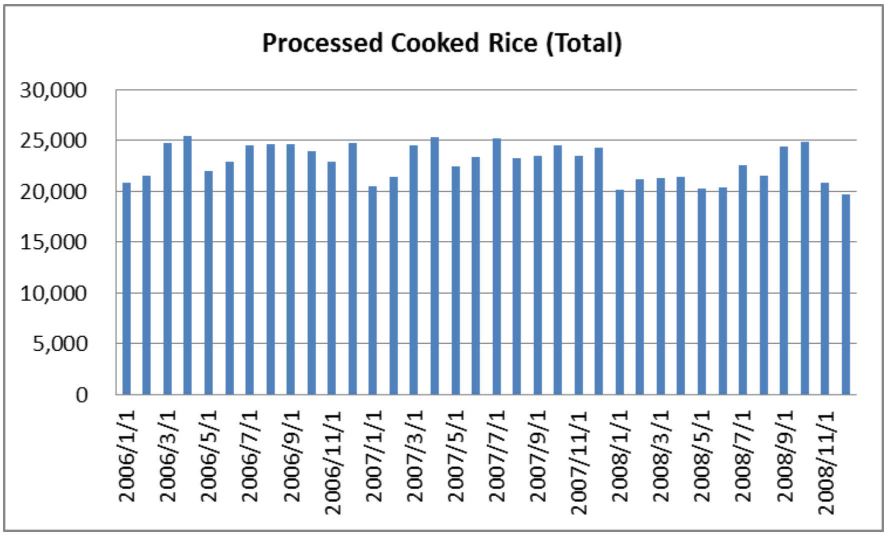

Figure 7-1: Data of Processed Cooked Rice (Total)

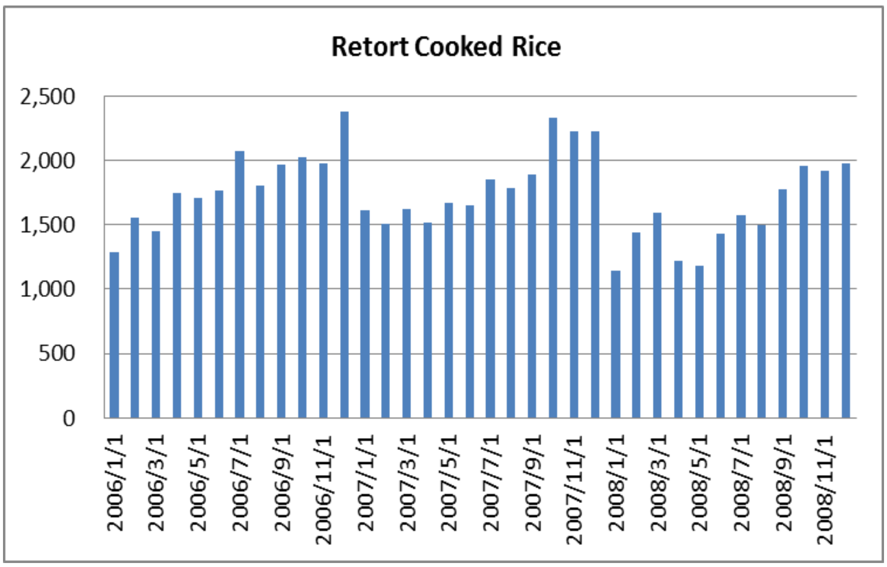

Figure 7-2: Data of Retort Cooked Rice

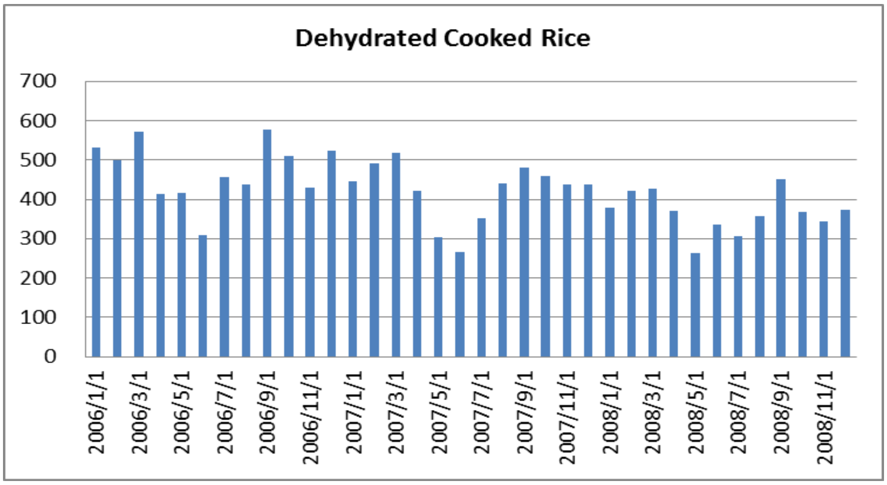

Figure 7-3: Data of Dehydrated Cooked Rice

7.2 Execution Results

GA execution condition is exhibited in Table 7-1.

Table7-1: GA Execution Condition

GA Execution Condition	
Population	100
Maximum Generation	50
Crossover rate	0.7
Mutation ratio	0.05
Scaling window size	5
The number of elites to retain	2
Tournament size	2

We made 10 times repetition and the maximum, average, minimum of the variance of forecasting error and the average of convergence generation are exhibited in Table 7-2 and 7-3.

Table7-2: GA execution results(Monthly ratio is not used)

Food	The variance of forecasting error			Average of convergence generation
	Maximum	Average	Minimum	
Processed Cooked Rice (Total)	6254952.198	4773756.223	4587880.739	10.3
Retort Cooked Rice	216829.0258	155165.7454	148127.8054	15.3
Dehydrated Cooked Rice	4989.709619	3404.034352	3189.945469	16.9

Table7-3: GA execution results(Monthly ratio is used)

Food	The variance of forecasting error			Average of convergence generation
	Maximum	Average	Minimum	
Processed Cooked Rice (Total)	5279063.541	2765813.356	2588932.107	11.1
Retort Cooked Rice	143425.3998	60059.74106	53198.87798	16.9
Dehydrated Cooked Rice	4464.665018	2147.230534	1948.942757	16

The case monthly ratio is used is smaller than the case monthly ratio is not used concerning the variance of forecasting error in every case.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

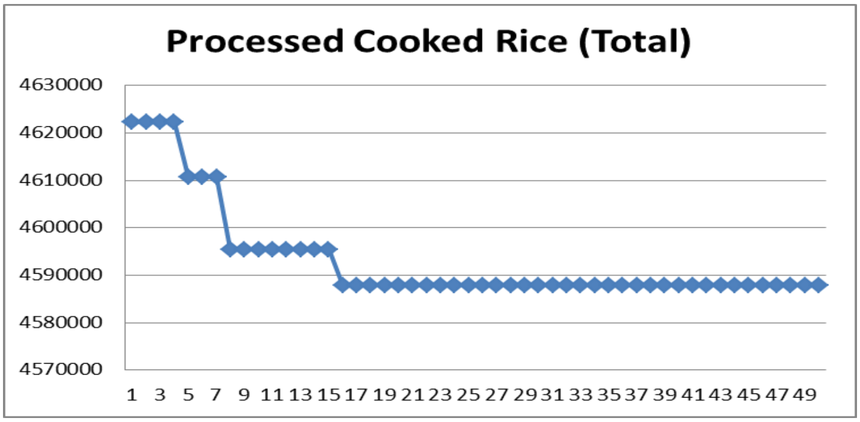

Figure7-4: Convergence Process in the case of
Processed Cooked Rice (Total) (Monthly ratio is not used)

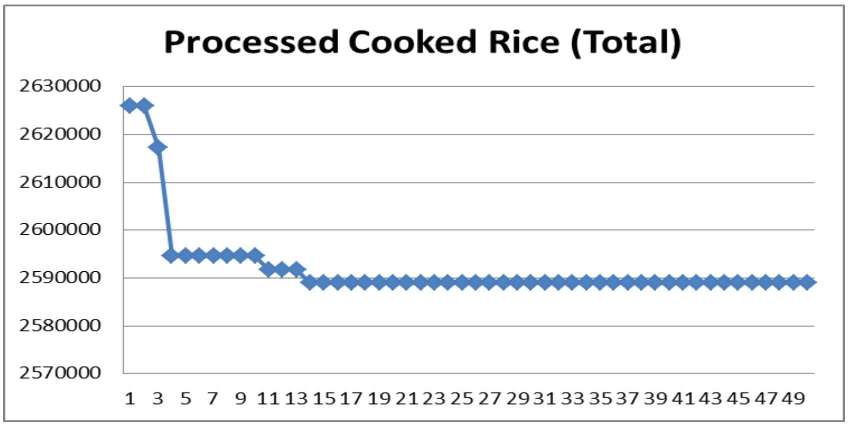

Figure7-5: Convergence Process in the case of
Processed Cooked Rice (Total) (Monthly ratio is used)

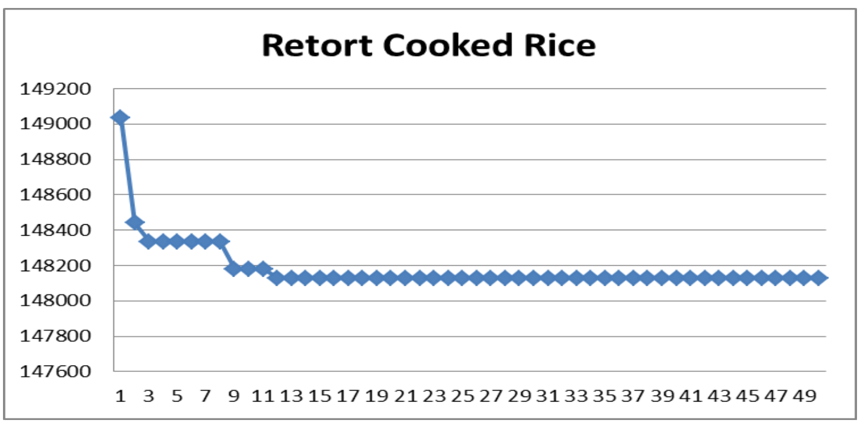

Figure7-6: Convergence Process in the case of
Retort Cooked Rice (Monthly ratio is not used)

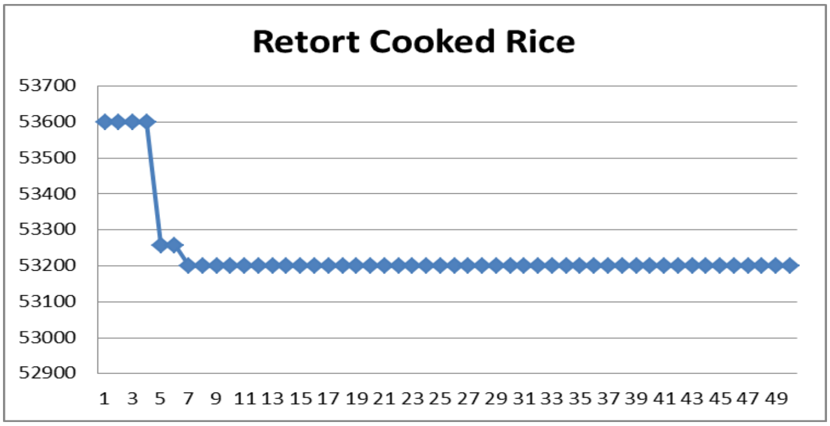

Figure7-7: Convergence Process in the case of
Retort Cooked Rice (Monthly ratio is used)

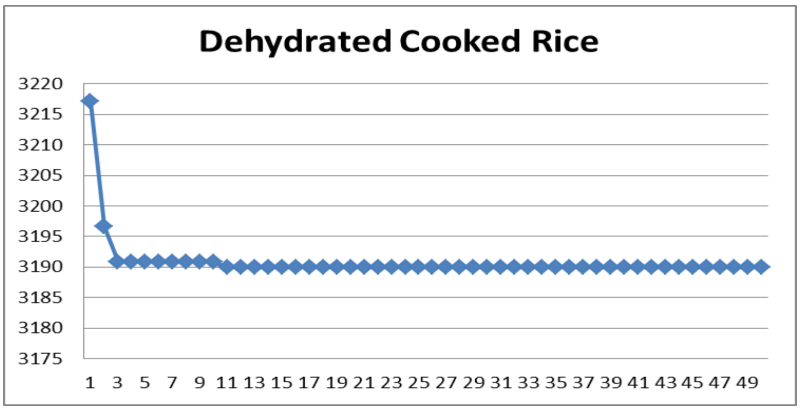

Figure7-8: Convergence Process in the case of
Dehydrated Cooked Rice (Monthly ratio is not used)

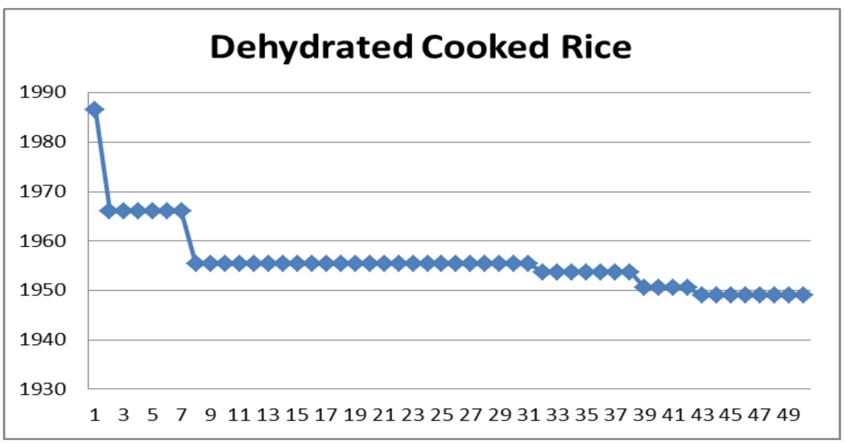

Figure7-9: Convergence Process in the case of
Dehydrated Cooked Rice (Monthly ratio is used)

Next, optimal weights and their genes are exhibited in Table 7-4,7-5.

Table7-4: Optimal weights and their genes (Monthly ratio is not used)

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
Processed Cooked Rice (Total)	1.00	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Retort Cooked Rice	1.00	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Dehydrated Cooked Rice	0.50	0.50	1	0	0	0	0	0	0	0	0	1	1	1	1	1

Table7-5: Optimal weights and their genes (Monthly ratio is used)

Data	α_1	α_2	position of the bit													
			13	12	11	10	9	8	7	6	5	4	3	2	1	0
Processed Cooked Rice (Total)	1.00	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Retort Cooked Rice	1.00	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Dehydrated Cooked Rice	1.00	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0

In the case monthly ratio is not used, the linear function model is best in two cases for Processed Cooked Rice (Total) and Retort Rice, while Dehydrated Rice has chosen linear and 2nd order non-linear function as the best one. In the case monthly ratio is used, the linear function model is best in all cases. Parameter estimation results for the trend of equation (21) using least square method are exhibited in Table 7-6 for the case of 1st to 24th data.

Table7-6: Parameter estimation results for the trend of equation (21)

Data	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
Processed Cooked Rice (Total)	42.32	22999.63	-3.26	123.83	22646.4	1.66	-65.34	757.38	21193.73
Retort Cooked Rice	19.04	1579.91	0.04	18.07	1584.12	0.69	-25.88	282.56	977.67
Dehydrated Cooked Rice	-3.36	489.34	0.21	-8.6	512.04	0.02	-0.67	0.36	491.5

Trend curves are exhibited in Figure 7-10 - 7-12.

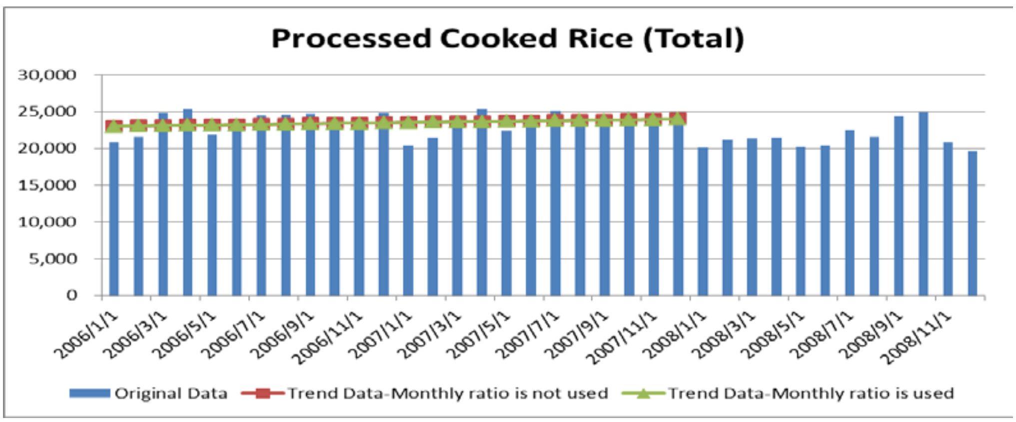

Figure7-10: Trend of Processed Cooked Rice (Total)

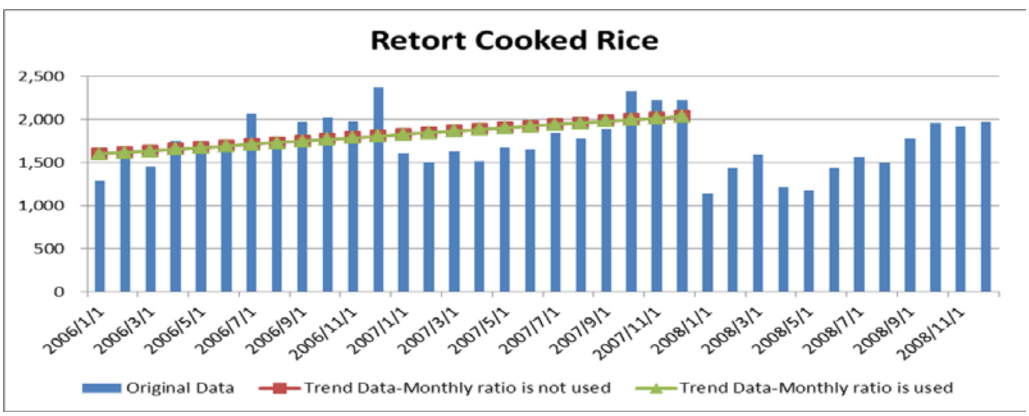

Figure7-11: Trend of Retort Cooked Rice

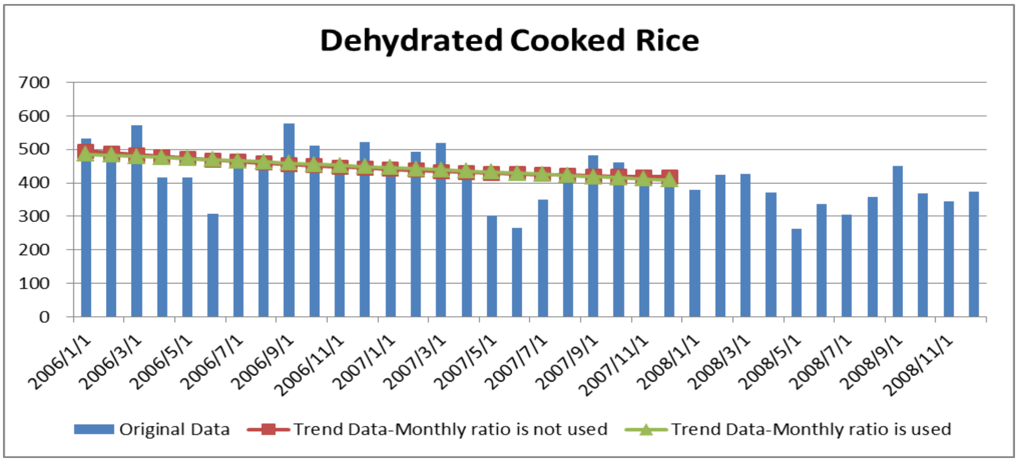

Figure7-12:Trend of Dehydrated Cooked Rice

Calculation results of Monthly ratio for 1st to 24th data are exhibited in Table 7-7.

Table7-7: Parameter Estimation result of Monthly ratio

Data	1	2	3	4	5	6	7	8	9	10	11	12
Processed Cooked Rice (Total)	0.89	0.92	1.05	1.08	0.95	0.98	1.05	1.01	1.02	1.03	0.98	1.03
Retort Cooked Rice	0.84	0.89	0.88	0.93	0.95	0.95	1.08	0.98	1.04	1.15	1.11	1.20
Dehydrated Cooked Rice	1.05	1.07	1.19	0.92	0.80	0.64	0.90	1.00	1.20	1.11	1.01	1.12

Estimation result of the smoothing constant of minimum variance for the 1st to 24th data are exhibited in Table 7-8, 7-9.

Table 7-8: Smoothing constant of Minimum Variance of equation (17) (Monthly ratio is not used)

Data	ρ1	α
Processed Cooked Rice (Total)	-0.210408	0.779348
Retort Cooked Rice	-0.382185	0.535275
Dehydrated Cooked Rice	-0.199611	0.791730

Table 7-9: Smoothing constant of Minimum Variance of equation (17)
(Monthly ratio is used)

Data	ρ1	α
Processed Cooked Rice (Total)	-0.192750	0.799501
Retort Cooked Rice	-0.465967	0.316076
Dehydrated Cooked Rice	-0.327017	0.627642

Forecasting results are exhibited in Table 7-13 - 7-15.

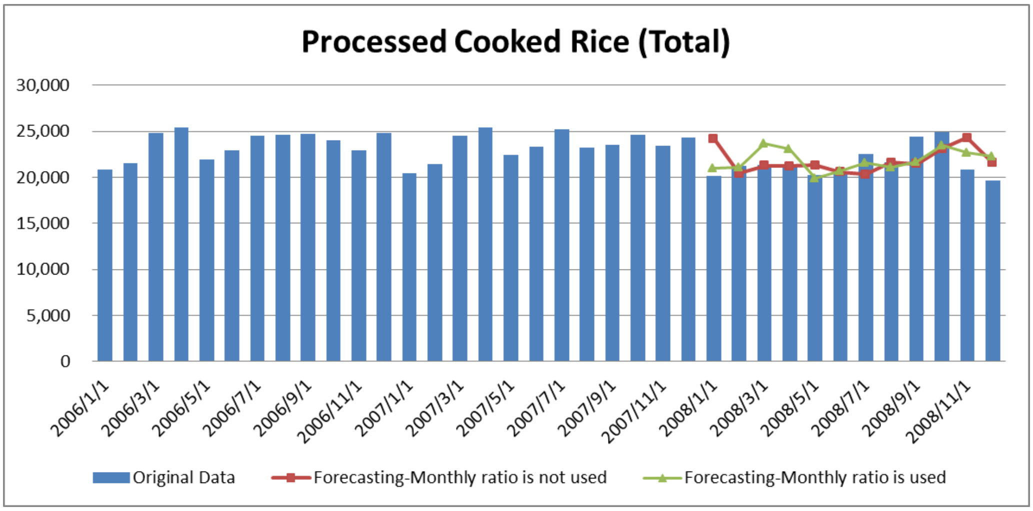

Figure 7-13: Forecasting Result of Processed Cooked Rice (Total)

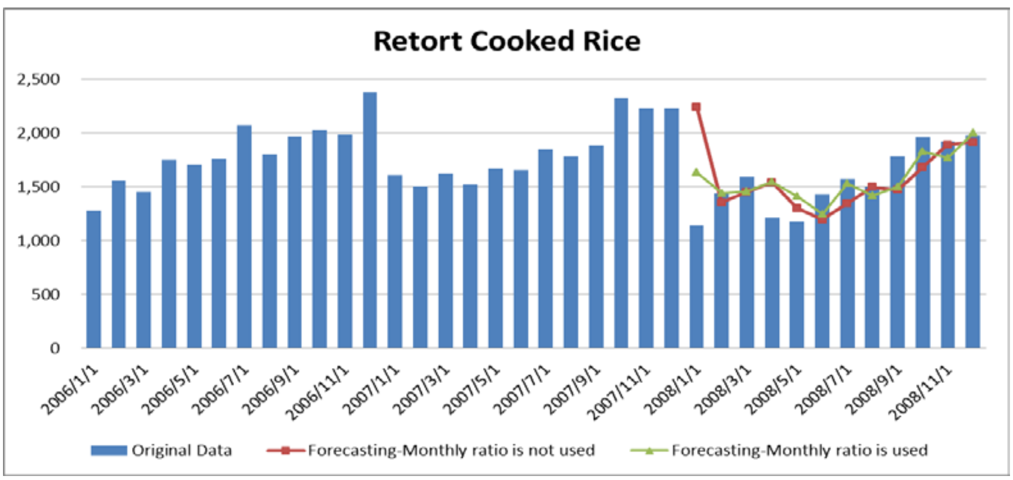

Figure 7-14: Forecasting Result of Retort Cooked Rice

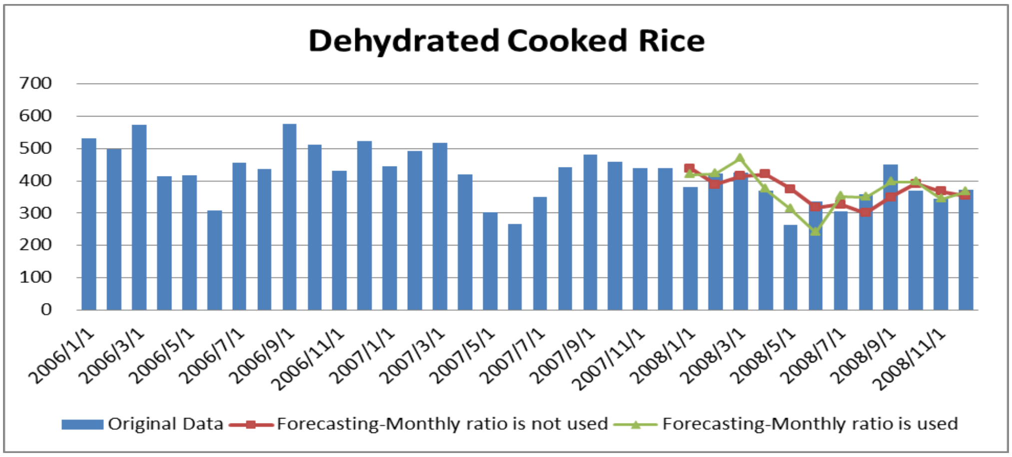

Figure 7-15: Forecasting Result of Dehydrated Cooked Rice

7.3 Remarks

In the case monthly ratio is not used, the linear function model is best in two cases for Processed Cooked Rice (Total) and Retort Rice, while Dehydrated Rice has chosen linear and 2^{nd} order non-linear function as the best one. In the case monthly ratio is used, the linear function model is best in all cases.

The case monthly ratio is used is smaller than the case monthly ratio is not used concerning the variance of forecasting error in every case.

The minimum variance of forecasting error of GA coincides with those of the calculation of all considerable cases and it shows the theoretical solution. Although it is a rather simple problem for GA, we can confirm the effectiveness of GA approach. Further study for complex problems should be examined hereafter.

8 Conclusion

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrary. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the stock market price data of Processed Cooked Rice (Total), Retort Rice and Dehydrated Rice. The combination of linear and non-linear function was also introduced in trend removal. Genetic Algorithm was utilized to search the optimal weight for the weighting parameters of linear and non-linear function. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. The new method shows that it is useful for the time series that has various trend characteristics. The effectiveness of this method should be examined in various cases.

References

[1] Box Jenkins, *Time Series Analysis Third Edition*, Prentice Hall, 1994.

[2] R.G. Brown, *Smoothing, Forecasting and Prediction of Discrete –Time Series*, Prentice Hall, 1963.

[3] Hidekatsu Tokumaru et al., *Analysis and Measurement –Theory and Application of Random data Handling*, Baifukan Publishing, 1982.

[4] Kengo Kobayashi, *Sales Forecasting for Budgeting*, Chuokeizai-Sha Publishing, 1992.

[5] Peter R. Winters, Forecasting Sales by Exponentially Weighted Moving Averages, *Management* Science, **6**(3), (1984), 324-343.

[6] Katsuro Maeda, Smoothing Constant of Exponential Smoothing Method, Seikei University, *Report Faculty of Engineering*, **38**, (1984), 2477-2484.

[7] M. West and P.J. Harrison, *Baysian Forecasting and Dynamic Models*, Springer-Verlag, New York, 1989.

[8] Steinar Ekern, Adaptive Exponential Smoothing Revisited, *Journal of the Operational Research Society*, **32**, (1982), 775-782.

[9] F.R. Johnston, Exponentially Weighted Moving Average (EWMA) with Irregular Updating Periods, *Journal of the Operational Research Society*, **44**(7), (1993), 711-716.

[10] Spyros Makridakis and Robeat L. Winkler, Averages of Forecasts; Some Empirical Results, *Management Science*, 29(9), (1983), 987-996.

[11] Naohiro Ishii et al., Bilateral Exponential Smoothing of Time Series, *Int. J. System Sci*., **12**(8), (1991), 997-988.

[12] Kazuhiro Takeyasu, *System of Production, Sales and Distribution*, Chuokeizai-Sha Publishing, 1996.

[13] Kazuhiro Takeyasu and Kazuko Nagao, Estimation of Smoothing Constant of Minimum Variance and its Application to Industrial Data, *Industrial Engineering and Management Systems*, **7**(1), (2008), 44-50.

[14] Masatosi Sakawa and Masahiro Tanaka, *Genetic Algorithm*, Asakura Pulishing Co., Ltd, 1995.

[15] Hitoshi Iba, *Genetic Algorithm*, Igaku Publishing, 2002.