Journal of Computations & Modelling, vol.3, no.2, 2013, 67-81
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2013

# Research on Minimum Safety Distance in Free Flight Based on CNS Performances

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## Abstract

Free flight is one of effective methods to solve airspace congestion in the future. In order to guarantee safety of flight in free flight environment, the minimum safety distance was studied. Within circumstance that collision avoidance system hasn't started to make TCAS logic judgment to flight nearby, communication, navigation and surveillance (CNS) performances play a decisive role to minimum safety distance. The position errors, which were affected by CNS performances, were regarded as Brownian motion along the coordinate direction respectively. Then a model for collision risk in free flight environment was established basing stochastic differential equations. Minimum safety distance between flights can be obtained using dichotomy to optimize under the given Target Level of Safety (TLS). The example shows that the model is feasible.

**Mathematics Subject Classification:** 60H10

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Article Info: *Received* : April 18, 2013. *Revised* : May 21, 2013
*Published online* : June 20, 2013

**Keywords:** free flight, safety distance, position error, dichotomy, target level of safety

## 1 Introduction

With the rapid development of civil aviation industry, air traffic volumes are increasing and the current air traffic control mode already can't satisfy the needs of the traffic, so Radio Technical Commission for Aeronautics (RTCA) defined the concept of free flight [1]. In free flight environment, the fixed routes and the constraints of air lines will be reduced because of operators having the freedom to choose optimal flight path, speed and altitude in real time. It alleviates airspace tension greatly, but the variability of paths also increases collision risk. Therefore, in order to ensure flight safety, the research on minimum safety distance in free flight environment is particularly important.

The scholars had made achievements on research of free flight, but most of them focused on the collision risk, the conflict detection and resolution. But there was few researches on minimum safety distance in free flight environment. Rick Cassell established the risk assessment model for free flight terminal area reduced separation and the model made quantitative analyze to the relations between separation reduction and collision risk [2]. Daniel established separation minima model and analyzed the effects of factors on safety separation [3]. Mariken modeled lateral spacing and separation for airborne separation assurance using dynamically colored Petri Nets and used Monte Carlo method to simulate [4]. Zhang Zhaoning established collision risk assessment models of parallel route and crossing track based on CNS in current air traffic control mode and assessed safety separation of routes [5-7]. Cai Ming, Zhang Zhaoning established collision risk model in free flight environment using the method of probability theory [8]. Zhang Zhaoning, Zuo Jiangli and Zhang Zhaoning, Wang Yiming established collision risk model based on stochastic differential equations in free flight environment and used Runge-Kutta method [10] and Euler method [9] respectively to solve stochastic differential equations rapidly. All were to strive for collision risk under a certain distance, but there was few researches on minimum safety distance between flights for a given TLS in the actual application.

In free flight environment, collision risk between aircrafts is mainly affected by CNS performances, human factors and collision avoidance system. These uncertainties lead to randomness of the states of flight, and such randomness will affect collision risk and minimum safety distance directly. Stochastic differential equations can integrate the uncertainty factors that affect flight into model and solve the problem of randomness better, so it's suitable for collision risk model in free flight environment. Among the uncertainty factors, the most effective one on position errors is CNS [10]. The author does not consider the influence of human factors on aircraft position error because of complexity of researching. Collision avoidance system is essential to free flight. When the distance between aircrafts is less than $6\,nmile$ and the height difference is less than 1200 ft, collision avoidance system starts to make TCAS logic judgment to the flight nearby. This article takes the influence of CNS on aircraft position error into account and studies minimum safety distance within the circumstance that the collision avoidance system hasn't started to make TCAS logic judgment to the flight nearby.

The references [9-10] just overall general regarded affect of random factors on position error as Brownian motion, this paper regards the influence of CNS performances on position coordinates as Brownian motion along coordinates directions respectively and a model for collision risk in free flight environment was established using stochastic differential equations. Because stochastic differential equation can't be worked out directly, we can obtain the minimum safety distance between flights using dichotomy to optimize under the situation that the Target Level of Safety (TLS) was given.

## 2 Establishment of collision Risk model based on Stochastic Differential Equations in Free Flight

Because civil aircraft maintains a fixed height on route, considering passengers' comfort and fuel consumption when aircraft climbs or descents, aircraft usually prefers to maneuver in the horizontal direction rather than change their vertical height to avoid collision, so we can convert the three-dimensional problem to two-dimensional problem that calculate the minimum safety distance between aircrafts in free flight environment.

In order to establish collision risk model, this paper first regards the influence of CNS performances on position coordinates as Brownian motion along coordinate directions respectively. Then use stochastic differential equations to represent position state of aircrafts. And then study the effect of CNS performances on $\sigma^2$ of Brownian motion variance. Finally a collision risk model is established based on stochastic differential equations.

## 2.1 Stochastic differential equations representing position state of aircrafts

Assume that

*1) Coordinates of aircraft 1 is* $\left(x_1\left(t\right),\ y_1(t)\right)$,

*2) Coordinates of aircraft 2 is* $\left(x_2\left(t\right),\ y_2(t)\right)$,

*3) Initial coordinates of aircraft 1 is* $\left(x_{10,}y_{10}\right)$,

*4) Initial coordinates of aircraft 2 is* $\left(x_{20},y_{20}\right)$,

*5) Airspeed of aircraft 1 is* $\mathrm{V}_1$,

*6) Airspeed of aircraft 2 is* $\mathrm{V}_2$,

*7) Decompositions of* $\mathrm{V}_1$ *along axis directions respectively are* $v_{1x},v_{1y}$,

*8) Decompositions of* $\mathrm{V}_2$ *along axis directions respectively are* $v_{2x},v_{2y}$,

Use stochastic differential equations to represent the position state of two aircrafts at moment $t$ in free flight environment

$$\begin{cases} dx_i\left(t\right)=v_{ix}dt+dw_{ix}\left(t\right),i=1,2 \\ dy_i\left(t\right)=v_{iy}dt+dw_{iy}\left(t\right),i=1,2 \end{cases} \tag{1}$$

$x_i\left(t\right)$ represents abscissa state of aircraft $i$ at moment $t$ and $y_i\left(t\right)$ represents ordinate state of aircraft $i$ at moment $t$. Both $w_{ix}$ and $w_{iy}$ are basic wiener process whose mean value is 0 and variance is $\sigma^2 t$. Probability density function is

$$f\left(w\right)=\frac{1}{\sigma\sqrt{2\pi t}}\exp\left(-\frac{w^2}{2\sigma^2 t}\right)$$

## 2.2 Affect on position error by CNS performances

Assume that position error $X_c$ , $X_n$ , $X_s$ is resulted from communication, navigation and surveillance performance respectively and all met normal distribution.

$$X_c \sim \mathrm{N_c}\left(0,\ \sigma_c^{\ 2}\right)$$

$$X_n \sim \mathrm{N_n}\left(0,\ \sigma_n^{\ 2}\right)$$

$$X_s \sim \mathrm{N_s}\left(0,\ \sigma_s^{\ 2}\right)$$

References [12-14] have already showed RCP(Required communication performances), RNP(Required navigation performances), RSP(Required surveillance performances) types, but they did not point out the influence on position errors variance by CNS performances, then this paper will work out $\sigma_c$ , $\sigma_n$ , $\sigma_s$ .

### 2.2.1 Calculation of $\sigma_n^{\ 2}$

RNP is defined as: when aircraft operates in a certain route, airspace or area, RNP is determined by the value to achieve the expected navigation performance accuracy at least 95% of flight time. The accuracy of navigation has direct affect on position error. Table 1 shows RNP types.

Table 1: RNP types

| Parameter | RNP1 | RNP4 | RNP10 | RNP12.6 | RNP20 |
|---|---|---|---|---|---|
| Accuracy | $\pm 1.0\, nmile$ | $\pm 4.0\, nmile$ | $\pm 10\, nmile$ | $\pm 12.6\, nmile$ | $\pm 20.0\, nmile$ |

Probability density function of position error resulted from RNP is

$$f_n(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{x^2}{2{\sigma_n}^2}\right)$$

From the definition of RNP, we know that

$$\int_{-n}^{n} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{x^2}{2{\sigma_n}^2}\right) dx = 0.95 \quad (2)$$

From formula (2), we can get $\sigma_n = \frac{n}{1.96}$.

### 2.2.2 Calculation of ${\sigma_c}^2$

Reference [12] gave out RCP types and described RCP from processing time, continuity, availability and integrity aspects. We can get that RCP mainly depends on communication processing time, so this paper mainly considers position error caused by processing time. Table 2 shows RCP types.

Table 2: RCP types

| Parameter | RCP10 | RCP60 | RCP120 | RCP240 | RCP400 |
|---|---|---|---|---|---|
| Processing time | 10 | 60 | 120 | 240 | 400 |

Probability density function of position error resulted from RCP is

$$f_c(x) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{x^2}{2{\sigma_c}^2}\right)$$

The definition of RCP is similar to RNP, so

$$\int_{-n'}^{n'} \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{x^2}{2\sigma_c{}^2}\right) dx = 0.95 \qquad (3)$$

$n'$ is the accuracy when processing time is $n$, so $n' = n \times v$.

From formula (3), we can get $\sigma_c = \dfrac{n \times v}{1.96}$.

### 2.2.3 Calculation of $\sigma_s{}^2$

Reference [14] mainly described accuracy (refresh rate and reaction time), continuity, availability and integrity of surveillance performance. The accuracy of surveillance affects position error directly, so this paper mainly takes position error resulted from accuracy into account. With reference to the division method of RNP, table 3 divided RSP into three types according to refresh rate. Table 3 shows RSP types.

Table 3: RSP types

| Parameter | RSP5 | RSP10 | RSP20 |
|---|---|---|---|
| Refresh rate | ≤5 | ≤10 | ≤20 |
| Reaction time | 2 | 2 | 2 |

Probability density function of position error resulted from the accuracy of RSP is

$$f_s(x) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{x^2}{2\sigma_s{}^2}\right)$$

Similar to RNP,

$$\int_{-n''}^{n''} \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{x^2}{2\sigma_s{}^2}\right) dx = 0.95 \qquad (4)$$

$n''$ representing the accuracy of RSP, so

$$n'' = (\text{refresh rate} + \text{reaction time}) \times speed.$$

From formula (4) we can get

$$\sigma_s = \frac{n''}{1.96} = \frac{(\text{refresh rate} + \text{reaction time}) \times speed}{1.96}.$$

## 2.3 Establishment of collision risk model

After period $t$, position coordinates of two aircrafts are:

$$\begin{cases} x_1(t) = x_{10} + v_{1x}t + \Delta w_{1x} \\ y_1(t) = y_{10} + v_{1y}t + \Delta w_{1y} \end{cases}$$

$$\begin{cases} x_2(t) = x_{20} + v_{2x}t + \Delta w_{2x} \\ y_2(t) = y_{20} + v_{2y} + \Delta w_{2y} \end{cases}$$

$\Delta w_{1x} \sim N(0, \sigma_{1x}{}^2 t)$, $\Delta w_{1y} \sim N(0, \sigma_{1y}{}^2 t)$, $\Delta w_{2x} \sim N(0, \sigma_{2x}{}^2 t)$, $\Delta w_{2y} \sim N(0, \sigma_{2y}{}^2 t)$

$$\begin{aligned} \Delta X &= x_2(t) - x_1(t) \\ &= x_{20} - x_{10} + (v_{2x} - v_{1x})t + \Delta w_x \sim \mathrm{N}\left((x_{20} - x_{10}) + (v_{2x} - v_{1x})t, (\sigma_{1x}{}^2 + \sigma_{2x}{}^2)t\right) \\ \Delta Y &= y_2(t) - y_1(t) \\ &= y_{20} - y_{10} + (v_{2y} - v_{1y})t + \Delta w_y \sim \mathrm{N}\left((y_{20} - y_{10}) + (v_{2y} - v_{1y})t, (\sigma_{1y}{}^2 + \sigma_{2y}{}^2)t\right) \end{aligned}$$

The distance between aircrafts is

$$d(t) = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{(x_2(t) - x_1(t))^2 + (y_2(t) - y_1(t))^2}$$

The collision risk $P$ is the probability of $d(t)$ less than $\frac{\lambda_1}{2} + \frac{\lambda_2}{2}$ , so $P = P\{d(t) \le \frac{\lambda_1}{2} + \frac{\lambda_2}{2}\}$. $\lambda_i$ is the average geometric length of aircraft $i$. The collision probability at moment $t$ is

$$P = \int_{-\frac{\lambda_1+\lambda_2}{2}}^{\frac{\lambda_1+\lambda_2}{2}} f_d(d) d_d \tag{5}$$

## 3 Calculation of minimum safety distance

Given CSN performances and initial condition of aircrafts, this paper can analysis collision probability at different time in different distance and also can get minimum safety distance of aircrafts in different TLS.

### 3.1 Dichotomy

The advantages of dichotomy are that error is very small and speed is fast, so this paper uses dichotomy to solve approximate solution of non-linear equation. Assume the accuracy rating is $\varepsilon$ and given TLS is $P_{TLS}$.

The following are steps:

1. Determine the interval $[t_{start},\ t_{end}]$,

2. Seek midpoint $t_{mid}$ of the interval $[t_{start},\ t_{end}]$,

3. Calculate $P_{t_{mid}}$

(1) If $P_{TLS} - P_{t_{mid}} = 0$, $P_{t_{mid}}$ is the required point,

(2) If $P_{TLS} - P_{t_{mid}} \rangle 0$, then let $t_{start} = t_{mid}$,

(3) If $P_{TLS} - P_{t_{mid}} \langle 0$, then let $t_{end} = t_{mid}$,

4. Judge whether accuracy rating reaches $\varepsilon$ : if $\left|P_{t_{start}} \text{-} P_{t_{end}}\right| \langle \varepsilon$, then approximate solution is $P_{t_{start}}$ (or $P_{t_{end}}$),or to repeat steps 2-4.

Finally output the distance which corresponds to approximate solution.

## 3.2 Algorithm of minimum safety distance

Because formula (1) is stochastic differential equation and formula (5) is difficult to solve directly, so it's impossible to solve $d$ from these formulas. This paper calculates $d$ through optimization iteration algorithm. The accuracy rating is $\varepsilon$ and given TLS is $P_{TLS}$. Figure 1 shows algorithm flow chart.

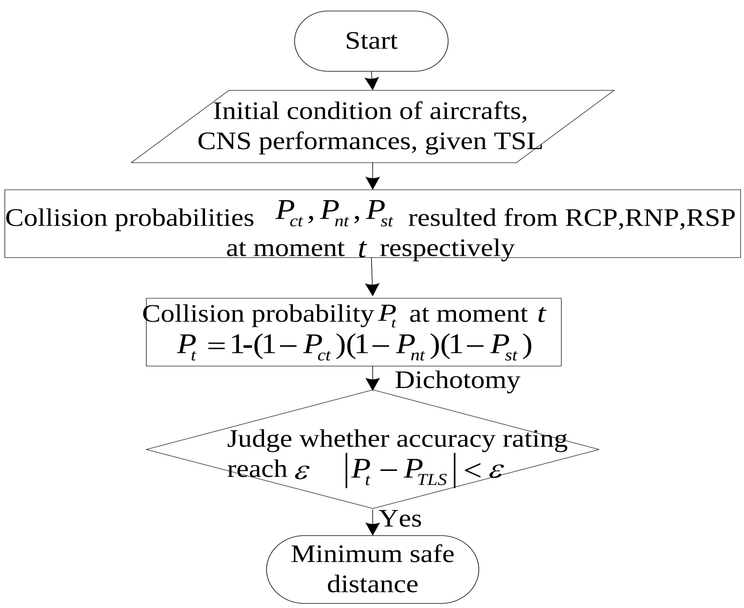


Figure1: Algorithm flow chart

## 4 Numerical example

However, free flight has not been implemented yet, so relevant parameters can't be obtained accurately and this paper references parameters of current flight. Assume in certain airspace, an incrossing event will occur with the increasing of time. With $P_t$ representing collision probability at moment $t$, there will be different minimum safety distance under different collision probability. Parameter values are shown in Table 4.

Table 4: Calculation parameters and their values

| | | | |
|---|---|---|---|
| Airspeed $V_1$ of aircraft 1 | 480 knots | Airspeed $V_2$ of aircraft 2 | 480 knots |
| Heading angel of aircraft 1 | $60^o$ | Heading angel of aircraft 2 | $120^o$ |

Assume that collision probability is $1.5\times10^{-8}$ (times/per flight hour), here we just take CNS performances (RCP10, RNP4, RSP10) as example and calculate the minimum safety distance under different collision probability using MATLAB. Figure 2 shows relations between collision probability and minimum safety distance.

From Figure 2, we can get that minimum safety distance will decrease when collision probability increase between aircraft 1 and aircraft 2 in free flight environment. Minimum safety distance is 8 *nmile* when TSL is $1.5\times10^{-8}$ (times/per flight hour). The result shows that minimum safety distance decreases apparently with the development of CNS performances under a given TSL and this is consistent with actual situation. We also get that collision probability of incrossing aircrafts will increase with the decreasing of distance through MATLAB simulating.

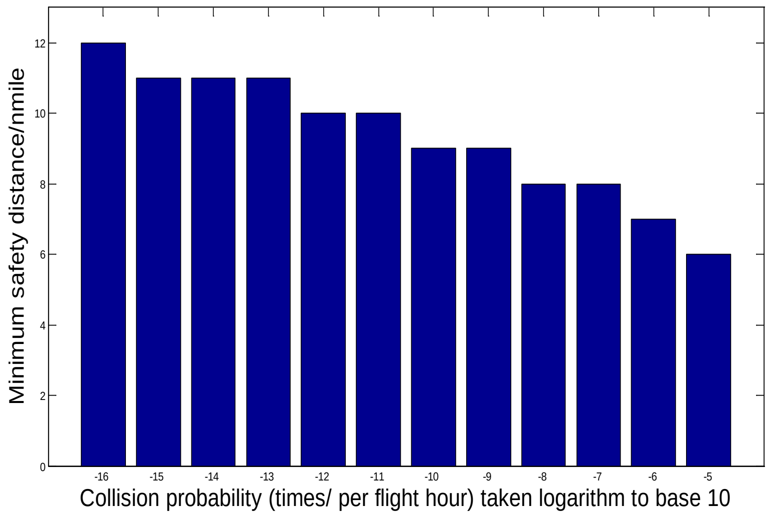


Figure 2: Relations between collision probability and minimum safety distance

## 5 Conclusion

In view of aircraft in free flight environment, position error affected by CNS performances and minimum safety distance are studied. And this paper establishes collision risk model based on stochastic differential equations and uses dichotomy to optimize to get minimum safety distance.

(1) Study the effect of CNS performances on $\sigma^2$ of Brownian motion variance and establish collision risk model using stochastic differential equations.

(2) Optimizing minimum safety distance using dichotomy is fast.

(3) Giving TLS, CSN performances and initial condition of aircrafts, this paper can calculate minimum safety distance between aircrafts.

**ACKNOWLEDGEMENTS.** The authors were supported by the National Natural Science Foundation of China (No.71171190).

## References

[1] Final Report of RTCA Task Force 3-Free Flight Implementation, *RTCA Inc*, Washington DC, (1995).

[2] Rick Cassell, Alex Smith and Roger Shepherd, A Risk Assessment Model for Free Flight Terminal Area Reduced Separation, *Digital Avionics Systems Conference*, 15th AIAA/ IEEE, Atlanta, ( 1996), 73-80.

[3] D.M. Benitez, A.R. Groskreutz and L. Fucke, Separation minima model: How change in Contributing Factors Could Affect Current Standards, *Eighth USA/Europe Air Traffic Management Research and Development Seminar (ATM2009)*, United States, Napa, (2009), 94.

[4] Mariken H.C. Everdij, Henk A.P. Blom and Bert G.A.J. Bakker, Modelling Lateral Separation and Separation for Airborne Separation Assurance Using Petri Nets, *Simulation*, **83**(5), (2003), 401-414.

[5] Zhang Zhaoning, Shen Jinwei and Liu Jimin, Lateral collision risk model based on CNS position error, *Journal of Traffic and Transportation Engineering*, **9**(6), (2009), 110-113.

[6] Zhang Zhaoning, Liu Jimin and Wang Lili, Assessment of Longitudinal

Collision Risk on Parallel Routes Based on Communication, Navigation, and Surveillance Performances, *Journal of Southwest Jiaotong University*, **44**(6), (2009), 918-925.

[7] Liu Jimin, Zhang Zhaoning and Shen Jinwei, Assessment of Lateral Collision Risk on Parallel Routes Based on Communication, Navigation, and Surveillance Performances, *Aeronautical Computing Technique*, **39**(5), (2009), 14-18.

[8] Cai Ming, Zhang Zhaoning and Wang Lili, Research on Collision Risk in Free Flight, *Aeronautical Computing Technique*, **41**(1), (2011), 51-56.

[9] Zhang Zhaoning, Zuo Jiangli, Study on Free Flight Collision Risk Based ON Brownian Motion, *China Safety Science Journal*, **22**(8), (2012), 43-47.

[10] Zhang Zhaoning, Wang Yiming, Research on Collision Risk in Free Flight Based on Fuzzy Stochastic Differential Equations, *China Safety Science Journal*, **22**(10), ( 2012), 14-18.

[11] Zhang Zhaoning and Wang Lili, *Introduction to Security Assessment on Airborne Separation*, Science Press, Beijing, 2009.

[12] Manual on required communication performance (RCP), ICAO, *Doc.9869.AN/462*, (2006).

[13] Manual on required navigation performance (RNP), Montreal, ICAO, Second Edition, *Doc 9613.AN/937*, (1999).

[14] Pavicevic F., Euro control perspectives on required surveillance performance (RSP), ICAO, *WP/05*, (2005).