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Bulk service Markovian queue with service batch size dependent and accessible and non accessible service batches and with vacation

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Abstract

In this paper we consider a bulk service Markovian queue with service batch size dependent and accessible and non accessible service batches and with server's vacation. The initial batch size is assumed to be one and the size of successive batches are governed by Markov chain rule with transition probability matrix $P = p_{ij}$, ($i, j = 1, 2, \ldots, b$). The server starts service if there is at least one customer in the waiting room. Late entries can enter service station without affecting the service time, if the size of the batch being served is less than accessible limit determined by Markov chain rule. In addition after completion of service if there is no customer in queue then the server can avail two types of vacations, multiple vacation (Model I) and single vacation (Model II).

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1 Introduction

A simple queue model of infinite length with waiting room in which the arrival of customers are according to homogenous Poisson process with rate λ to a single server which serves in batches according to Markov chain rule with transition probability matrix $P = p_{ij}$, $i, j = 1, 2, \ldots, \mathrm{b}$. The maximum accessible limit for bulk service is '*b*' and minimum a single customer is enough to start the service. According to Markov chain rule the initial batch size to be served is assumed to be 1 and the next consecutive batches are being served if the size of the batch is 'n_i' where ($1 \leq n_i \leq \mathrm{b}$). Late entries are allowed to join a batch in course of ongoing service as long as batch size is less than that determined by Markov chain rule.

According to the general bulk service rule introduced by Neuts (1967), the server starts service only when a minimum of 'a' customers in the buffer (waiting room) and a maximum service capacity is 'b'. The service rule further allows the later entries to join the batch without affecting the service time, of ongoing service as long as the number of customers in the batch is less than the maximum accessible limit 'd' ($\mathrm{a} \leq \mathrm{d} \leq \mathrm{b}$). The same concept for accessibility in batches while receiving service was discussed by Gross and Harris (1985). Ayyappan and Renganathan (1997) discussed about the service batch size dependence on bulk service Markovian queue.

Queuing model consists of bulk service has been discussed by several authors and the general consideration was if more than 'b' customers are waiting only the first 'b' customers are taken for service and the remaining will have to wait. If the number of customers is less than 'a' then the server remains idle until the batch size reaches 'a'. In this paper the server serves according to Markov chain rule where there is no minimum batch size required and however it has the maximum batch size 'b' ($1 \leq X \leq b$), where x is the number of customer waiting in queue. In addition to that vacation is introduced for server in two models (Model I and Model II) with parameter α.

2 Model I

In this model, after completion of service, the server can avail vacation if there is no succeeding customer in the queue and as soon as the server returns from vacation succeeding monitors the queue size, again if there is no customer in the queue, then the server immediately takes another vacation. Server continues in this manner exponentially until the queue size gets at least a single customer for service.

2.1 Steady State Probability Vector

The process can be formulated as a continuous time Markov chain with state space

$$S = \{(0, 0, k);\ 1 \leq k \leq b\}\ U\ \{(0, j, k);\ 1 \leq j \leq k,\ 1 \leq k \leq b\}$$
$$U\ \{(i, 0, k);\ i \geq 1,\ 1 \leq k \leq b\}\ U\ \{(i, k, k);\ 1 \leq k \leq b\} \quad (1)$$

where i denote number of customer waiting in waiting room at time t, j denotes number of customers being in service at time t, k denotes the maximum accessible limit for the service for customers being in service at time t (determined by the Markov chain rule). If the j = 0 then the server is in vacation otherwise the server is in busy state.

The corresponding generator Q of the Markov process is given by

$$
Q = \begin{array}{c}
\\ \underline{0'} \\ \underline{1'} \\ \underline{2'} \\ \vdots \\ \underline{b'} \\ \underline{1} \\ \underline{2} \\ \underline{3} \\ \vdots \\ \underline{b} \\ \underline{b+1} \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc}
\underline{0'} & \underline{1'} & \underline{2'} & \underline{3'} & \cdots & \underline{b'} & \underline{1} & \underline{2} & \underline{3} & \underline{\cdots} & \underline{b} & \underline{b+1}
\end{array} \\
\left[\begin{array}{cccccccccccc}
B_0 & A_0 & & & & & & & & & & \\
B_{10} & B_{01} & A_{01} & & & & A_{10} & & & & & \\
B_{20} & & B_{02} & A_{02} & & & A_{20} & & & & & \\
\vdots & & & \ddots & & & \vdots & & & & & \\
B_{b0} & & & & & B_{0b} & A_{b0} & & & & & \\
 & N & & & & & A_1 & A_2 & & & & \\
 & & N_{01} & & & & N_{10} & A_1 & A_2 & & & \\
 & & & N_{02} & & & N_{20} & N_{10} & A_1 & A_2 & & \\
 & & & & \ddots & & & & & \ddots & \ddots & \\
 & & & & & N_{0(b-1)} & N_{(b-1)0} & \cdots & & N_{10} & A_1 & A_2 \\
 & & & & & & N_{b0} & N_{(b-1)0} & & & & A_1 \\
 & & & & & & \ddots & \ddots & & & &
\end{array}\right]
\end{array}
$$

where B_0, A_0, B_{10}, B_{20}, . . . , B_{b0}, B_{01}, B_{02}, . . . , B_{0b}, A_{01}, A_{02}, . . . , A_{0b}, A_{10}, A_{20}, . . . , A_{b0} are the sub matrices of orders (b x b) are given as follows:

$A_0 = \lambda I_b$, $B_0 = - \lambda I_b$, where I_b is the identity matrix of order b.

$A_{01} = \text{diag}(0, \lambda, \lambda, \ldots, \lambda)$, $A_{02} = \text{diag}(0, 0, \lambda, \ldots, \lambda)$, . . . ,

$A_{0(b-1)} = \text{diag}(0, 0, \ldots, 0, \lambda)$,

$A_{10} = \text{diag}(\lambda, 0, \ldots, 0)$, $A_{20} = \text{diag}(0, \lambda, 0, \ldots, 0)$, . . . ,

$A_{b0} = \text{diag}(0, 0, \ldots, 0, \lambda)$,

$B_{01} = \text{diag}(0, -\lambda-\mu, -\lambda-\mu, \ldots, -\lambda-\mu)$, $B_{02} = \text{diag}(0, 0, -\lambda-\mu, \ldots, -\lambda-\mu)$, . . . ,

$B_{0b} = \text{diag}(0, 0, \ldots, 0, -\lambda-\mu)$,

$$B_{10} = \begin{bmatrix} \mu\, p_{1,1} & \mu\, p_{1,2} & \cdots & \mu\, p_{1,b} \\ \mu\, p_{1,1} & \mu\, p_{1,2} & \cdots & \mu\, p_{1,b} \\ \vdots & \vdots & & \vdots \\ \mu\, p_{1,1} & \mu\, p_{1,2} & \cdots & \mu\, p_{1,b} \end{bmatrix}, \quad B_{20} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mu\, p_{2,1} & \mu\, p_{2,2} & \cdots & \mu\, p_{2,b} \\ \vdots & \vdots & & \vdots \\ \mu\, p_{2,1} & \mu\, p_{2,2} & \cdots & \mu\, p_{2,b} \end{bmatrix},$$

$$\ldots, B_{b0} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \mu\, p_{b,1} & \mu\, p_{b,2} & \cdots & \mu\, p_{b,b} \end{bmatrix}$$

and A_1, A_2, N, N_{01}, N_{02}, . . . , $N_{0(b-1)}$, N_{10}, N_{20}, . . . , N_{b0} are the sub matrices of order (2b x 2b) are given as follows:

$$A_1 = -(\lambda + \mu)\, I_{2b}, \qquad A_2 = \lambda I_{2b},$$

$$N = \begin{bmatrix} \alpha & & & \\ & \alpha & & \\ & & \ddots & \\ & & & \alpha \\ \mu\, p_{1,1} & \mu\, p_{1,2} & \cdots & \mu\, p_{1,b} \\ \mu\, p_{2,1} & \mu\, p_{2,2} & \cdots & \mu\, p_{2,b} \\ \vdots & \vdots & & \vdots \\ \mu\, p_{b,1} & \mu\, p_{b,2} & \cdots & \mu\, p_{b,b} \end{bmatrix}, \quad N_{01} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \alpha & & \\ \vdots & & \ddots & \\ & & & \alpha \\ 0 & \mu\, p_{1,2} & \cdots & \mu\, p_{1,b} \\ 0 & \mu\, p_{2,2} & \cdots & \mu\, p_{2,b} \\ \vdots & \vdots & & \vdots \\ 0 & \mu\, p_{b,2} & \cdots & \mu\, p_{b,b} \end{bmatrix},$$

$$\ldots, N_{b0} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ & & & \alpha \\ 0 & 0 & \cdots & \mu\, p_{1,b} \\ 0 & 0 & \cdots & \mu\, p_{2,b} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \mu\, p_{b,b} \end{bmatrix}$$

$$N_{10}=\begin{bmatrix} 0 & 0 & \cdots & 0 & \alpha & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ & & & & 0 & & & \\ 0 & 0 & \cdots & 0 & \mu\, p_{1,1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \mu\, p_{2,1} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \mu\, p_{b,1} & 0 & \cdots & 0 \end{bmatrix},$$

$$N_{20}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \alpha & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ & & & & 0 & & & \\ 0 & 0 & \cdots & 0 & 0 & \mu\, p_{1,2} & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \mu\, p_{2,2} & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \mu\, p_{b,2} & \cdots & 0 \end{bmatrix}$$

$$N_{b0}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ & & & & 0 & & & \alpha \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \mu\, p_{1,b} \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \mu\, p_{2,b} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \mu\, p_{b,b} \end{bmatrix}$$

For N_{10}, N_{20}, . . . , N_{b0} have elements only at $(b + k)^{th}$ column only remaining elements are zero, where $1 \le k \le b$. The infinitesimal generator Q of the infinite state Markov model under consideration has the block partitioned structure given by

$\underline{0}$' = {(0, 0, 1), (0, 0, 2), . . . , (0, 0, b)}, $\underline{1}$' = {(0, 1, 1), (0, 1, 2), . . . , (0, 1, b)},
$\underline{2}$' = { -,(0, 2, 2), (0, 2, 3), . . . , (0, 2, b)}, . . . , $\underline{b}$' = {-, -, . . . , -, (0, b, b)}.

where '-'indicates impossible events. They are introduced to tackle the dimensionality problem.

i = {(i, 0, 1), (i, 0, 2), . . . , (i, 0, b), (i, 1, 1), (i, 2, 2), . . . , (i, b, b)}, $i \geq 1$.

The vector $\underline{X}$ of steady state probability associated with Q, then

$$\underline{X}\,Q = \underline{0} \quad \text{and} \quad \underline{X}\,\underline{e} = 1. \tag{2}$$

where $\underline{e} = (1,1, \ldots, 1)^T$. Now partition X as

$$\underline{X} = (X_{001}, X_{002}, \ldots, X_{00b}, X_{011}, X_{012}, \ldots, X_{01b}, X_{022}, X_{023}, \ldots, X_{02b}, \ldots, X_{0bb}, X_{i01}, X_{i02}, \ldots, X_{i0b}, X_{i11}, X_{i22}, \ldots, X_{ibb}). \tag{3}$$

Following Neuts (1981), we examine the existence of a solution of the form

$$\underline{X}_i = \underline{X}_1 R^{i-1}, \quad \text{for} \quad i \geq 1 \tag{4}$$

Deriving

$$A_0 + RA_1 + R^2 N_{10} + \cdots + R^{b+1} N_{b0} = 0 \tag{5}$$

by using the matrix Q in the equation (2).

It may be noted that $A_0 + A_1 + N_{10} + \cdots + N_{b0}$ is reducible. The equation (5) can be written as

$$R = A_0 A_1^{-1} - R^2 N_{10} A_1^{-1} - \cdots - R^{b+1} N_{b0} A_1^{-1} \tag{6}$$

The matrix R can be computed using the recurrence relation (6) with $R(0) = 0$ and for i, j = 1, 2, . . . , b. The $(i, j)^{th}$ element of R is

$$R_{i,j} = \frac{\lambda}{\lambda+\mu}\delta_{i,j} + \frac{\mu}{\lambda+\mu}\sum_{j=1}^{b} R_{ij}^2 p_{ij} + \frac{\mu}{\lambda+\mu}\sum_{j=1}^{b} R_{ij}^3 p_{ij} + \ldots + \frac{\mu}{\lambda+\mu}\sum_{j=1}^{b} R_{ij}^{b+1} p_{ij} \tag{7}$$

A necessary and sufficient condition for the stability of the system can be obtained as follows:

The expected number of customers taken for service in a batch (assuming that at least the number of customers determined by the Markov chain rule is available) is given by $\sum_{i=1}^{b}\sum_{j=1}^{b} jp_{i,j}$. Hence the service rate is $\mu\sum_{i=1}^{b}\sum_{j=1}^{b} jp_{i,j}$. If the average number λ of arrivals per unit time is less than the service rate then the system is stable. Therefore the system is stable if and only if the traffic intensity is less than one or equivalently

$$\frac{\lambda}{\mu} < \sum_{i=1}^{b}\sum_{j=1}^{b} jp_{i,j} \tag{8}$$

under this condition $\underline{X}_k \to 0$ as $k\to\infty$, and so the special radius of R must be less than one. In the case of $M/M^{1,\ b}/1/\infty$ queue with accessible batch service with server vacation model without the Markov dependence of the service batch sizes, we have the stability condition $\frac{\lambda}{\mu} < b$. Thus the problem under the consideration includes $M/M^{1,\ b}/1/\infty$ queue with accessible batch service with server vacation model as a particular case.

To find ($\underline{X}_0$', $\underline{X}_1$', . . . , $\underline{X}_b$', $\underline{X}_1$') by defining Q* as

$$Q^* = \begin{bmatrix} B_0 & A_0 & 0 & & \cdots & 0 \\ B_{10} & B_{01} & A_{01} & & \cdots & A_{10} \\ B_{20} & 0 & B_{02} & A_{02} & \cdots & A_{20} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ B_{b0} & 0 & 0 & & \cdots B_{0b} & \\ 0 & N & RN_{01} & R^2N_{02} & \cdots R^{b-1}N_{0(b-1)} & B_{01}+RN_{10}+R^2N_{20}+\cdots+R^bN_{b0} \end{bmatrix}$$

It can be proved that $Q^*\underline{e} = \underline{0}$ where e is the column vector of appropriate dimension with all the elements equal to 1.

Let $\underline{X}^*$ = ($\underline{X}_0$', $\underline{X}_1$', . . . , $\underline{X}_b$',$\underline{X}_1$') be a solution of $\underline{X}^*Q^* = \underline{0}$.

Then the below equation obtain from above relation,

$$\underline{X}_0' B_0 + \underline{X}_1' B_{10} + \underline{X}_2' B_{20} + \ldots + \underline{X}_b' B_{b0} = 0$$

$$\underline{X}_0' A_0 + \underline{X}_1' B_{01} + \underline{X}_1 N = 0$$

$$\ldots \quad \ldots \quad \ldots$$

$$\underline{X}_1' A_{10} + \underline{X}_2' A_{20} + \ldots + \underline{X}_1(B_{01} + RN_{10} + R^2N_{20} + \ldots + R^bN_{b0}) = 0 \tag{9}$$

The vectors $\underline{X}_0$', $\underline{X}_1$', . . . , $\underline{X}_b$' can be expressed in terms of $\underline{X}_1$ using the above set of equations and $\underline{X}_1$ can be normalized using

$$\sum_{i=0}^{b} \underline{X}_i \underline{e_1} + \underline{X}_1 (I-R)^{-1} \underline{e_1} = 1 \tag{10}$$

where $\underline{e}_1 = (1, 1, \ldots, 1)^T$ is a b – component column vector.

2.2 Optimality problem

The cost related to waiting time of customers is considered to search for the Markov chain rule that will minimize the cost from among a given set of such Markov chain rules.

Cost due to waiting time of customers while the server is in vacation:

The number of customers waiting could be 0, 1, 2, . . . , and the steady state probability are $\underline{X}(0)$, $\underline{X}(1)$, $\underline{X}(2)$, . . . , where

$$\underline{X}(0) = \sum_{i=1}^{b} x_{(0,0,i)} + \sum_{i=1}^{j} x_{(0,i,j)}\,, \quad \underline{X}(n) = \sum_{j=1}^{b} x_{(n,j,j)}\ ; \quad n = 1, 2, 3, \ldots$$

The expected number of customer waiting for service is based on the number available in the waiting room number of arrivals at the time of service or server is in vacation and the number determined by Markov chain rule. Let n be the number of customer waiting in waiting room of the system, K be the number of arrivals at the time of service and j be the number determined by the Markov chain rule (with probability p_{ij}). Then the number of waiting is $(n + k + j)^+$ with probability $\underline{X}(n + k)$. Therefore the expected number of waiting customer

$$= \sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{j=1}^{b}\sum_{i=1}^{b} (n+k+j)^+ p_{i,j} \underline{X}(n+k)$$

Let C be the cost per unit associated with the waiting time. Therefore expected cost of waiting customer per unit time

$$= C\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{j=1}^{b}\sum_{i=1}^{b} \frac{(n+k+j)^+}{\mu} p_{i,j} \underline{X}(n+k) \tag{11}$$

3 Model II

It is more similar to model I and the only difference is a server in model II takes only a single vacation at a time. When the server returns to the main system, server starts service immediately if there is X (X ≥ 1) customers in the queue. If there is no customer in the queue, then the server waits until the queue reaches X ≥ 1.

3.1 Steady State Probability Vector

The process can be formulated as a continuous time Markov chain with state space

$$S = \{(0, 0, k, 0); 1 \le k \le b\} \cup \{(0, 0, k, I); 1 \le k \le b\}$$
$$\cup \{(0, j, k); 1 \le j \le k, 1 \le k \le b\}$$
$$\cup \{(i, 0, k); i \ge 1, 1 \le k \le b\} \cup \{(i, k, k); 1 \le k \le b\} \qquad (12)$$

all the notations are same as the first model but the difference is (0, 0, k, 0) denotes the server is in vacation and (0, 0, k, I) denotes the server stay idle in the service station. The technique used for the analysis of Model I is successfully applied for the above described Model II and the results are verified below.

4 Numerical study for Model I and Model II

By Theorem 1 of Latouche and Neuts (1980), R is the limit of the sequence of matrices $\{R(n)\}$, $n \ge 0$ defined by

$$R(0) = 0$$

$$R(n+1) = A_0 A_1^{-1} - R^2(n) N_{10} A_1^{-1} - \cdots - R^{b+1}(n) N_{b0} A_1^{-1}; \quad n \ge 0 \qquad (13)$$

For illustration, let us choose the parameter values as $\lambda = 1$, $\mu = 1$, $b = 5$, C = 1000, α = 2 and the 10 different Markov chain rules; the cost given in the following Table 1 is minimum for the Markov chain rule MC_5.

$$MC_1 = \begin{bmatrix} 0.05 & 0.20 & 0.30 & 0.25 & 0.20 \\ 0.15 & 0.10 & 0.25 & 0.35 & 0.15 \\ 0.20 & 0.15 & 0.15 & 0.30 & 0.20 \\ 0.30 & 0.25 & 0.20 & 0.15 & 0.10 \\ 0.35 & 0.30 & 0.20 & 0.10 & 0.05 \end{bmatrix} MC_2 = \begin{bmatrix} 0.05 & 0.15 & 0.40 & 0.30 & 0.10 \\ 0.10 & 0.20 & 0.30 & 0.35 & 0.05 \\ 0.15 & 0.20 & 0.30 & 0.30 & 0.05 \\ 0.10 & 0.15 & 0.35 & 0.35 & 0.05 \\ 0.05 & 0.15 & 0.30 & 0.40 & 0.10 \end{bmatrix}$$

$$MC_3 = \begin{bmatrix} 0.03 & 0.12 & 0.40 & 0.35 & 0.10 \\ 0.05 & 0.10 & 0.45 & 0.35 & 0.05 \\ 0.02 & 0.08 & 0.40 & 0.40 & 0.10 \\ 0.07 & 0.10 & 0.43 & 0.35 & 0.05 \\ 0.04 & 0.06 & 0.50 & 0.38 & 0.02 \end{bmatrix} MC_4 = \begin{bmatrix} 0.02 & 0.08 & 0.40 & 0.43 & 0.07 \\ 0.03 & 0.07 & 0.45 & 0.40 & 0.05 \\ 0.04 & 0.09 & 0.42 & 0.41 & 0.04 \\ 0.05 & 0.07 & 0.43 & 0.42 & 0.03 \\ 0.06 & 0.09 & 0.45 & 0.39 & 0.01 \end{bmatrix}$$

$$MC_5 = \begin{bmatrix} 0.03 & 0.12 & 0.38 & 0.39 & 0.08 \\ 0.01 & 0.14 & 0.37 & 0.36 & 0.12 \\ 0.02 & 0.11 & 0.36 & 0.39 & 0.12 \\ 0.03 & 0.15 & 0.35 & 0.37 & 0.10 \\ 0.02 & 0.14 & 0.36 & 0.39 & 0.09 \end{bmatrix}$$

$$MC_6 = \begin{bmatrix} 0.01 & 0.08 & 0.47 & 0.39 & 0.05 \\ 0.02 & 0.07 & 0.50 & 0.40 & 0.01 \\ 0.01 & 0.06 & 0.60 & 0.30 & 0.03 \\ 0.03 & 0.06 & 0.55 & 0.34 & 0.02 \\ 0.03 & 0.05 & 0.68 & 0.21 & 0.03 \end{bmatrix}$$

$$MC_7 = \begin{bmatrix} 0.02 & 0.12 & 0.70 & 0.13 & 0.03 \\ 0.03 & 0.14 & 0.68 & 0.11 & 0.04 \\ 0.01 & 0.10 & 0.78 & 0.10 & 0.01 \\ 0.04 & 0.09 & 0.73 & 0.11 & 0.03 \\ 0.03 & 0.10 & 0.75 & 0.09 & 0.03 \end{bmatrix}$$

$$MC_8 = \begin{bmatrix} 0.03 & 0.18 & 0.58 & 0.20 & 0.01 \\ 0.05 & 0.15 & 0.60 & 0.13 & 0.07 \\ 0.04 & 0.14 & 0.62 & 0.12 & 0.08 \\ 0.07 & 0.13 & 0.61 & 0.14 & 0.05 \\ 0.06 & 0.12 & 0.62 & 0.15 & 0.05 \end{bmatrix}$$

$$MC_9 = \begin{bmatrix} 0.05 & 0.26 & 0.24 & 0.36 & 0.09 \\ 0.09 & 0.24 & 0.30 & 0.29 & 0.08 \\ 0.07 & 0.26 & 0.31 & 0.28 & 0.08 \\ 0.06 & 0.25 & 0.35 & 0.27 & 0.07 \\ 0.08 & 0.28 & 0.29 & 0.29 & 0.06 \end{bmatrix}$$

$$MC_{10} = \begin{bmatrix} 0.18 & 0.22 & 0.21 & 0.19 & 0.20 \\ 0.19 & 0.21 & 0.22 & 0.20 & 0.18 \\ 0.17 & 0.20 & 0.22 & 0.22 & 0.19 \\ 0.19 & 0.21 & 0.21 & 0.21 & 0.18 \\ 0.16 & 0.23 & 0.20 & 0.21 & 0.20 \end{bmatrix}$$

Table 1: Cost comparison

Markov chain rule	Non - accessible service sysyetm			Accessible service sysyetm			Multiple vacation		Single vacation	
	Expected waiting	Expected idleness of server	Total cost	Expected waiting	Expected idleness of server	Total cost	Expected waiting	Total cost	Expected waiting	Toatl cost
MC1	1.124397	0.304129	1276.46	0.312549	0.445146	535.12	0.352395	352.4	0.47184	471.84
MC_2	0.844376	0.307786	998.27	0.235529	0.446597	458.83	0.184745	184.75	0.25002	250.02
MC_3	0.616233	0.314698	773.58	0.134773	0.459292	364.42	0.102929	102.92	0.14368	143.68
MC_4	0.597535	0.315333	755.2	0.12359	0.461913	354.55	0.09321	93.21	0.13303	133.03
MC_5	0.572538	0.315777	730.42	0.119317	0.461494	350.06	0.089758	89.75	0.12674	126.74
MC_6	0.58198	0.315741	739.85	0.119596	0.462168	350.68	0.090387	90.38	0.12654	126.54
MC_7	0.746799	0.311091	902.34	0.19203	0.451226	417.64	0.147389	147.38	0.19717	197.17
MC_8	0.804514	0.309569	959.3	0.217808	0.448303	441.96	0.178726	178.73	0.24005	240.05
MC_9	0.875394	0.307672	1029.23	0.249102	0.445352	471.78	0.19371	193.71	0.24866	248.66
MC_{10}	1.124075	0.30093	1274.54	0.373551	0.431193	589.15	0.308674	308.67	0.39597	395.97

By comparing all the total costs in the above table the total cost for expected waiting cost while the server is in multiple vacations is less than the other costs.

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