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Investigating the Effect of Capital Flight on the Economy of a Developing Nation via the NIG Distribution

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Abstract

The effect of the abnormal capital outflow (capital flight) on the economy of a developing nation is investigated herein via a class of distribution known as the Normal Inverse Gaussian (NIG) distribution. The capital outflow is first modeled as an infinitely divisible process to adapt to the operational time which is an important characteristic of the NIG. Data of capital flight from 1973-1989; source from the IFS (1990) year book fitted to the NIG as illustrative test of the model.

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1 Introduction

The development and economy growth in each country has direct relation with the investments done in that country. So that in the developing country, capital formation is considered as an important factor in economic growth and making the process for development ([1]).

There is relative importance for good capital formation in any country. For example stock exchange is an official organization for capital in the developed countries. It attracts investment and leads to increase in economic growth. However in the developing country with bad capital organization or bad investment policy the capital “runs away” or “flees” from such country. This abnormal movement of capital is refers to as “Capital flight”.

The term capital flight connotes the illegal movement of capital from one country to another. This connotation implies that there may be “normal” or “legal” and “abnormal” or “illegal “flows [2]. Abnormal capital flows are those which are not sanction by the government. The question of the legality of capital flows, then implies that the country in question imposes exchange or capital controls.

So far attempts have been made to measure this abnormal outflow. [3], [4], [5], [6],[7], [8], etc are all attempts to measure capital flight from the developing nations to more developed ones.

Our aim however in this paper is to investigate instead the effect of capital flight in the economy of a developing nation. We first consider the capital outflow process as a Brownian motion process with drift that evolves according to an operational time instead of the physical time (this is an important characteristic of the generalized Hyperbolic processes).

Applications of NIG have been reported in several papers since the introduction by [9]. [10] applied this distribution to finance. [11] and [12] applied it to financial risk management .[13]used the NIG to price synthetic Collateralized Debt Obligations (CDO). This paper applies same model as a tool to investigate the effect in future, the economy of a developing nation with poor financial policy, leading to capital flight. Capital flight data from 1972-1989 drawn mainly from the IFS (1990) year book are fitted for the Normal Inverse Gaussian distribution.

2 The mathematical formulation

To investigate the effect of capital flight on developing nation, the first approach is to recognize that capital flight is speculative capital (Hot money fund on the wind).This speculative capital responds to various forms of losses.

Taking this approach means that capital flight refers essentially to capital export by the private non-bank sector. Also in some cases, banks and some official non entities may engage in it. Since capital flight is essentially concealed, it shows up in the errors and omissions of balance of payment entry. We therefore define capital flight as the sum of short-term private capital flows with errors and omissions. Let the capital flows at time t with errors and omissions be L_t and L_s respectively.

Denote therefore as N(t) the cumulative loss on a given outflow and C(t) the cumulative loss on reference portfolio, then

$$N(t) = \begin{cases} 0 & , \text{ if } C(t) \le L_t \\ C(t) - L_t & , \text{ if } L_t \le C(t) \le L_s \\ L_s - L_t & , \text{ if } C(t) \ge L_t \end{cases} \tag{1}$$

The determination of the incurred capital loss N(t) is therefore the essential part in order to calculate the loss in private capital flow and loss in reference portfolio, which in turn affects the growth rate of the economy of a nation under poor economic policy. The process

$$N = \{N(t)\}_{t \ge 0} \tag{2}$$

is a Levy process with independent and stationary increment which is F_t measurable and hence connected to the infinitely divisible laws. It is reasonable to relate this cumulative loss to the Levy process through

$$\frac{dN(t)}{N(t)} = \alpha dt + \sigma dW_t \tag{3}$$

where W_t is a standard Brownian motion on a probability space $\{\Omega, F, p\}$. F is a σ- algebra generated by W_t , $t \geq 0$. Let the economy of the developing country, X_t , follows a diffusion process given by (with time suppressed)

$$dX = HdN + r(X - HN)dt \tag{4}$$

Here the financial policy of the nation is defined by an adapted process H_t , $t \geq 0$ representing the proportion of incurred capital loss at time t. The remaining $(1 - H_t)$ proportion represents the growth rate of the developing nation's economy. Putting (3) into (4) gives

$$dX = [rX + HN(\alpha - r)]dX + HN\sigma dW \tag{5}$$

Where α-r is the risk premium and r the risk free rate of returns.

$$G(t) = V(k, t) \tag{6}$$

would track the value of the economy of the developing nation if H (the financial policy) is a replicating.

Ito's formula on (6) gives [14]

$$\frac{dV}{dt} + \frac{1}{2}\sigma^2 N^2 \frac{d^2V}{dN^2} + rN\frac{dV}{dk} - rV = 0 \tag{7}$$

Let

$$u = \frac{\sqrt{2}}{\sigma} N \Rightarrow N = \frac{\sigma}{\sqrt{2}} u \tag{8}$$

then (7) becomes

$$u^2 \frac{d^2V}{dN^2} + u\frac{dV}{du} + (u^2 - \alpha^2)V = N \tag{9}$$

where r is now a quadratic function of u, $\tilde{\alpha} = \frac{\alpha\sqrt{2}}{\sigma}$.

Equation (9) is the Bessel differential equation with solutions [14] for

$$u >> \left|\alpha^2 - \tfrac{1}{4}\right|$$

as;

$$V = \frac{1}{\sqrt{2\pi u}}[e^u + \pi e^{-u}] + \frac{\sqrt{2\alpha}}{\alpha(2\alpha - 1)} \tag{10}$$

and

$$V = \frac{1}{2^\alpha 2u^\alpha \alpha\Gamma(\alpha)}[\alpha 2^{2\alpha}\Gamma^2(\alpha) + 2u^{2\alpha}] + \frac{\sqrt{2\alpha}}{\alpha(2\alpha - 1)} \tag{11}$$

For $0 < u << \sqrt{\alpha + 1}$.

Notice that $u(N) \to \infty$ as $N \to \infty$, thus $V \to \frac{\sqrt{2\alpha}}{\alpha(2\alpha - 1)}$ for $\alpha < \tfrac{1}{2}$, the economic value of the nation becomes negative. A great crash has just occurred with depression and economic meltdown. There will be increase in the migration of capital due to domestic inflation. This means that the growth rate of the economy depends on the parameter α. Given $N(t) = L_s - L_t$ in equation (1), it is reasonable to relate the cumulative capital outflow to the Levy process L_t through

$$N(t) = N(0)\exp(L_t) \tag{12}$$

known as a Geometric Levy process. The ratio of the outflows at t and s is now, [15]

$$\ln\left(\frac{N(s)}{N(t)}\right) = L_s - L_t \tag{13}$$

and we pass in a discrete time to an ordinary random walk based on independent increments $L_s - L_t$, the distribution of which being our modeling tool. Herein the distribution of N(t) is assumed to belong to a specific parameter form.

3 Definition and Properties of the Normal Inverse Gaussian Distribution

The Normal inverse Gaussian (NIG) distribution with parameters

$\alpha>0$, $-\alpha<\beta<\alpha$ and $\delta>0$, NIG(α,β,δ), has a characteristic function [9] given by

$$\varnothing_{NIG}(u,\alpha,\beta,\delta) = \exp\left(-\delta\sqrt{\alpha^2-(\beta+iu)^2}-\sqrt{\alpha^2-\beta^2}\right) \qquad (14)$$

This is an infinitely divisible characteristic function [16].The NIG process can be defined hence

$$x^{NIG} = \{x_t^{NIG},\ t \geq 0\} \qquad (15)$$

with $x_0^{NIG} = 0$ stationary and independent NIG distributed increments. x_t^{NIG} has an NIG (α, β, $t\delta$) law. The levy measure for the NIG process is given by

$$\upsilon_{NIG}(dx) = \delta\alpha \exp(\beta x)\frac{k_{\lambda(\alpha|x|)}}{|x|}dx \qquad (16)$$

where $k_\lambda(x)$ denotes the modified Bessel function of the third kind with index λ. It is shown in [17] that if u tends to one of the values α-β or β-α, the expression $\delta\sqrt{\alpha^2-(\beta-u)^2}$ converges to zero. In this case the behavior of the expression $\frac{k_{\lambda(x)}}{x^\lambda}$, for $x \to 0$. From [18], equations 9.6.6, 9.6.8 and 9.6.9, we know that, as $x \to 0$,

$$k_\alpha(x) \sim \begin{cases} \dfrac{\Gamma(\lambda)2^{\alpha-1}}{x^\alpha}, & \text{for } \alpha > 0 \\ -\ln x, & \text{for } \alpha = 0 \\ \dfrac{\Gamma(-\alpha)}{x^{\lambda+1}}x^\alpha, & \text{for } \alpha < 0 \end{cases} \qquad (17)$$

An NIG process has no Brownian component and its Levy triplet is given by $[\gamma, 0, V_{NIG} d(x)]$, where

$$\gamma = \frac{2\delta_\alpha}{\pi} \int_0^1 \sinh(\beta x) k_1(\alpha x) dx \tag{18}$$

Then the density of the NIG(α,β,δ) distribution is given by;

$$f_{NIG}(x; \alpha, \beta, \mu, \delta) = \frac{\alpha\delta}{\pi} \exp\left(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu)\right) \frac{K_\alpha(\alpha\sqrt{\delta^2 + (x-\mu)^2})}{\sqrt{\delta^2 + (x-\mu)^2}} \tag{19}$$

where $x, \mu \in R, 0 \le |\beta| \le \alpha$ and as in (15) is the modified Bessel function of the third kind.

- $\alpha > 0$ determines the slope
- β with $0 \le |\beta| < \alpha$ the skewness
- μ the location, and
- δ>0 is a scaling parameter. The mean, the variance, the skewness and excess kurtosis of $X \sim NIG(\alpha, \beta, \mu, \delta)$ are given by

$$m = \mu + \delta\beta \big/ \sqrt{\alpha^2 - \beta^2},$$

$$\theta = \alpha^2 \delta (\alpha^2 - \beta^2)^{-\frac{3}{2}}$$

$$S = 3\beta\alpha^{-1}\delta^{-1/2}(\alpha^2 - \beta^2)^{-1/4}$$

and

$$K = 3\left(1 + \frac{4\alpha^2 + 4\beta}{\delta\alpha^2\sqrt{\alpha^2 - \beta^2}}\right)$$

3.1 The Fitting of the Normal Inverse Gaussian Model

Instead of modeling the log capital out flow with the normal distribution, we now replace it a more sophisticated infinitely divisible probability distribution (the NIG). The NIG family has the nice properties that it is flexible, and parameters that can produce fat tails and skewness and can be solved in closed form. It is convolution stable under certain conditions. The NIG is defined as the normal variance-mean mixture.

Let $C_1, C_2, ...$ be a sequence of independent and identically distributed random variables with mean 0 and variance 1. For each $n \geq 1$ define a continuous time stochastic process $\{B_n(t)\}_{t \geq 0}$ by

$$B_n(t) = \frac{1}{\sqrt{n}} \sum_{1 \leq j \leq [nt]} C_j \tag{20}$$

This is the random step functions with jumps of size $\pm \frac{1}{\sqrt{n}}$ at times $\frac{k}{n}$, where $k \in Z_+$. Since the random variables C_j are independent, the increments of $B_n(t)$ are independent. Given (18), the economic process X_t can be considered a NIG process written as;

$$X_t = \beta\delta^2 N_{\tau(t)} + \delta B_{N_{\tau(t)}} \tag{21}$$

with parameters α, β and δ [16]. $B_t \sim N(0, \xi)$ is a Brownian motion and the "operational time" τ(t) is an increasing Levy process (i.e. a subordinator).

There is a simple expression for the moment generating function which makes the expression of all moments available. These moments' expressions [12]:

$$\left.\begin{aligned} m_1 &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ m_2 &= \frac{1}{n}\sum_{i=1}^{n} (x_i - m_1)^2 \\ m_3 &= \frac{1}{n}\sum_{i=1}^{n} (x_i - m_1)^3 \\ m_4 &= \frac{1}{n}\sum_{i=1}^{n} (x_i - m_1)^4 \end{aligned}\right\} \tag{22}$$

help us to calculate the skewness and kurtosis of NIG, thus;

$$\left.\begin{aligned} S &= \frac{m_3}{(\sqrt{m_2})^3} \\ K &= \frac{m_4}{(\sqrt{m_2})^2} \end{aligned}\right\} \tag{23}$$

However the following theorem makes the calculation of the four parameters easier as one can easily calculate the parameters;

Theorem: [20] Given an NIG(α,β,μ,δ) distributed random variable, if its sample mean, sample variance, sample skewness, and sample excess kurtosis are M, ν, S, and K respectively and $3K > 5S^2 > 0$, then the method of moment estimators for the parameters are;

$$\left.\begin{aligned} \tilde{\alpha} &= 3\rho^{1/2}(\rho-1)^{-1}\theta^{-1/2}|S|^{-1} \\ \tilde{\beta} &= 3(\rho-1)^{-1}\theta^{-1/2}S^{-1} \\ \tilde{\mu} &= \mathrm{M} - 3\rho^{-1}\theta^{1/2}|S|^{-1} \\ \tilde{\delta} &= 3\rho^{-1}(\rho-1)^{1/2}\theta^{1/2}|S|^{-1} \end{aligned}\right\} \tag{24}$$

Where $\rho = \left|3KS^{-1} - 4\right| > 1$. With the aid of the above parameters, one can easily calculate the mean, variance, skewness and kurtosis of NIG distribution.

4 Empirical example

Exchange rate misalignment, financial sector constrains, fiscal deficits and external incentives (Ajayi, 1995) are some factors grouped under relative risks that cause capital flight. The real exchange rate plays a significant role in the direction and magnitude of capital flight from highly-indebted countries (Pastor, 1990). Domestic wealth owners normally shift out of domestic assets into foreign assets, if currency depreciation is expected. We have collected the external debt, capital flight, change in external debt and capital flight external debt data from 1972 until 1989 for an empirical evaluation of the normal inverse Gaussian model. Data source: IMF (1990), IFS statistic year book. These data are in millions of the US Dollars and without loss of generality, we normalize these data by the equation;

$$P(N(t)) = \frac{N_i(t)}{N_C(t)} \tag{25}$$

Where $N_C = \frac{1}{T}\sum_i N_i$, such that it can be seen as probability. Table 1 below shows these normalized data. The moments under the NIG assumption have been calculated with equation (22) through (24). For all data sets whose results are summarized in figures 1-4, the parameter estimation has been done first under the assumption of the NIG.

The parameter α controls the steepness of the density of NIG, in the sense that the steepness increases monotonically with an increasing α. This also has implications for the tail behavior, by the fact that large values of α implies light tails, while smaller values of α implies heavier tails as illustrated in Figures 1-4.

Table 1: Normalized estimate ratio of capital flight to change in External Debt.

Year	External Debt	Capital Flight	Change in External Debt	Capital Flight External Debt
1972	0.836	0.10652	0.10784	0.1
1973	0.2866	0.2824	0.292	0.477
1974	0.2566	1.33799	1.3855	0.3899
1975	0.0939	1.33657	1.30226	0.12802
1976	0.0979	1.23297	1.1999	0.16206
1977	1.966	1.567	1.5593	1.82647
1978	0.4011	0.6015	0.54937	0.2034
1979	-0.067	1.2631	1.2774	0.52027
1980	2.1421	2.8956	2.9103	2.240586
1981	1.6833	1.3715	1.2529	1.1633
1982	-0.0303	-0.4978	0.5306	-3.2968
1983	1.5917	0.6916	0.68833	1.72242
1984	-0.1334	0.3559	0.3556	0.158099
1985	2.81800	1.2020	1.254	2.58979
1986	4.3442	1.5269	1.568	4.4438
1987	4.6379	1.67884	1.7600	4.72378
1988	0.82352	0.5533	0.5674	0.780766
1989	-0.23608	0.49378	0.5	0.3

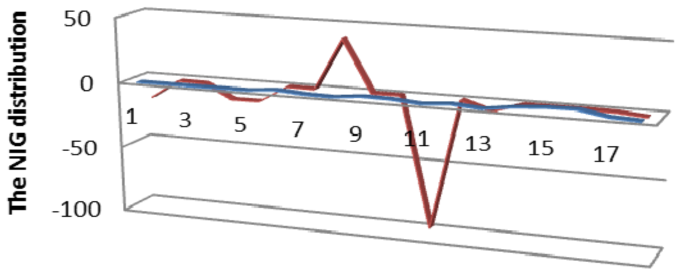


Figure 1: The NIG captured effect of extended debt data on the economy

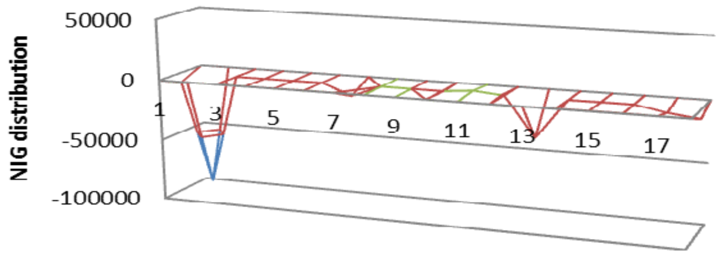


Figure 2: The NIG captured effect of Capital Flight on the economy.

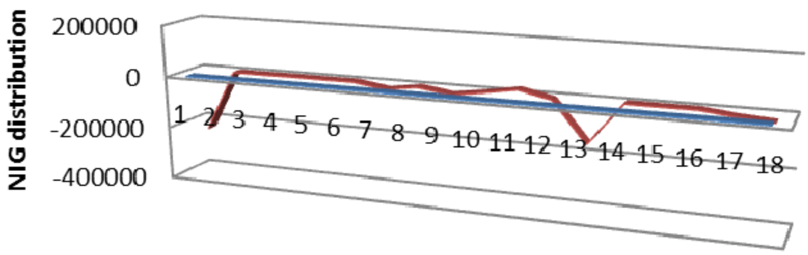
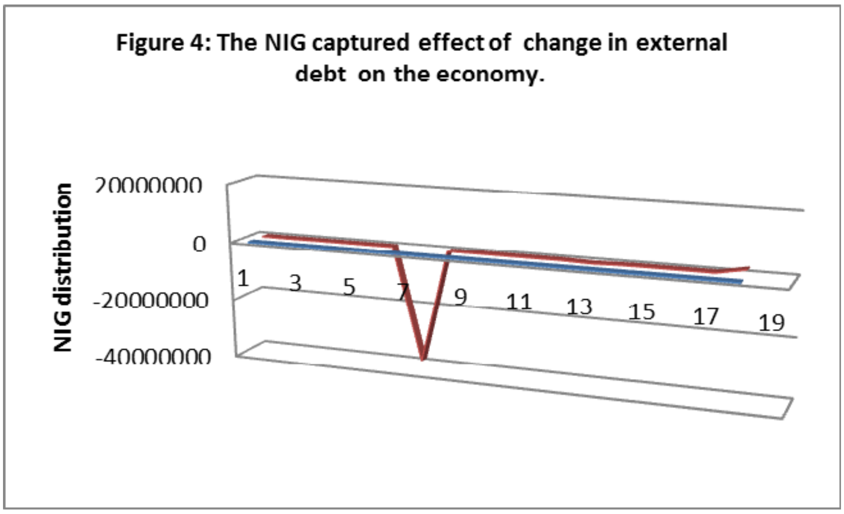


Figure 3: The NIG captured effect of capital flight external debt on the economy.

Table 2: Estimated parameters of NIG distribution via the empirical normalized estimate ratio of capital flight to change in External Debt.

Data	α	β	μ	δ	γ
External Debt	0.837494	0.3014	0.6245	2.2093	0.7813
Capital Flight	3.9	1.2	0.3999	1.8465	3.7
Change in External Debt	5.13	3.66586	0.3	0.7944	3.58629
Capital Flight and External Debt	6.19	-2.68696	-5.5	13.78	5.576



5 Conclusion

The NIG distribution with her properties seems a very useful tool in the investigation of the effect of the capital flight in economic growth of a developing nation (eg Nigeria). Obviously the NIG distribution provides a very good fit to the empirical distribution in the center as well as in the tails. With only four parameters it has captured facts like heavy tails observed in the capital flight to change on the extended Debt data of table 1. The years with negative signs in table 1 are perceived as years of capital repatriation or capital inflow. This period is associated with the period of oil wealth in Nigeria. Thus, one can see the episodic nature of capital flight. The macroeconomic environment is said to be favorable to capital inflow as a result of the buoyant economic policy associated with the oil boom years.

The peaked positive triangular shapes in figures 1-4 represent the economic indices of the Nigerian nation with capital inflow (i.e. year with negative signs), probably with buoyant economic policy. While the dipped negative triangular shapes are associated with the years of capital outflow with poor economic policy per se. These triangular shapes are basically due to the conversion of the estimates of the shape parameters α and β to an alternative but more pictorial form

$$\xi = \left(1 - \sqrt{\alpha^2 - \beta^2}\right)^{-\frac{1}{2}} \quad , \quad \chi = \frac{\beta}{\alpha}\xi$$

where $0 \leq |\chi| < \xi < 1$, since $\beta < \alpha$.

If a random variable X_t as in (19) is following an NIG(α,β,δ) distribution, we have that $-X_t$ is NIG(α,-β,δ) distributed. Thus given $N_{\tau(t)}$ and B_t, the growth rate of the economy X_t is positive or negative depending whether the capital flows in or flows out according to the economic policy. Hence, a better policy measures should be instituted to make the domestic economy more attractive for private investment if capital flight is to be confronted and flight capital recaptured.

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