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A Fault detection scheme for a biregional and multisectoral output growth model

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Abstract

The problem of fault detection, isolation and identification are for the case of faulty output growth models describing biregional and multisectoral economies. The model is in discrete time descriptor form including non measurable total outputs and uncertain final demands. For this model the problems of fault detection, isolation and identification are solved on the basis of the design of a bank of unknown input discrete time observers provide a residual index. The present results are successfully applied to a biregional 12 sector economy where 4 different fault scenarios including single and multi faults are considered.

Keywords: Biregional and Multisector Input Output models, Fault Detection, Fault Isolation

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1 Introduction

Multiregional dynamic input output models reveal that significant changes in the growth process of economies have to be addressed to local agglomerate processes [1]. The early multiregional, multisectoral models were static [2]. In [3], [4], [5] and in order to develop tools facilitating the analysis of the interregional linkages between spatially defined economies subdivided into productive sectors, modifications of static models have been proposed. In [6] the theoretical difficulties concerning the forward in time projection of the growth path have been proposed. The positiveness of the dynamic trajectory as well as the causal indeterminacy problem, have attracted considerable attention. In [7], [8] and [9] the relative stability of the balanced growth has been studied for non singular models while in [6] and [10] singular models have been studied. For continuous time models, some first results regarding fault detection in single region models have been derived in [11] and [12].

In this paper, the case of a bioregional multisector model input output model in discrete time descriptor form is considered. The vector of the total outputs corresponding to intermediate total outputs is considered to be partially known or measurable. Furthermore, the final demand vector is considered to be partially uncertain. To study the fault detection problem a bank of unknown input discrete time descriptor observers has been designed. Based on the outputs of this bank the nonmeasurable part of the total output vector is estimated accurately. The observers are of reduced order. They are causal and have arbitrary poles. Using the outputs of the observer bank a residual index is composed. Based on this index the problems fault detection, fault isolation and fault identification are solved. The results are successfully applied to the twelve sector biregional model in [6] and [13], and four fault scenarios are considered.

2 A biregional and multisectoral Input Output Model

Consider a biregional and n- sector economy ([6], [13]). According to the input output theory, for every region $r \in \{1,2\}$ there exist a $n \times 1$ total output vector $x_r(k)$ and a $n \times 1$ vector of final demands $f_r(k)$. The $n \times n$ matrix A_r is direct consumption coefficient matrix of the r region and B_r is the respective $n \times n$ capital coefficient matrix. According to [6] and [13] the dynamic model of the biregional economy is given by the following discrete time vector equation:

$$x(k) = TAx(k) + TB\left[x(k+1) - x(k)\right] + Tf(k) \tag{1}$$

where $x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$ is the $2n \times 1$ vector including the total output of all sectors in both regions of the economy, $f(k) = \begin{bmatrix} f_1(k) \\ f_2(k) \end{bmatrix}$ is the vector including the final demands in both regions and where $T = \begin{bmatrix} T_1 & I_n - T_2 \\ I_n - T_1 & T_2 \end{bmatrix}$, $A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$. T is the interregional trade coefficient matrix where T_1, T_2 are appropriate $n \times n$ positive diagonal matrices and I_n is the $n \times n$ identity matrix. A is the total direct consumption coefficient matrix and B is the total capital coefficient matrix. According to [6] and [13] many sectors do not capitalize goods and thus the rows of B_1 and B_2, corresponding to these sectors, are equal to zero. The matrix TB is singular. Hence, after a row rearrangement the matrix TB can be expressed as $\begin{bmatrix} \hat{T}B \\ \tilde{T}B \end{bmatrix} = \begin{bmatrix} \hat{T}B \\ 0 \end{bmatrix}$ where $\hat{T}$ is the matrix including the rows of T corresponding to the no zero rows of B and $\tilde{T}$ is the matrix including the rows of T corresponding to the zero rows of B. To produce a solution forward in time and similar to that in [14], it is assumed that (see [6] and [13]) the non zero rows of B are linearly independent among themselves. Thus $\hat{T}B$ is of full row rank. Let $C = I - TA + TB$. Due to the diagonal structure of T_1 and T_2, equation (1) can be expressed as follows

$$\begin{bmatrix} \hat{T}B \\ 0 \end{bmatrix} x(k+1) = \begin{bmatrix} \hat{C} \\ \tilde{C} \end{bmatrix} x(k) + \begin{bmatrix} \hat{T} \\ \tilde{T} \end{bmatrix} f(k) \tag{2}$$

where $\hat{C}$ includes the rows of C corresponding to the non zero rows of B and $\tilde{C}$ includes the rows of C corresponding to the zero rows of B. Similarly to [6] and [13], it is assumed that $\begin{bmatrix} \hat{T}B \\ \tilde{C} \end{bmatrix}$ is invertible. Thus, the following forward in time solution of equation (2) can be derived:

$$x(k+1) = \Theta x(k) + \Gamma f(k) + \Delta f(k+1) \tag{3}$$

where $\Theta = \begin{bmatrix} \hat{T}B \\ \tilde{C} \end{bmatrix}^{-1} \begin{bmatrix} \hat{C} \\ 0 \end{bmatrix}$, $\Gamma = -\begin{bmatrix} \hat{T}B \\ \tilde{C} \end{bmatrix}^{-1} \begin{bmatrix} \hat{T} \\ 0 \end{bmatrix}$ and $\Delta = \begin{bmatrix} \hat{T}B \\ \tilde{C} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \tilde{T} \end{bmatrix}$. For (3) to be equivalent to (2) the initial total output has to be consistent to the initial demand vector, i.e. $\tilde{C}x(0) = \tilde{T}f(0)$.

3 A faulty biregional and multisectoral Input Output Model with measured and non measured variables

3.1 Measured and non measured variables

As already mentioned in [11], the total outputs of some sectors can not be considered to be known or measured accurately. Let $m(k) = Mx(k)$ be the $p \times 1$ vector including the measured total outputs. Clearly, $\operatorname{rank} M = p$. In most cases, M is of the form $\begin{bmatrix} I_p & 0 \end{bmatrix} \tilde{J}$ where I_p is the $(p \times p)$ identity matrix and $\tilde{J}$ is a column rearrangement matrix. Writing first the measured total outputs, the state vector is transformed to

$$\begin{bmatrix} m(k) \\ w(k) \end{bmatrix} = \begin{bmatrix} M \\ M_O^T \end{bmatrix} x(k) \tag{4}$$

where $w(k)$ denotes the $(2n-p)\times 1$ vector including the total outputs that are not considered to be known or measured. Let M_O be the $(2n-p)\times n$ orthogonal of M. In the case $M = \begin{bmatrix} I_p & 0 \end{bmatrix}\tilde{J}$ it holds that $M_O = \begin{bmatrix} 0 & I_{2n-p} \end{bmatrix}\tilde{J}$.

3.2 Faults in the final demand

Let $f(k)$ denotes the known part of the final demand vector, the difference between the nominal and the real demand is an uncertainty vector. According to [11], the real final demand vector is of the form $f(k)+N\delta(k)$ where the $q\times 1$ vector $\delta(k)$ corresponds to the unknown part of some of the demands and N denotes the uncertainty distribution matrix. Clearly, $\mathrm{rank}N = q$.

3.3 The faulty model

As a result of Subsections 3.1 and 3.2, system (3) can be written as follows:

$$\begin{aligned} \begin{bmatrix} m(k+1) \\ w(k+1) \end{bmatrix} &= \begin{bmatrix} \Theta_{1,1} & \Theta_{1,2} \\ \Theta_{2,1} & \Theta_{2,2} \end{bmatrix} \begin{bmatrix} m(k) \\ w(k) \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} f(k) + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} f(k+1) + \\ &+ \begin{bmatrix} \mathrm{E}_1 \\ \mathrm{E}_2 \end{bmatrix} \delta(k) + \begin{bmatrix} \mathrm{Z}_1 \\ \mathrm{Z}_2 \end{bmatrix} \delta(k+1) \end{aligned} \tag{5}$$

where

$$\begin{bmatrix} \Theta_{1,1} & \Theta_{1,2} \\ \Theta_{2,1} & \Theta_{2,2} \end{bmatrix} = \tilde{J}\Theta\tilde{J}^T, \quad \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \tilde{J}\Gamma, \quad \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \tilde{J}\Delta, \quad \begin{bmatrix} \mathrm{E}_1 \\ \mathrm{E}_2 \end{bmatrix} = \tilde{J}\Gamma N, \quad \begin{bmatrix} \mathrm{Z}_1 \\ \mathrm{Z}_2 \end{bmatrix} = \tilde{J}\Delta N.$$

4 A Fault Detection Scheme

4.1 Unknown input observer design

The design goal is to estimate $w(k)$ accurately enough despite the presence of the fault $\delta(k)$. Assume that the following conditions are satisfied:

$$\mathrm{rank}\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \\ \mathrm{E}_2 & \mathrm{Z}_2 \end{bmatrix} = \mathrm{rank}\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix} \tag{6a}$$

$$\mathrm{rank}\begin{bmatrix} zI_{2n-p} - \Theta_{2,2} & \mathrm{E}_2 & \mathrm{Z}_2 \\ -\Theta_{1,2} & \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix} = 2n - p + \mathrm{rank}\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix},\ \forall z \in \mathbb{C} \tag{6b}$$

Let L satisfies the equation $\begin{bmatrix} \mathrm{E}_2 & \mathrm{Z}_2 \end{bmatrix} = L\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix}$. Let P the special solution of (7). Let N_O be $(p - \mathrm{rank}\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix}) \times p$ full row rank matrix being orthogonal to $\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix}$, i.e. $N_O\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix} = 0$. The general solution of L is:

$$L = P + \Lambda N_O \tag{7}$$

where Λ is a $(2n-p) \times (p - \mathrm{rank}\begin{bmatrix} \mathrm{E}_1 & \mathrm{Z}_1 \end{bmatrix})$ arbitrary matrix. If (6a and b) are satisfied then there exists an unknown input observer [15] of the form:

$$z(k+1) = Fz(k) + Gm(k) + Hf(k) + Kf(k+1) \tag{8a}$$

$$\hat{w}(k) = z(k) + Lm(k) \tag{8b}$$

where $\hat{w}(k)$ is the estimation of the vector $w(k)$ and

$$F = \Theta_{2,2} - (P + \Lambda N_O)\Theta_{1,2},\ \ G = F(P + \Lambda N_O) + \Theta_{2,1} - (P + \Lambda N_O)\Theta_{1,1} \tag{9a}$$

$$H = \Gamma_2 - (P + \Lambda N_O)\Gamma_1,\ \ K = \Delta_2 - (P + \Lambda N_O)\Delta_1 \tag{9b}$$

F : arbitrary (or stable) eigenvalues via appropriate choice of Λ (9c)

Let $e_w(k) = w(k) - \hat{w}(k)$ be the estimation error. The error is governed by the equation $e_w(k+1) = Fe_w(k)$ and $\lim_{k \to +\infty} e_w(k) = 0$.

4.2 Design of a bank of unknown input observers

If the conditions described above are satisfied, then a bank of $q+1$

observers, let $\{O, O_1, \ldots, O_q\}$ will be designed. The observer O is the one designed in Section 3.1. To design the rest observers, consider the "minus one fault" models

$$\begin{aligned}\begin{bmatrix} m(k+1) \\ w(k+1) \end{bmatrix} &= \begin{bmatrix} \Theta_{1,1} & \Theta_{1,2} \\ \Theta_{2,1} & \Theta_{2,2} \end{bmatrix}\begin{bmatrix} m(k) \\ w(k) \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} f(k) + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} f(k+1) + \\ &+ \begin{bmatrix} \mathrm{E}_1^{(i)} \\ \mathrm{E}_2^{(i)} \end{bmatrix}\delta^{(i)}(k) + \begin{bmatrix} \mathrm{Z}_1^{(i)} \\ \mathrm{Z}_2^{(i)} \end{bmatrix}\delta^{(i)}(k+1) \quad , \quad i = 1, \ldots, q\end{aligned} \tag{10}$$

where

$$\delta^{(i)}(k) = \begin{bmatrix} \delta_1(k) & \cdots & \delta_{i-1}(k) & \delta_{i+1}(k) & \cdots & \delta_q(k) \end{bmatrix}^T \tag{11a}$$

$$\mathrm{E}_1^{(i)} = \begin{bmatrix} (\varepsilon_1)_1 \cdots (\varepsilon_1)_{i-1} (\varepsilon_1)_{i+1} \cdots (\varepsilon_1)_q \end{bmatrix} \tag{11b}$$

$$\mathrm{E}_2^{(i)} = \begin{bmatrix} (\varepsilon_2)_1 \cdots (\varepsilon_2)_{i-1} (\varepsilon_2)_{i+1} \cdots (\varepsilon_2)_q \end{bmatrix} \tag{11c}$$

$$\mathrm{Z}_1^{(i)} = \begin{bmatrix} (\zeta_1)_1 \cdots (\zeta_1)_{i-1} (\zeta_1)_{i+1} \cdots (\zeta_1)_q \end{bmatrix} \tag{11d}$$

$$\mathrm{Z}_2^{(i)} = \begin{bmatrix} (\zeta_2)_1 \cdots (\zeta_2)_{i-1} (\zeta_2)_{i+1} \cdots (\zeta_2)_q \end{bmatrix} \tag{11e}$$

and where $\delta_i(k)$ is the $i-$th element of $\delta(k)$, $(\varepsilon_1)_j$ is the $j-$th column of E_1, $(\varepsilon_2)_\lambda$ is the $\lambda-$th column of E_2, $(\zeta_1)_j$ is the $j-$th column of Z_1 and $(\zeta_2)_\lambda$ is the $\lambda-$th column of Z_2. From (6) it is observed that

$$\mathrm{rank}\begin{bmatrix} \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \\ \mathrm{E}_2^{(i)} & \mathrm{Z}_2^{(i)} \end{bmatrix} = \mathrm{rank}\begin{bmatrix} \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \end{bmatrix} \tag{12a}$$

$$\mathrm{rank}\begin{bmatrix} zI_{2n-p} - \Theta_{2,2} & \mathrm{E}_2^{(i)} & \mathrm{Z}_2^{(i)} \\ -\Theta_{1,2} & \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \end{bmatrix} = 2n - p + \mathrm{rank}\begin{bmatrix} \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \end{bmatrix}, \ \forall z \in \mathbb{C} \tag{12b}$$

Let $P^{(i)}$ the special solution of $L^{(i)} = P^{(i)} + \Lambda^{(i)} N_O^{(i)}$. Let $N_O^{(i)}$ be the orthogonal of $\begin{bmatrix} \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \end{bmatrix}$. Then the general solution of $L^{(i)}$ is:

$$L^{(i)} = P^{(i)} + \Lambda^{(i)} N_O^{(i)} \tag{13}$$

where $\Lambda^{(i)}$ is a $(2n-p) \times (p - \mathrm{rank}\begin{bmatrix} \mathrm{E}_1^{(i)} & \mathrm{Z}_1^{(i)} \end{bmatrix})$ arbitrary matrix. The observer $O_i \ (i = 1, \ldots, q)$ being the unknown input observer of model (10), is of the form

$$z^{(i)}(k+1) = F^{(i)}z^{(i)}(k) + G^{(i)}m(k) + H^{(i)}f(k) + K^{(i)}f(k+1) \qquad (14a)$$

$$\hat{w}^{(i)}(k) = z^{(i)}(k) + L^{(i)}m(k) \qquad (14b)$$

where

$$F^{(i)} = \left[\Theta_{2,2} - P^{(i)}\Theta_{1,2}\right] - \Lambda^{(i)}N_O^{(i)}\Theta_{1,2} \qquad (15a)$$

$$H^{(i)} = \Gamma_2 - (P^{(i)} + \Lambda^{(i)}N_O^{(i)})\Gamma_1, \quad K^{(i)} = \Delta_2 - (P^{(i)} + \Lambda^{(i)}N_O^{(i)})\Delta_1 \qquad (15b)$$

$$G^{(i)} = F^{(i)}(P^{(i)} + \Lambda^{(i)}N_O^{(i)}) + \Theta_{2,1} - (P^{(i)} + \Lambda^{(i)}N_O^{(i)})\Theta_{1,1} \qquad (15c)$$

$$F^{(i)} : \text{arbitrary eigenvalues via appropriate } \Lambda^{(i)} \qquad (15e)$$

Typically, the estimation error of the observer O_i is given by the relation $e_w^{(i)}(k) = w(k) - \hat{w}^{(i)}(k)$ and is governed by the equation

$$e_w^{(i)}(k+1) = F^{(i)}e_w^{(i)}(k) + [-(\varepsilon_2)_i + L^{(i)}(\varepsilon_1)_i]\delta_i(k) + [-(\zeta_2)_i + L^{(i)}(\zeta_1)_i]\delta_i(k+1) \qquad (16)$$

and tends to zero as fast is allowed by the arbitrary eigenvalues of $F^{(i)}$.

4.3 Fault detection index

If the observer is applied to the original model (2) the error tends to zero only if the fault $\delta_i(k)$ is equal to zero. All observers $\{O, O_1, \ldots, O_q\}$ have the same initial conditions at $k = k_0$ $\hat{w}(k_0) = \hat{w}_1(k_0) = \cdots = \hat{w}_q(k_0)$. Thus, the fault detection index $r^{(i)}(k) = e_w(k) - e_w^{(i)}(k)$ can be defined. It is important to mention that the fault detection index is equal to $\hat{w}^{(i)}(k) - \hat{w}(k)$ and thus it can directly be computed by the outputs of the bank of the observer. Then the fault detection index will be governed by the equation:

$$\begin{aligned} r^{(i)}(k+1) = F^{(i)}r^{(i)}(k) + (L^{(i)} - L)(\varepsilon_1)_i\delta_i(k) + (L^{(i)} - L)(\zeta_1)_i\delta_i(k+1) + \\ + (L^{(i)} - L)\Theta_{1,2}e_w(k) \end{aligned} \qquad (17)$$

If the poles of F and $F^{(i)}, \forall i \in \{1, \ldots, q\}$ are chosen to be equal to zero. Then for $k > k_0 + 2n - p$ it holds that $e_w(k) = 0$. Thus for $k > k_0 + 2n - p$ the equation (17) becomes

$$r^{(i)}(k+1) = F^{(i)}r^{(i)}(k) + (L^{(i)} - L)(\varepsilon_1)_i \delta_i(k) + (L^{(i)} - L)(\zeta_1)_i \delta_i(k+1) \quad (18)$$

If the faults are considered to be applied for $k > k_0 + 2n - p + 1$ then it holds that $r^{(i)}(k_0 + 2n - p + 1) = 0$ and $\delta_i(k_0 + 2n - p + 1) = 0$. Thus, for $k > k_0 + 2n - p + 1$ The fault's value could be calculated through the difference equation (18). In (18) the fault δ_i has been calculated by using the residual. From the proposed algorithm we derive that a fault could be detected when one from the q residuals becomes different than zero. In fact we define a threshold, namely a small nonnegative number θ_i (e.g. $\theta_i = 0$),

where:

$$\begin{cases} \left\| r^{(i)}(k) \right\| \leq \theta_i, & \text{no fault case} \\ \left\| r^{(i)}(k) \right\| > \theta_i, & \text{faulty case} \end{cases}$$

Also after the fault detection we could locate the fault's source. Thus an isolation of the faulty element could be performed just by isolating the residual that exceed the predefined threshold. Finally the value of the fault could be identified by using (18). Hence the faulty element of the systems has been detected, isolated and identified by using the proposed bank of observers and the proposed faulty index.

5 Application to a 12 sector and 2 region economy

The fault detection method analyzed in the previous section will be applied to a 12-sector bioregional economy given in [6] and [13]. The first region is the North, while the second is the South. In Table 1 the 12 sectors of the economy are given.

Table 1: Sectors of the economy

No.	Sector
1	Agriculture, forestry and fishing
2	Energy
3	Ferrous and non-ferrous metals
4	Non-metallic minerals
5	Food, beverages and tobacco
6	Chemical and pharmaceutical products
7	Mechanical, autovehicles, textiles and other manufacturing
8	Construction
9	Trade, hotels, restaurants, scrap
10	Transportation and communication
11	Credit, finance and insurance
12	Retail and non-retail services

The 24×24 matrices A, B and T can be found in [6] and [13]. The final demands, used here, are given in Table 2. The initial values of the total outputs are:

$$x_1(0) = \begin{bmatrix} 55585.5 \\ 108200.5 \\ 45763.9 \\ 23664.3 \\ 76850.5 \\ 64158.7 \\ 260078.9 \\ 80034.6 \\ 184417.7 \\ 72448.6 \\ 46085.1 \\ 224788.7 \end{bmatrix} \quad \text{and} \quad x_2(0) = \begin{bmatrix} 26205.3 \\ 91314.8 \\ 8811.3 \\ 7502.1 \\ 27976.2 \\ 16257.7 \\ 62429.4 \\ 25433.4 \\ 63367.8 \\ 21352.3 \\ 9138.8 \\ 83650.2 \end{bmatrix}$$

The initial values of the total outputs as well as the values in Table 2 are in thousand million of Italian lires. The unknown (non measured) total outputs are considered to be the 7^{th} and the 12^{th} sectors (see Table 1) for both the North and the South. Thus, $p = 20$. The uncertain final demands are assumed to be the 1st, 3rd, 5th, 8th and 9th sectors for both the North and the South. Thus, $q = 10$. All of the 11 observers of the bank start with the same initial vector coming from the static model of the economy, i.e.,

$$z^{(i)}(0) = z(0) = \begin{bmatrix} 0 & I_{20} \end{bmatrix} \tilde{J}(I_{24} - TA)^{-1} Tf(0) - Lm(0),$$

where $m(0)$ and $f(0)$ are consistent. The poles of all the observers are chosen to be equal to zero. From (18) it is concluded that the bank needs four time instants to perfectly follow the model. The residuals are equal to zero, $r^{(i)}(5) = 0$. This can also be observed from the simulation plots. Four faults have been introduced to the system in different time instants. Firstly a fault in the final demand of the first sector, which corresponds to the Agriculture of the North, is introduced at $k = 6$. In Figure 1, we observe that for $k = 6$ only the residual $r_1(k)$ is different than zero. Thus the fault has not only been detected but also it has been isolated. Next a fault in the final demand of the fifth sector, which corresponds to the Food beverage and tobacco of the North, and in the thirteenth sector, which corresponds to the Agriculture of the South, have been introduced at $k = 7$. Finally a fault in the final demand of the eighteenth sector, which corresponds to the Food beverage and tobacco of the South, has been introduced at $k = 8$. The faults are considered to be

$\delta_1(0) = \delta_1(1) = \delta_1(2) = \delta_1(3) = \delta_1(4) = \delta_1(5) = 0$, $\delta_1(6) = 193.873$, $\delta_1(7) = 196.227$, $\delta_1(8) = 198.545$, $\delta_1(9) = 200.865$ and $\delta_1(10) = 203.07$.

$\delta_3(0) = \delta_3(1) = \delta_3(2) = \delta_3(3) = \delta_3(4) = \delta_3(5) = \delta_3(6) = 0$, $\delta_3(7) = 1680.409$, $\delta_3(8) = 1676.013$, $\delta_3(9) = 1735.964$ and $\delta_3(10) = 1726.854$.

$\delta_6(0) = \delta_6(1) = \delta_6(2) = \delta_6(3) = \delta_6(4) = \delta_6(5) = \delta_6(6) = 0$, $\delta_6(7) = 194.865$, $\delta_6(8) = 197.996$, $\delta_6(9) = 201.066$ and $\delta_6(10) = 204.043$.

$\delta_8(0) = \delta_8(1) = \delta_8(2) = \delta_8(3) = \delta_8(4) = \delta_8(5) = \delta_8(6) = \delta_8(7) = 0$, $\delta_8(8) = 1605.056$, $\delta_8(9) = 1002.867$ and $\delta_8(10) = 1151.195$.

To study fault detection for the present fault scenarios, only five of the 11 observers are used. To evaluate (18) the following quantities are computed

$$F^{(1)}=\begin{bmatrix} -7.002 & -0.07 & 73.498 & 3.513 \\ 0.347 & 0.049 & -0.299 & -0.0001 \\ -0.661 & -0.006 & 6.944 & 0.331 \\ -0.013 & -0.0001 & 0.163 & 0.007 \end{bmatrix}, F^{(3)}=\begin{bmatrix} -6.975 & -0.074 & 72.626 & 3.473 \\ 0.166 & 0.028 & -0.298 & -0.0001 \\ -0.666 & -0.007 & 6.939 & 0.331 \\ -0.015 & -0.0001 & 0.163 & 0.007 \end{bmatrix}$$

$$F^{(6)}=\begin{bmatrix} -11.609 & -0.141 & 199.808 & 20.813 \\ 0.166 & 0.028 & -0.292 & 0.001 \\ -0.671 & -0.008 & 11.58 & 1.207 \\ -0.015 & -0.0001 & 0.123 & 0.0003 \end{bmatrix},$$

$$F^{(8)}=\begin{bmatrix} -6.974 & -0.079 & 72.345 & 3.457 \\ 0.166 & 0.028 & -0.298 & -0.0001 \\ -0.668 & -0.007 & 6.938 & 0.331 \\ -0.015 & -0.0001 & 0.163 & 0.007 \end{bmatrix}$$

$$(L^{(1)}-L)(\varepsilon_1)_1=\begin{bmatrix} 7.127 \\ 15.862 \\ 0.681 \\ 0.141 \end{bmatrix}, \quad (L^{(3)}-L)(\varepsilon_1)_3=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (L^{(6)}-L)(\varepsilon_1)_6=\begin{bmatrix} 11193.425 \\ 0.863 \\ 650.045 \\ -5.531 \end{bmatrix},$$

$$(L^{(8)}-L)(\varepsilon_1)_8=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (L^{(1)}-L)(\zeta_1)_1=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (L^{(3)}-L)(\zeta_1)_3=\begin{bmatrix} -5784.321 \\ -0.002 \\ 0.161 \\ -0.001 \end{bmatrix},$$

$$(L^{(6)}-L)(\zeta_1)_6=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (L^{(8)}-L)(\zeta_1)_8=\begin{bmatrix} -449.628 \\ 0.0006 \\ -0.147 \\ 0.002 \end{bmatrix}$$

The two residuals $r_3(k)$ and $r_6(k)$ leave zero at $k=7$. Thus two simultaneous faults have detected and isolated by the method. At $k=8$ the residual $r_8(k)$ leaves zero.

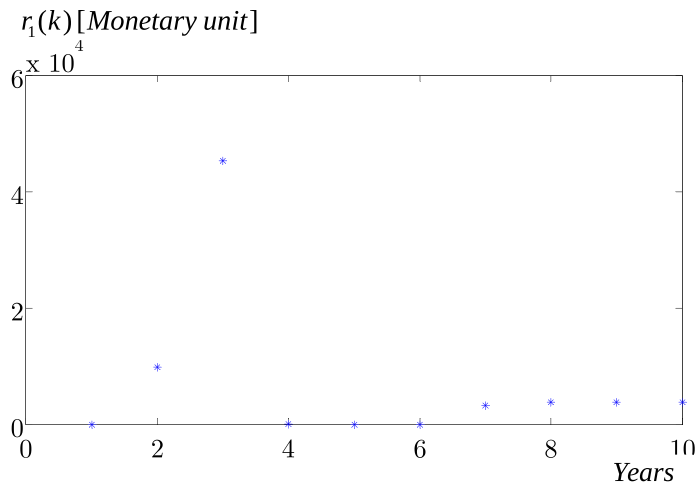

Figure 1: Residual of the observer O_1

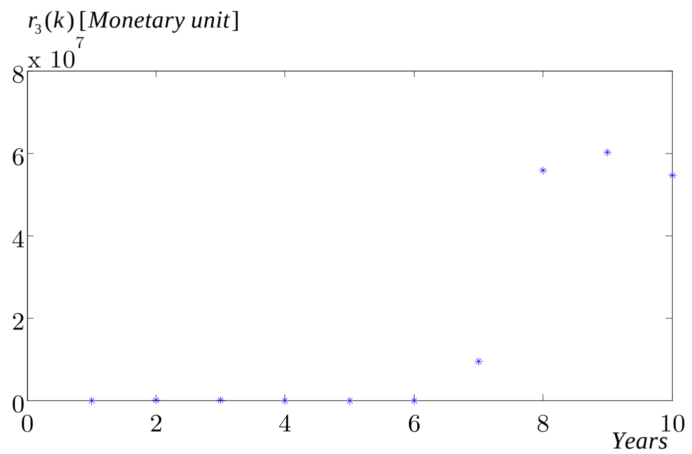

Figure 2: Residual of the observer O_3

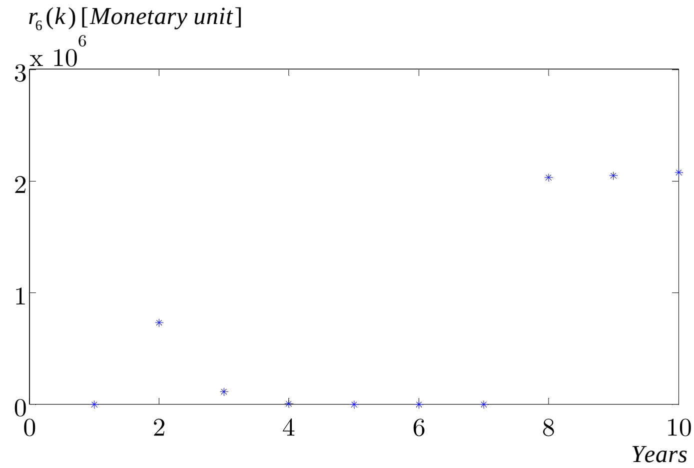

Figure 3: Residual of the observer O_6

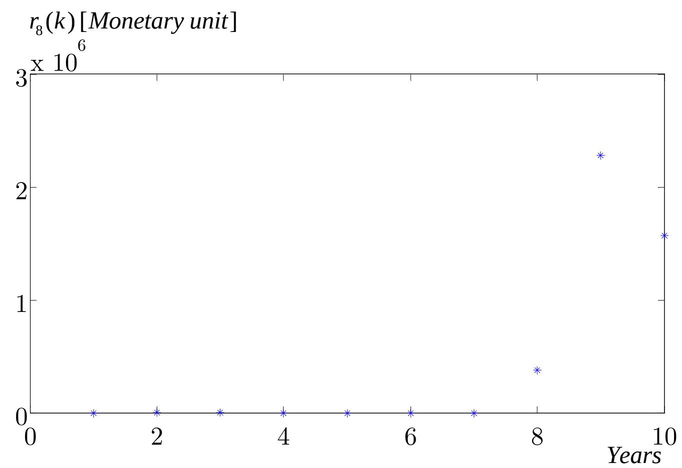

Figure 4: Residual of the observer O_8

6 Conclusion

In this paper a method for fault detection, fault isolation and fault identification for biregional and multisectoral input output and discrete time economic models have been proposed. The problems have been solved on the basis of the design of a bank of unknown input discrete time observers providing a residual index. The present results have successfully applied to a biregional 12-sector economy where 4 different fault scenarios including single and multi faults have been considered.

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Table 2: The final demand vector $f(k)$ from $k=0$ till $k=10$

Regions	Sectors	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
North	1	21579.3	21841.4	22108.2	22377.1	22656.9	22941.5	23211.	23492.2	23777.1	24028.2	24304.8
	2	57291.5	58974.9	60562.8	62135.4	63741.9	65322.2	66963.2	68592.6	70228.2	71834.1	73480.1
	3	203.718	194.231	199.966	200.398	215.204	232.451	253.913	280.641	313.736	357.888	403.695
	4	4527.87	4663.15	4797.4	4931.11	5068.86	5205.18	5343.79	5481.34	5620.87	5758.2	5898.33
	5	43250.	44665.6	46110.6	47593.3	49028.8	50450.7	51862.2	53316.6	54714.6	56196.	57618.7
	6	19440.9	19753.8	20067.9	20402.9	20716.3	21023.7	21326.8	21642.	21944.5	22256.4	22589.3
	7	40597.3	38323.9	36025.9	33673.5	31348.9	29009.6	26808.	24622.6	22246.9	20030.5	17794.1
	8	20298.8	18993.3	18141.	16171.4	15384.5	14005.8	13583.4	12789.	11058.1	10639.5	9730.36
	9	121000.	127346.	133504.	139714.	145872.	152071.	158332.	164552.	170720.	176949.	183235.
	10	31539.5	33102.5	34705.2	36271.4	37838.4	39413.1	41012.2	42612.5	44192.4	45795.3	47387.
	11	3560.72	3749.28	3937.31	4120.	4309.59	4495.49	4681.42	4861.49	5051.83	5234.36	5416.61
	12	163036.	172639.	182341.	191990.	201893.	211719.	221608.	231319.	240970.	250796.	260438.
South	13	9941.39	10118.7	10279.3	10435.5	10608.2	10767.4	10942.2	11094.9	11248.	11420.7	11583.4
	14	27584.	28186.6	28899.7	29595.2	30274.4	30942.	31566.3	32233.6	32885.3	33563.5	34166.8
	15	7456.33	7831.99	8198.38	8573.91	8938.19	9302.43	9665.69	10028.1	10389.3	10737.8	11088.7
	16	2898.97	2965.97	3031.6	3100.65	3165.37	3233.82	3300.47	3367.25	3433.96	3500.89	3566.85
	17	27024.2	27995.4	28961.8	29903.8	30865.1	31850.5	32814.4	33770.4	34778.2	35744.4	36716.
	18	8265.43	8734.61	9217.04	9682.51	10170.2	10650.4	11130.2	11602.9	12087.8	12576.1	13037.4
	19	91727.5	93698.	95686.3	97626.5	99718.2	101663.	103660.	105648.	107554.	109624.	111615.
	20	3085.84	3104.78	3081.68	2914.51	3043.44	3118.07	3138.06	3365.3	3169.58	3381.29	3251.63
	21	50955.6	53776.2	56623.7	59502.7	62385.1	65259.1	68081.1	70929.8	73784.2	76644.8	79459.8
	22	12966.	13720.3	14479.5	15245.7	16013.7	16767.8	17528.7	18271.8	19019.8	19787.3	20537.7
	23	1348.17	1442.86	1535.27	1625.12	1718.4	1809.08	1897.39	1987.98	2079.	2169.73	2260.59
	24	69875.6	74171.3	78443.2	82735.9	87079.7	91407.3	95719.5	100027.	104262.	108549.	112807.