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Modelling the fluctuations of Brent oil prices by a probabilistic Markov chain

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Abstract

Modelling of crude oil prices has been extensively made. In this paper, we concluded that the Brent oil prices follow a Markov chain. Moreover, we predict the model of fluctuating these prices from January 2004 to July 2010 by integrating the limit probability distribution of a Markov chain and Gumbel Max distribution. In this model, we analyze the trends of Brent oil prices from the short term to middle and long terms.

Mathematics Subject Classification: 60J10

Keywords: Brent oil price, Markov chain, Gumbel Max distribution

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1 Introduction

The industrialized world relies on crude oil as a central source of energy supply. Crude oil is the main feedstock for products and as such, its fundamental price level has an important effect on wholesale-refined product prices. The price of crude oil is one of the world’s most influential global economic indicators. It is precisely observed by policy-makers, producers, consumers and financial market participants. It is clear that oil prices are highly volatile and sometimes experience extreme shocks. One of the most significant marker crude oil grades is Brent. Several studies have examined the oil market from different perspectives, leading to several new insights on a market that has significance to global economic growth. Burbidge and Harrison [3] used a vector auto regression (VAR) model and compute impulse responses to oil price changes and find a casual relationship from oil price shocks to economic variables. W. Xie and et al [10] proposed a method based on support vector machine for the task of crude oil price time series prediction. D. Wang and et al [9] introduced the method of data mining combined with statistic knowledge to analysis oil price time series; furthermore, there have been various studies on oil prices based on the application of a Markov chain. A. Abazi [1] proposed a non-linear stochastic mean and volatility model for crude oil prices dynamics and applied Markov chain Monte Carlo algorithms to estimate the parameters of the model. A. Dafas [4] proposed a mean-reverting Markov switching jump diffusion model to characterize the stochastic behavior of the crude oil spot prices. K. Larsson and M. Nossman [8] examined the empirical performance of affine jump diffusion models with stochastic volatility in a time series study of daily WTI spot prices using the Markov chain Monte Carlo method. X. Haiyan and Z. Zhongxiang [6] applied Markov chain and lognormal distribution to model the prices of OPEC basket of crude oils. In this paper we examine the modeling of the Brent oil prices by combining the limit probability distribution of Markov chain and Gumbel Max distribution.

2 Oil price State Transition Chain as a Markov Chain

Table 1 gives the monthly prices of BRENT crude oil prices from January 2004 to July 2010, [11].

Table 1: Monthly prices of BRENT crude oil prices

	2004	2005	2006	2007	2008	2009	2010
January	31.18	44.28	63.57	54.3	91.92	44.86	76.37
February	30.87	45.56	59.92	57.76	94.82	43.24	74.31
March	33.8	53.08	62.25	62.14	103.28	46.84	79.27
April	33.36	51.86	70.44	67.4	110.44	50.85	84.93
May	37.92	48.67	70.19	67.48	123.94	57.94	76.25
June	35.19	54.31	68.86	71.32	133.05	68.59	74.84
July	38.37	57.58	73.9	77.2	133.9	64.92	74.74
August	43.03	64.09	73.61	70.8	113.85	72.5	
September	43.38	62.98	62.77	77.13	99.06	67.69	
October	49.77	58.52	58.38	83.04	72.84	73.19	
November	43.05	55.53	58.48	92.53	53.24	77.04	
December	39.65	56.75	62.31	91.45	41.58	74.67	

Figure 1 shows the trend and stochastic fluctuations of crude oil prices. We catagorised the prices as five states:

(0, 40) [40, 65) [65, 90) [90,125) [125,150)

There are 78 state transitions which form an oil price transition process.

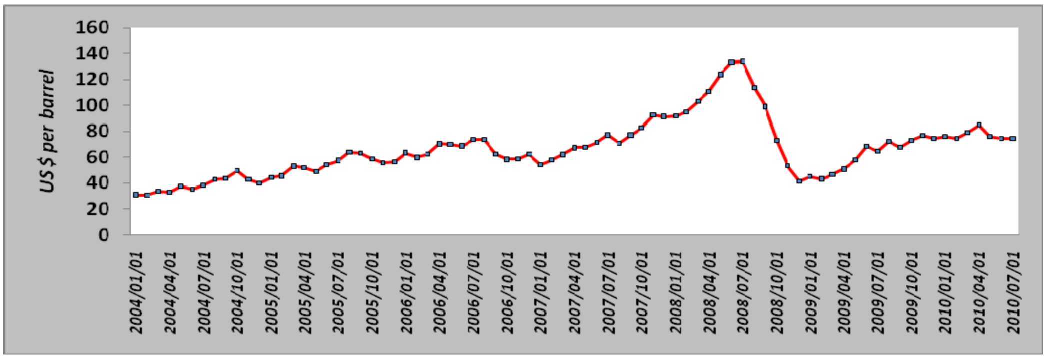

Figure 1.Trend of BRENT oil prices from January 2004 to July 2010.

Table 2 shows the state transition frequency matrix of oil price five state transition chains. We test that whether this sequence of sample follows a Markov chain or not by a chi-square test as Bartlett [2] and Hoel [7].

Table 2: State transition frequency of oil prices

	(0,40)	[40,65)	[65,90)	[90,125)	[125,150)	$n_{i\cdot}$
(0,40)	6	2	0	0	0	8
[40,65)	1	29	4	0	0	34
[65,90)	0	3	21	1	0	25
[90,125)	0	0	1	7	1	9
[125,150)	0	0	0	1	1	2
$n_{\cdot j}$	7	34	26	9	2	$n = 78$

Table 3 shows the testing results. Using a 5% significance level, we find that. The observed value of the sample statistics is 161.5638 which is higher than (16, 0.05) =26.296. Therefore, we reject the null hypothesis that states are independent. Finally, it approved that a state transition chain of BRENT crude oil prices follows a Markov chain.

Table 3: Testing results of oil prices

$\frac{(n_{\cdot j} - n_{i\cdot} n_{\cdot j} / n)^2}{n_{i\cdot} n_{\cdot j} / n}$	(0,40)	[40,65)	[65,90)	[90,125)	[125,150)
(0,40)	38.8642	0.6342	2.6667	0.9231	0.2051
[40,65)	1.3790	13.5662	4.7451	3.9231	0.8718
[65,90)	2.2436	5.7233	19.2534	1.2313	0.6410
[90,125)	0.8077	3.9231	1.3333	34.2224	2.5638
[125,150)	0.1795	0.8718	0.6667	2.5638	17.5451

3 Limit Probability of a Markov chain for changing trends of BRENT oil prices

The Ergodic theorem of a Markov chain is studied by R. Douc and et al [5]. We induce the changing trends of BRENT oil prices in the middle and long terms by the limit probability of a Markov chain. The values are as follows:

	(0, 40)	[40,65)	[65,90)	[90,125)	[125,150)
Limit Probability Value of Markov Chain	0.0465	0.3953	0.3876	0.1353	0.0310

This vector denotes the ultimate probability of five states in the crude oil price series. We observe that the probability of (0,40) is 0.0465 , the proportion of [40,65) and [65,90) in the series are 39.53% and 38.76% respectively. Figure 2 shows the ultimate states of BRENT oil prices which are exhibited by a Markov chain.

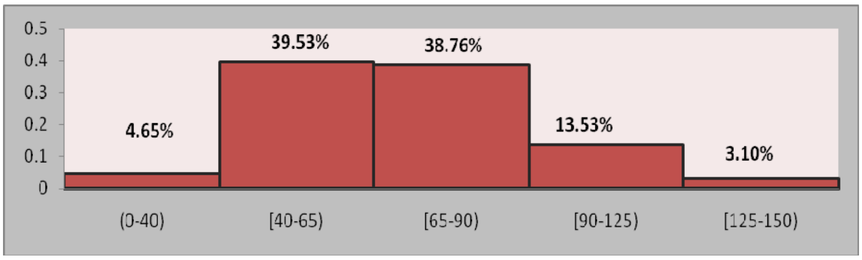

Figure 2: The Limit Probability of BRENT monthly oil prices

4 The Probability Distribution of Changing Trends of Oil Prices

Now we have substituted an actual distribution by a probability simulation of actual oil price distribution for the short term. The hypothesis test of distribution approved that our data conform to a Gumbel Max distribution. The test results and distribution fitting curve are shown in Table 4 and Figure 3, espectively

Table 4: Fitting distribution test results

Kolmogorov-Smirnov					
Sample Size	79				
Statistic	0.07944				
P-Value	0.67123				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.1186	0.13551	0.15052	0.16832	0.1806
Reject?	No	No	No	No	No
Anderson-Darling					
Statistic	0.4171				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No
Chi-Squared					
Deg. of freedom	6				
Statistic	5.8616				
P-Value	0.43887				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	8.5581	10.645	12.592	15.033	16.812
Reject?	No	No	No	No	No

The function of a Gumbel Max distribution is as follows:

$$f(x) = \frac{1}{\sigma}\exp(-z - \exp(-z)), \quad -\infty < x < \infty,$$

where $z \equiv \dfrac{x-\mu}{\sigma}$. We estimate the parameters as $\mu = 56.134$ and $\sigma = 17.333$.

Then we calculate the probability value of each interval as follows:

$$p_i = \int_{\text{the lower limit of interval}}^{\text{the upper limit of interval}} \frac{1}{\sigma} \exp(-z - \exp(-z))dx, \quad \text{for } x = 0, 15, 40, 65, \ldots, 125, \ldots$$

The probability of Gumble Max distribution and accumulated probability values are shown in Figure 4.

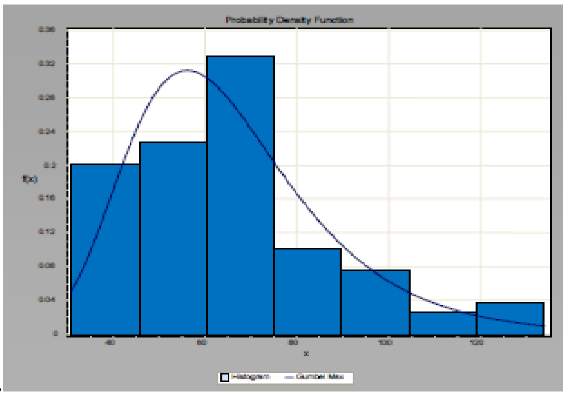

Figure 3. A distribution of BRENT oil prices

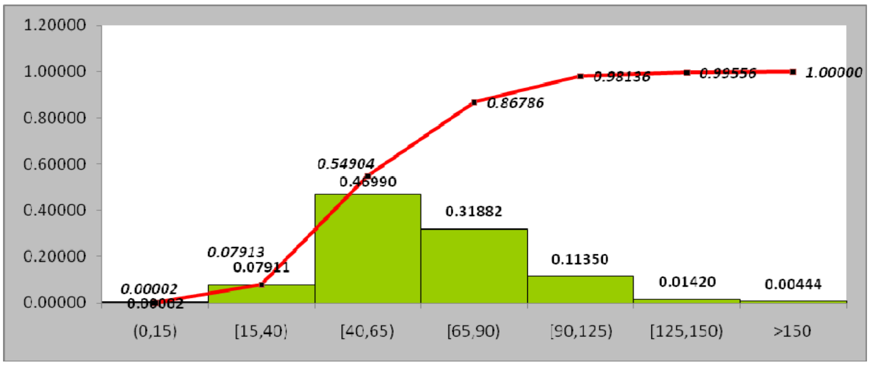

Figure 4: The probability of Gumble Max distribution and accumulated probability values of BRENT oil prices.

Table 5: Comparisions between Gumbel Max distribution and Limit probability of a Markov chain of BRENT oil prices

Price interval	(0,40)	[40,65)	[65,90)	[90,125)	[125,150)
Probability value of a Gumble Max distribution	0.0791	0.4699	0.3188	0.1135	0.0186*
Limit probability value of Markov chain	0.0465	0.3953	0.3876	0.1353	0.0310
The difference between the two probabilities	-0.0326	-0.0746	0.0688	0.0218	0.0124
The percentage of the above difference (%)	-41.21	-15.88	21.58	19.21	66.67
Theoretical frequency of Gumble Max distribution	6.2489	37.1221	25.1852	8.9665	1.4694*
Limit frequency of a Markov chain	3.6735	31.2287	30.6204	10.6887	2.449
The difference between the two frequencies	-2.5754	-5.8934	5.4352	1.7222	0.9796
The sum of positive and negative numbers in the above rows	-8.4688		8.137		

*The value is calculated for prices higher than 125US$/barrel.

From the general distribution of oil prices, the probability of oil prices below 125US$/barrel is 0.98136, indicating that this price or less would prevail in the market. The probability of oil prices below 90 US$/barrel is 0.86786 which would have a key role in making oil price steady. Next we integrate the results of the Markov chain model and the probability distribution function model and deduce the changing trends of BRENT oil prices from short term to middle and long terms. The results are shown in Table 5. The probability of oil prices less than 40 U$/barrel is 0.0791 in the short term and 0.0465 in the middle and long terms, 41.21% less than that in the short term. The probability of oil prices being in the

[40,65) interval is reduced from 0.4699 in the short term to 0.3953 in the middle and long terms by 15.88%. The probability of oil prices falling in the [65,90), [90,125) and [125,150) are increased by 21.58%,19.21% and 66.67%, respectively. We can see that the probability of BRENT oil prices below 65US$/barrel will be reduced by 10.7% while the oil prices being higher than 65US$/barrel will increase by 10.3%. It is inferred that in the next 79 months the frequency of oil prices falling in the (0,65) interval will decrease by -8.4688, that is approximately 8 months. On the other hand, the number of months in which oil prices are in the [65,150) interval will increase approximately 8 months.

Table 6: Monthly BRENT oil prices (US$/barrel)

August 2010	September 2010	October 2010	November 2010	December 2010	January 2011	February 2011	March 2011	April 2011
76.69	77.79	82.92	85.67	91.8	96.29	103.96	114.44	123.13

Finally, we verify by the actual changing trends of BRENT oil prices from August 2010 to April 2011 as shown in Table 6. All these prices fall in [40,125) interval. This approves our valuation that oil prices fall in to this interval by a probability of 0.9022.

5 Conclusions

We investigated that monthly BRENT oil prices follow a Markov chain and formed a model of fluctuating these prices by integrating the probability distribution of oil price series with the limit probability distribution of a Markov chain. This model presents changes in different price states from the short term to middle and long terms. Our results confirm the BRENT oil prices in the 9 months followed our sample period. The probability of oil prices below 40US$/barrel has been decreased by 41.2% while the oil prices more than 65 US$/barrel has increased.

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