

Numerical Simulation of Tsunami Wave Propagation In Indian Ocean

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Abstract

This paper aims to simulate the Tsunami wave propagation in Indian Ocean. The simulation is done using OPENGL (Open Graphics Library) in Linux. The Analysis area covers from 8.15 degrees (-ve North) to 26 degrees (+ve north) and 71.72 degrees (+ve East) to 104.78 degrees (+ve East). The Bathymetry data and Grid map are derived from GEBCO (General Bathymetric Chart Of Oceans). The Bathymetric Grid Map is divided in the form of Grids. The Grid Map using OPENGL is scaled, transformed and displaced to get the depth value corresponding to the point which is assumed as epicentre. It is found that the major factors which influence the travel pattern of the wave are depth profile of the Ocean, boundary effects and wave reflections. The other factors which also play a vital role while generating the boundary effects are the momentum and the dispersion of the Tsunami wave when it propagates.

Keywords: Tsunami, wave propagation, Indian ocean, Numerical simulation

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1 Introduction

All the earthquakes do not cause tsunamis, but many do. For a tsunami to hit Indian coast, it is necessary that a tsunamigenic earthquake occurs and its magnitude should be larger than M7. First tsunami reported was in the year 326 BC [2]. Since then as many as over 25 tsunamis have occurred in the vicinity of Indian region [1]. However, the extent of damage caused by 2004 Sumatra earthquake is quite high compared to other events occurred. Since then, major initiatives have started to model Tsunami characteristics. Indian government has taken the initiatives to set up National Tsunami Early Warning System at Indian National Center for Ocean Information Services (INCOIS) with prime motivation of disseminating the early warning.

This paper aims to simulate the Tsunami wave propagation in Indian Ocean. The simulation is done using OPENGL (Open Graphics Library) in Linux. The Analysis area covers from 8.15 degrees (-ve North) to 26 degrees (+ve north) and 71.72 degrees (+ve East) to 104.78 degrees (+ve East). The Bathymetry data and Grid map are derived from GEBCO (General Bathymetric Chart Of Oceans). The Bathymetric Grid Map is divided in the form of Grids. The Grid Map using OPENGL is scaled, transformed and displaced to get the depth value corresponding to the point which is assumed as Epicentre. From the Origin point with the incremental value along x and y directions the depth values of the surrounding points are read and the velocity at each of the point is calculated. In this manner, the time taken by the Tsunami wave to reach to the coastal areas is calculated as the total distance by velocity along that direction. The travel time of the Tsunami wave is calculated in all the directions from the origin point and also the propagation of the wave with respect to time is to be observed.

Depending on the relation between wavelength (λ) and water depth (d) the water waves are classified as shallow water (or long waves) and deep water (or short waves). The Indian Ocean is approximately 4km deep (d) but the wavelength (λ) of the sea floor deformation is 100km which is much larger than the water depth. So, shallow water approximation ($\lambda \gg d$) is done for tsunamis in Indian Ocean.

A finite amplitude wave theory appropriate for shallow water is the Cnoidal Wave Theory in which Cnoidal wave parameters are formulated in terms of

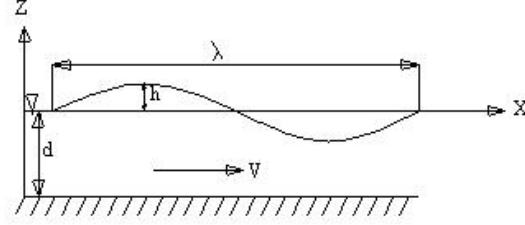


Figure 1: Wave propagation

elliptic cosine functions. This theory was developed from the governing equations for long waves using the assumption that square of the water surface or the wave steepness is small relative to unity. One of the important features of the Cnoidal waves is that they are periodic.

2 Basic Equations

2.1 Euler's Equation of Motion

In Euler's Equation, the forces due to gravity and pressure are taken into consideration. Consider a cylindrical element of cross-section dA and length ds . The forces acting on the cylindrical element are:-

- $p dA$: Pressure force acting in the direction of flow
- $(p + (\frac{p}{s} ds) dA$: acting in the opposite direction of flow
- $g dA ds$: Weight of the element
- θ : Angle between direction of flow and line of action of weight of the element

Resultant force in the direction of s must be equal to mass of fluid element and acceleration in the direction of s

$$p dA - (p + (p/s) ds) dA - g dA ds \cos \theta = dA ds a_s \quad (1)$$

Where a_s is acceleration in the direction of s

$$a_s = \frac{dV}{dt} \quad (2)$$

where V is a function of s and t

$$a_s = \left(\frac{\partial V}{\partial s}\right)\left(\frac{ds}{dt}\right) + \left(\frac{\partial v}{\partial t}\right) \quad (3)$$

$$= \left(\frac{\partial V}{\partial s}\right)V + \left(\frac{\partial v}{\partial t}\right) \quad (4)$$

$$= V \cdot \Delta V + \frac{\partial V}{\delta t} = \frac{DV}{Dt} \quad (5)$$

$a_s = \frac{DV}{dt}$ is total derivative. Then equation will be

$$\left(\frac{\partial p}{\partial s} ds\right)dA - \rho f dA ds \cos\theta = \rho dA ds \frac{DV}{Dt} \quad (6)$$

dividing the above equation with $\rho dA ds$ we get

$$-\left(\frac{\partial p}{\partial s}\right)\left(\frac{1}{\rho}\right) - g \cos\theta = \frac{DV}{Dt} \quad (7)$$

$$-\Delta p \left(\frac{1}{\rho}\right) - g \cos\theta = \frac{DV}{Dt} \quad (8)$$

when θ is small i.e., direction of flow is in the direction of cylindrical weight, then $\cos\theta = 1$, then

$$\frac{DV}{Dt} = -g - \frac{1}{\rho} \Delta p \quad (9)$$

This is the Euler's equation of motion.

2.2 Shallow Water Wave Equation

When the horizontal scale of motion or the wave length λ is much larger than the depth ($\lambda \gg d$), the vertical acceleration is negligible compared to gravity. This means the horizontal motion of water mass is almost uniform from bottom to surface. Such a wave is called Shallow Water wave or Long wave.

Let s & t be the horizontal and vertical components of V . Horizontal component of Euler's equation becomes by replacing the horizontal pressure gradient with the slope of water surface.

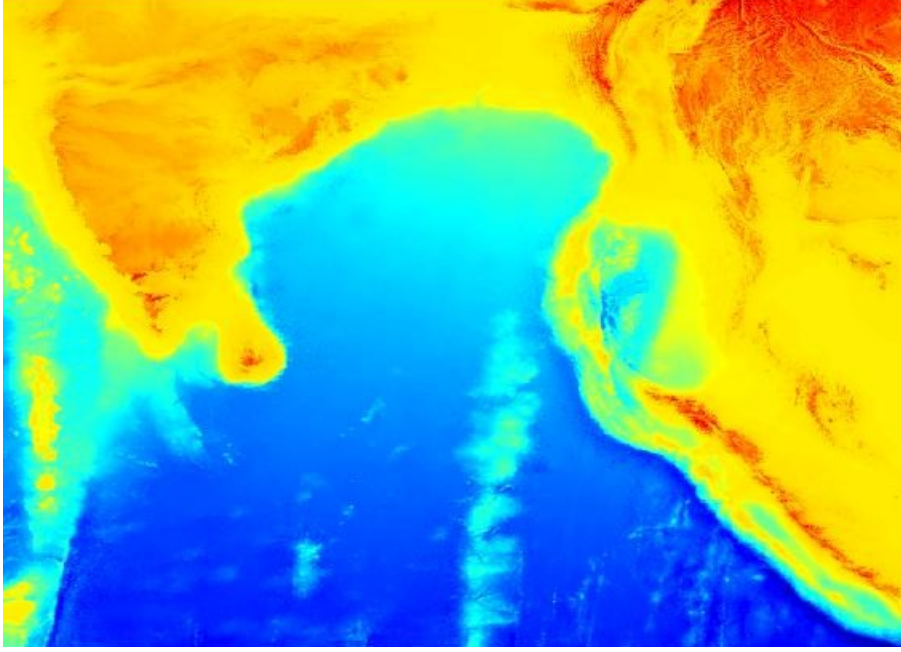


Figure 2: 2D Bathymetry Grid Map

$$\frac{Ds}{Dt} = -g\left(\frac{\partial h}{\partial x}\right) \quad (10)$$

$$\frac{Ds}{Dt} = \left(\frac{\partial s}{\partial x}\right)s + \left(\frac{\partial s}{\partial t}\right) \quad (11)$$

For Tsunami's convective acceleration $\frac{\partial s}{\partial t}$ is small, so it is neglected.

$$\frac{\partial s}{\partial t} = -g\frac{\partial h}{\partial x} \quad (12)$$

When amplitude is small compared to water depth $h \ll d$, equation of continuity can be written as (conservation of mass)

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}(d.s) \quad (13)$$

$$\frac{\partial s}{\partial t} = -g\frac{\partial h}{\partial x} \quad (14)$$

Differentiating w.r.t time t

$$\frac{\partial^2 s}{\partial t^2} = -g \frac{\partial}{\partial x} \frac{\partial h}{\partial t} \quad (15)$$

$$= -g \frac{\partial}{\partial x} \cdot - \frac{\partial}{\partial x} d.s \quad (16)$$

$$\frac{\partial^2 s}{\partial t^2} = g.d \frac{\partial^2 s}{\partial x^2} \quad (17)$$

The wave equation is of the form

$$\frac{\partial^2 s}{\partial t^2} = C^2 \frac{\partial^2 s}{\partial x^2} \quad (18)$$

$$C^2 = gd \quad (19)$$

Where C is the velocity which is determined by water depth d .

Now consider a water particle in water which is in equilibrium having y axis which specifies the vertical direction and z axis specifying the horizontal direction. The wave particle at time t has two components as vertical displacement and horizontal displacement as mentioned below.

$$\psi_y(y, z, t) = A_y(y) \cos(\omega t - kZ) \quad (20)$$

$$\psi_z(y, z, t) = A_z(y) \sin(\omega t - kZ) \quad (21)$$

For any values of y , z and t the displacements satisfy the below equation

$$\left(\frac{\psi_y}{A_y}\right)^2 + \left(\frac{\psi_z}{A_z}\right)^2 = 1 \quad (22)$$

This is equation of an Ellipse whose principal axes is along ψ_y and ψ_z meeting at point (y, z) . Thus, the path of each water particle is an upright ellipse centered on the particle's equilibrium position. As fluid motion is described in terms of the velocity of a fluid at a given point rather than the positions of water particles.

$$V_y(y, z, t) = -\omega A_y(y) \sin(\omega t - kz) \quad (23)$$

$$V_z(y, z, t) = \omega A_z(y) \cos(\omega t - kz) \quad (24)$$

2.3 Incompressibility Condition

Consider the surface enclosing a small rectangular volume $\Delta x \Delta y \Delta z$ at (y, z) where Δx is measured across the cross section. The net mass leaving the volume (inward flow) across the horizontal faces is $\Delta x \Delta z \frac{\partial V_y}{\partial y} \Delta y$. Similarly, the net outward flow across the vertical face is $\Delta x \Delta y \frac{\partial V_z}{\partial z} \Delta z$. The sum of these two quantities must be zero in incompressible water and so the required condition is:

$$\frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad (25)$$

2.4 No-viscosity condition

Rotation could be induced through the agency of shear forces which cannot exist in a non-viscous fluid. Circulation of any loop of water must remain zero at all times. Circulation means the line integral of the fluid velocity around the loop. Consider a small rectangular loop in which the contribution made to clockwise integral by horizontal sides of the loop having length Δz is namely $\Delta z \frac{\partial V_y}{\partial z} \Delta y$. The contribution from the vertical sides of the loop is $\Delta y \frac{\partial V_z}{\partial y} \Delta z$. Since the resultant circulation must be zero, we have

$$\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} = 0 \quad (26)$$

2.5 Ellipse Shape

We apply the incompressibility and no-viscous restrictions and a pair of boundary conditions to get the formulae for amplitudes $A_y(y)$ and $A_z(y)$ which fix the shape of the ellipses at a given depth. After applying incompressibility condition to the above equations and cancelling a factor $\sin(\omega t - kz)$ we get,

$$\frac{dA_y}{dy} - kA_z(y) = 0 \quad (27)$$

The no-viscosity condition leads to

$$\frac{dA_z}{dy} - kA_y(y) = 0 \quad (28)$$

Differentiate w.r.t y gives

$$\frac{d^2 A_y}{dy^2} - k \frac{dA_z}{dy} = 0 \quad (29)$$

$$\frac{d^2 A_y}{dy^2} - k^2 A_y = 0 \quad (30)$$

The general solution for the above differential equation is

$$A_y(y) = A' e^{ky} + B' e^{-ky} \quad (31)$$

Where A' and B' are arbitrary constants which can be found out by applying the boundary conditions. First boundary condition is that at top surface of the water where $y = 0$

$$\psi_y(0, z, t) = \psi(z, t) = A \cos(\omega t - kz) \quad (32)$$

Applying this boundary condition to the above equation

$$A_y(0) = A' + B' = A \quad (33)$$

second boundary condition is $y = -h$, as there can be no vertical movement, water on the bottom must remain on the bottom and so,

$$A_y(-h) = A' e^{-kh} + B' e^{kh} = 0 \quad (34)$$

Solving above two equations, we get the values of A' and B' as

$$A' = A \frac{e^{kh}}{e^{kh} - e^{-kh}} \quad (35)$$

$$B' = -A \frac{e^{-kh}}{e^{kh} - e^{-kh}} \quad (36)$$

Substituting the above values in equation (7) we get

$$A_y(y) = A \frac{e^{k(h+y)} - e^{-k(h+y)}}{e^{kh} - e^{-kh}} \quad (37)$$

The above expression can be written in the Hyperbolic form as

$$A_y(y) = A \frac{\sinh[k(h+y)]}{\sinh kh} \quad (38)$$

Substituting the $A_y(y)$ in above equation, we get the $A_z(y)$ as

$$A_z(y) = A \frac{(e^{k(h+y)} + e^{-k(h+y)})}{(e^{kh} - e^{-kh})} \quad (39)$$

In Hyperbolic form,

$$A_z(y) = A \frac{\cosh[k(h+y)]}{\sinh kh} \quad (40)$$

2.6 Motion in shallow water

In shallow water waves $kh \ll 1$ or $\lambda \gg d$ as the wavelength is much larger than the water depth.

$$\sinh kh = kh \quad (41)$$

$$\sinh[k(h+y)] = k(h+y) \quad (42)$$

$$\cos[k(h+y)] = 1 \quad (43)$$

Equation (1) becomes as

$$\psi_y(y, z, t) = A \frac{1+y}{h} \cos(\omega t - kz) \quad (44)$$

$$\psi_z(y, z, t) = A \frac{1}{kh} \cos(\omega t - kz) \quad (45)$$

2.7 Gravity waves in shallow water

$$\omega^2 = gk \tanh(kh) \quad (46)$$

from hyperbolic series expansion, $\tanh(kh) = kh - \frac{1}{3}kh^3 + \dots$. After approximation we get

$$\omega^2 \approx ghk^2 \left(1 - \frac{1}{3}h^2k^2\right) \omega \approx ck - dk^3 \quad (47)$$

where $c = \sqrt{gh}$ and $d = \frac{1}{6}ch^2$. Since dispersion $d > 0$, we can neglect d term then phase velocity $V_\theta \approx C = \sqrt{gh}$. Relation between longitudinal water velocity and surface displacement

$$\psi'_z(z, t) \approx \frac{\omega A}{kh} \cos(\omega t - kz) \quad (48)$$

$$= \frac{\omega}{kh} \psi_y(0, z, t) \quad (49)$$

$$= \frac{\omega}{kh} \psi(z, t) \quad (50)$$

$$\psi'_z \approx \frac{c}{h} \psi \approx \sqrt{\frac{g}{h}} \psi \quad (51)$$

where $\sqrt{\frac{g}{h}}$ is the characteristic impedance of the system. ψ is the amount by which the water surface is raised is a measure of the hydrostatic pressure increase underneath.

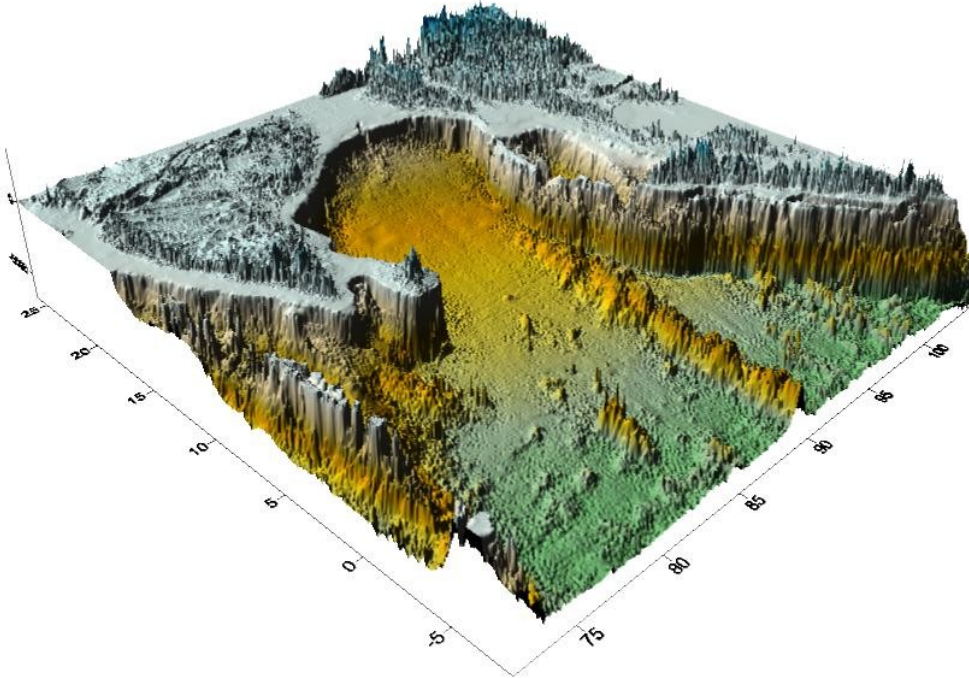


Figure 3: 3D Bathymetry Grid Map

3 Numerical Experimentation and Results

Firstly, the map constituting Indian Ocean which is in .grd format i.e is in grid file format is converted to a bitmap as .bmp format. Then the .bmp file is set to 256*256 pixels using GIMP software in Linux operation systems so that the image can be read while coding in Open Graphics Library (OPENGL) as the image is generally read in 256*256 or 512*512 pixels format only. (See Figures 3 & 4).

The inputs data file can be loaded in excel or document format in the OPENGL code. The sample data file is as

Latitude	Longitude	Depth
71.7167	-8.15	-1999
71.75	-8.15	-1991
71.7833	-8.15	-1961
71.8167	-8.15	-1979

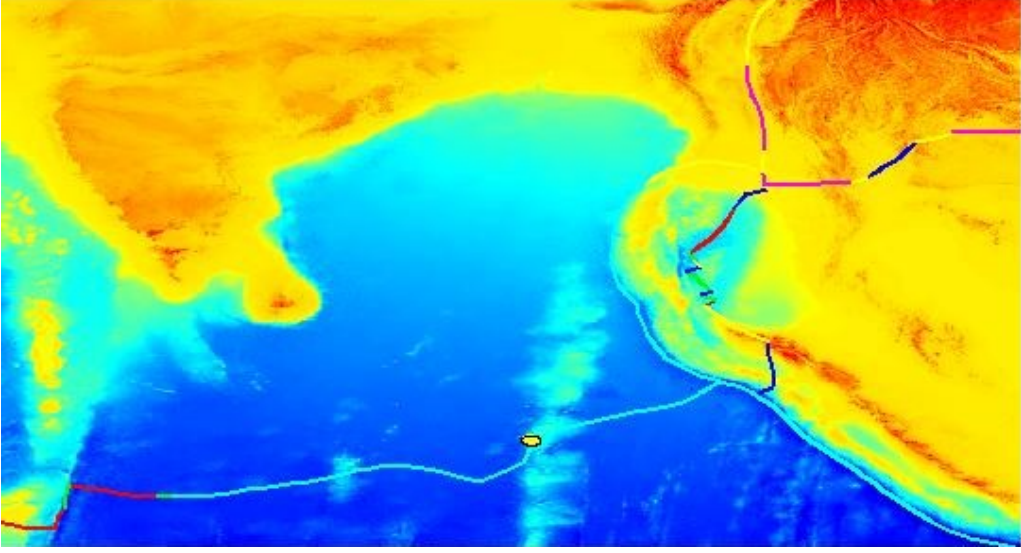


Figure 4: Location of Tsunami Source

The image is divided in the form of pixels as Longitude (in degrees) along x-direction, Latitude (in degrees) along y-direction and Depth values (in meters) along z-direction. The sea coastline is taken as reference depth value equal to zero. The positive depth values are taken for the land that is above the Mean Sea Level and negative depth values are taken below the Mean Sea Level.

Now when the map is read in the .bmp format it is necessary to scale the map, displace and then transform it to the window co-ordinates. The final window co-ordinates are:

$$X = \frac{x}{500}(33.0666) + 71.71666 \quad Y = \frac{y}{500}(-34.1333) + 25.9833 \quad (52)$$

Where (x, y) are the co-ordinates of the point at which the mouse is clicked and (X, Y) are the Latitude and Longitude of the map and from the input file we get the corresponding Depth value of the point at which the mouse was clicked. Assume the mouse click is the yellow point on the plate boundary seen in below map:

Once the Depth value of the point at which mouse click is known the velocity at that point can be calculated using the relation as square root times the product of gravitational constant and the known depth value. After that with an incremental value along longitudinal direction or Latitudinal direction or on both sides, the Depth values at each point can be retrieved from the

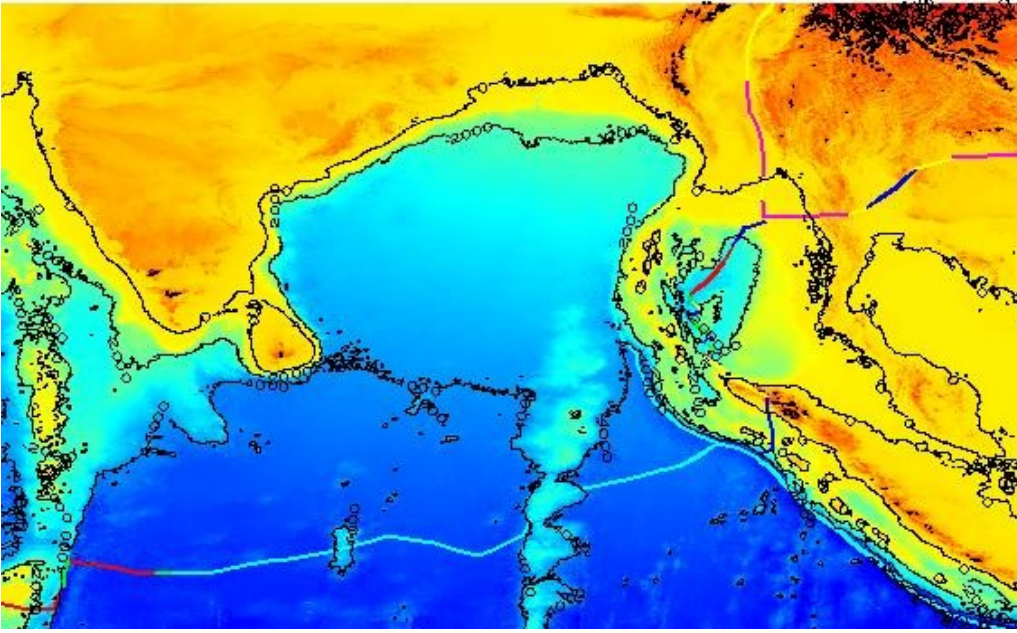


Figure 5: Depth Contour Map

input file and then the velocity can be calculated. Assuming unit time the distance of travel from the origin point to the sea coast line can be found out.

From the Contour Map (see Figure 5) it can be seen that how the waves are moving from the epicenter to the sea coastline, when the depth of the sea is less at a point the velocity decreases but when the depth of the sea is more the velocity will be more. The same rule is applied for calculating the distance traveled from one point to the other point.

The waves disperse as they move towards the sea coastline may produce significant differences in the coastal run-up (see Figure 6). Dispersive waves interacting with the natural frequencies of the bay or harbour tend more often to generate resonance. As the waves reach the coastline since there will be no sea depth as a result the velocity with which they will be zero, as a result the waves fall on the coastline with some height in the form of run-up.

As calculated from above equations, the particle velocity is

$$V_x = \psi \sqrt{\frac{g}{h}} \quad (53)$$

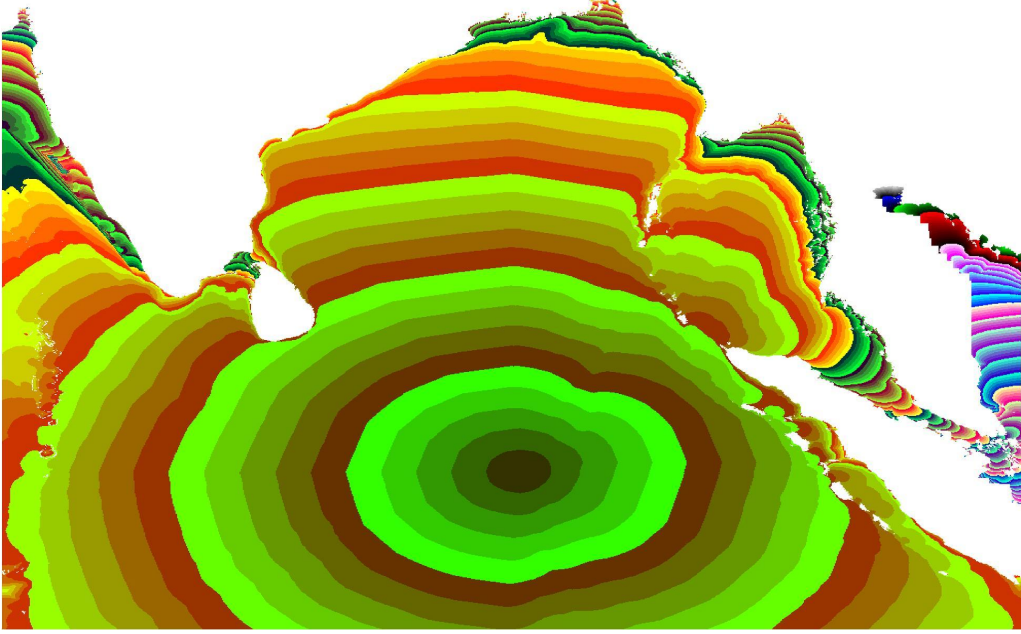


Figure 6: Depth Contour Map

Momentum/unit width is given by

$$M = (\psi + h)\psi\sqrt{\frac{g}{h}} \quad (54)$$

Taking only the 1st order term into consideration, $M = C\psi$ where $C = \sqrt{gh}$. The bottom friction is a force opposite to the direction of the particle velocity, V , which decreases the momentum in the part of the wave where is positive and increases the Momentum (i.e. decreases the negative momentum) where ψ is negative. However, particle velocities are small in deep water and this effect would be small. Only as the particle velocity becomes large as the wave enters shallow water or crosses the shoreline would this be significant.

Consider the total momentum contained in the first crest of a tsunami coming ashore. This momentum is scattered or reversed by forces on the moving water of objects in its path and of the ground itself. These forces are exactly the same but in opposite direction to the forces of the water on the objects, i.e. the destructive force of the tsunami. Thus, the total momentum is a measure of the destructive capability of the tsunami.

There are three kinds of forces to be considered:

1. The horizontal forces of the ground on the water (both friction and the horizontal component of pressure) are not destructive. They simply serve to slow the water down and to some extent, reverse the direction of the momentum.
2. The second kind of forces is those of irregular objects such as brush or trees and junk in general on the water. These forces tend to disorganize the water flow and, in effect, dissipate the organized momentum of the rushing water.
3. The third category of forces is those of buildings, cliff faces, and other large objects.

Considering the forces on the water which cause the momentum to reverse direction. This would require an impulse that is twice the momentum of the oncoming water. In the process of reversing direction, the wave amplitude is the sum of the incident wave and the reflected wave so that the water level is doubled in amplitude at the reflecting surface. This phenomenon can account for the reports of most large wave heights.

4 Conclusions

Tsunami wave propagation in Indian Ocean is simulated in this paper. The simulation is done using OPENGL (Open Graphics Library) in Linux. The Analysis area covers from 8.15 degrees (-ve North) to 26 degrees (+ve north) and 71.72 degrees (+ve East) to 104.78 degrees (+ ve East). The Bathymetry data and Grid map are derived from GEBCO (General Bathy-metric Chart Of Oceans). The Bathymetric Grid Map is divided in the form of Grids. The Grid Map using OPENGL is scaled, transformed and displaced to get the depth value corresponding to the point which is assumed as epicentre. It is found that the major factors which influence the travel pattern of the wave are depth profile of the Ocean, boundary effects and wave reflections. The other factors which also play a vital role while generating the boundary effects are the momentum and the dispersion of the Tsunami wave when it propagates.

References

- [1] R.K. Jaiswal, B.K. Rastogi and T.S. Murty, Tsunamigenic Sources in The Indian Ocean, *Science of Tsunami Hazards*, **27**(2), (2008).
- [2] E. Lisitzin, *Sea Level Changes*, Elsevier Oceanographic Series, No. 8, New York, pp. 273, 1974.
- [3] T.S. Murty, A.D. Rao, N. Nirupama and I. Nistor, Numerical Modeling Concepts for Tsunami Warning Systems, *Current Science*, **90**(8), (2006).
- [4] T.S. Murty, A. Bapat and P. Vinayak, Tsunamis on the coastlines of India, *Science of Tsunami Hazards*, **17**(3), (1999), 167-172.
- [5] B.K. Rastogi, Some facts about 26 Dec 2004 Sumatra earthquake & tsunami, (2005), [http://www/bestindia.com/jgsi.17pp](http://www.bestindia.com/jgsi.17pp).
- [6] Kenji Satake, *The Asian Tsunami of 2004: Observations and Numerical Simulation*, Active Fault Research Center, Tsukuba, Japan.
- [7] S. Yoshinori, *Wave Dispersion Study in Indian Ocean Tsunami of December 26, 2004*, National Defense Academy of Japan.
- [8] Xing cai and Hans Petter Langtangen, Making Hybrid Tsunami Simulators in a parallel Software Framework.
- [9] J. Foley, A. Dam, K.F. Steven and F.H. John, *Computer graphics (OPENGL)*, 2000.