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Synchronous Properties in Quantum Interferences Appearing in Simulated Double Path Experiments

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Abstract

Double slit experiments play a key role in Quantum Theory in distinct particle and wave interactions according to Feynman. In this paper, double path models together with variant logic principles are applied to establish a simulation system enabling the exhaustive testing of given targets. Using Einstein quanta interaction, different quaternion measures are investigated. Under conditions of Symmetry / Anti-symmetry and Synchronous / Asynchronous interaction, eight groups of statistical results are generated and presented as eight histograms showing the distributions. From this set of simulation results, it can be seen that not only is the synchronous condition the key factor in generating quantum wave interference patterns but also that the asynchronous condition is the key factor in classical particle distributions. Sample results are illustrated and explanations are discussed.

Keywords: Double Path, Interaction, Probability, Statistics

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1 Introduction

Feynman, having deeply explored the puzzling nature of quantum measurement [15,16] emphasized: "The entire mystery of quantum mechanics is in the double-slit experiment." This experiment directly illustrates both classical and quantum interactive results. Under single and double slit conditions, dual visual distributions are shown in particle and wave statistical distributions linked to von Neumann's measurement theory [28].

From the 1970s and piloted by CHSH [10], Aspect used experiments to test Bell inequalities [3-5]. After 40 years of development, many accurate experiments [26, 31-32] have been performed successfully worldwide using Laser, NMRI, large molecular, quantum coding and quantum communication approaches [1-7, 17-27, 29-30].

In this paper, a double path model is established using the Mach-Zehnder interferometer. Different approaches of quantum measures, namely those of Einstein, CHSH and Aspect are investigated by quaternion structures. Under multiple-variable logic functions and variant principles, logic functions can be transferred into variant logic expression as variant measures. Under such conditions, a variant simulation model is proposed. A given logic function f, can be represented as two meta-logic functions f_+ and f_- to simulate single and double path conditions. N bits of input vectors are exhausted by 2^N states for measured data, recursive data are organized into eight histograms. Results are determined by symmetry/anti-symmetry properties evident in these histograms. Results are obtained consistently from this model on synchronous/asynchronous conditions. Based on this set of simulation results, synchronous conditions show significant relationship linked to interference properties.

2 Double Path Model and Their Measures

2.1 Mach-Zehnder interferometer Model

The Mach-Zehnder interferometer is the most popular device [4, 22] to support a Young double slit experiment.

Figure 1(a) shows a double path interferometer. An input signal X under control function f causes Laser LS to emit the output signal ρ under BP (Bi-polarized filter) operation output as a pair of signals: ρ^+ and ρ^-. Both signals are processed by SW output ρ_L^+ and ρ_R^-, and then IM to generate output signals ($\mathrm{IM}(\rho_L^+, \rho_R^-)$. In Figure 1(b), a representation model has been described with the same signals being used.

2.2 Emission and Absorption Measures of Quantum Interaction

Einstein established a model to describe atomic interaction [8, 9, 13, 14] with radiation in 1916. For two-state systems, let a system have two energy states: the ground state E_1 and the excited state E_2. Let N_1 and N_2 be the average numbers of atoms in the ground and excited states respectively. The numbers of states are changed from emission state E_2 to E_1 with a rate $\frac{dN_{21}}{dt}$, in the same time; the numbers of ground states are determined by absorbed energies from E_1 to E_2 with a rate $\frac{dN_{12}}{dt}$ respectively. Let N_{12} be the number of atoms from E_1 to E_2 and N_{21} be the numbers from E_2 to E_1. In Einstein's model, a measurement quaternion is $\langle N_1, N_2, N_{12}, N_{21} \rangle$.

CHSH proposed spin measures testing Bell inequalities [4,10]. They applied $\perp \to +$ and $|| \to -$ to establish a measurement quaternion:

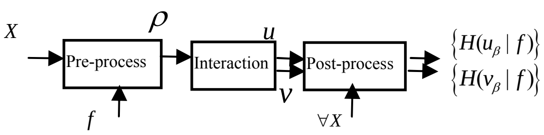
$$\langle N_{++}(a,b), N_{+-}(a,b), N_{-+}(a,b), N_{--}(a,b) \rangle .$$

Experimental testing of Bell inequalities were performed by Aspect [3] in 1982. Four parameters are measured: transmission rate N_t, reflection rate N_r, correspondent rate N_c and the total number N_ω in ω time period. This set of measures is a quaternion $\langle N_t, N_r, N_c, N_\omega \rangle$. Among these, N_c is a new data type not in Einstein and CHSH methods, this parameter could be an extension of synchronous/asynchronous time-measurement.

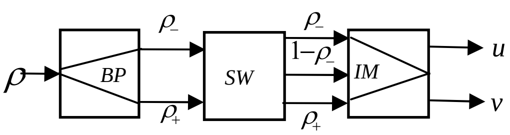
3 Simulation Systems

3.1 Simulation model

Using variant principle described in the following subsections, for a N bit 0-1 vector X and a given logic function f, all N bit vectors are exhausted and variant measures generate two groups of histograms. This variant simulation system is composed of three components: Pre-process, Interaction and Post-process as shown in Figure 2.



(a) Variant Simulation System



(b) Interaction Component

Figure 2: Variant Simulation System; (a) Variant simulation system; (b) Interactive Component

In Figure 2(a), three components of the variant simulation model are presented. At the pre-process stage, a N bit 0-1 vector X and a function f feed in to output a signal ρ. After an interactive component process, two groups of signals are output: u for symmetry group and v for anti-symmetry group. In the post-process stage, all N bit vectors are processed by pre-processing and interactive components until all of the 2^N data set has been processed to transform symmetry and anti-symmetry signals into eight histograms: four for symmetry distributions and another four for anti-symmetry distributions.

In Figure 2(b), only the interaction component is selected, input signal ρ processed by BP to generate two signals $\{\rho_-, \rho_+\}$. SW output triple signals $\{\rho_-, 1-\rho_-, \rho_+\}$ though IM to generate two groups of signals u and v.

3.2 Variant Principle

The variant principle is based on n-variable logic functions [33-35]. For any n-variables, $x = x_{n-1}...x_i...x_0$, $0 \le i < n$, $x_i \in \{0,1\} = B_2$. Let a position j be the selected bit $0 \le j < n$, $x_j \in B_2$ be the selected variable. Let output variable y and n-variable function $f, y = f(x)$, $y \in B_2, x \in B_2^n$. For all states of x, a set $S(n)$ composed of the 2^N states can be divided into two sets: $S_0^j(n)$ and $S_1^j(n)$.

$$\begin{cases} S_0^j(n) = \{x \mid x_j = 0, \forall x \in B_2^n\} \\ S_1^j(n) = \{x \mid x_j = 1, \forall x \in B_2^n\} \\ S(n) = \{S_0^j(n), S_1^j(n)\} \end{cases}$$

For a given logic function f, there are input and output pair relationships to define four meta-logic functions $\{f_\perp, f_+, f_-, f_T\}$:

$$\begin{cases} f_\perp(x) = \{ f(x) \mid x \in S_0^j(n), y = 0 \} \\ f_+(x) = \{ f(x) \mid x \in S_0^j(n), y = 1 \} \\ f_-(x) = \{ f(x) \mid x \in S_1^j(n), y = 0 \} \\ f_T(x) = \{ f(x) \mid x \in S_1^j(n), y = 1 \} \end{cases}$$

Two logic canonical expressions: AND-OR form is selected by $\{ f_+(x), f_T(x) \}$ as $y = 1$ items, and OR-AND form is selected from $\{ f_-(x), f_\perp(x) \}$ as $y = 0$ items. Considering $\{ f_T(x), f_\perp(x) \}$, $x_j = y$ items, they are invariant themselves.

To select $\{ f_+(x), f_-(x) \}$; $x_j \neq y$ forming variant logic expression.

Let $f(x) = \langle f_+ \mid x \mid f_- \rangle$ be a variant logic expression. Any logic function can be expressed as a variant logic form. In $\langle f_+ \mid x \mid f_- \rangle$ structure, f_+ selected 1 items in $S_0^j(n)$ as same as the AND-OR standard expression, and f_- selecting relevant parts the same as OR-AND expression 0 items in $S_1^j(n)$. For a convenient understanding of variant representation, 2-variable logic structures are illustrated for its 16 functions shown in Table 1.

E.g. Checking two functions $f = 3$ and $f = 12$:

$$\{ f = 3 := \langle 0 | 3 \rangle,\ \ f_+ = 11 := \langle 0 | \phi \rangle,\ \ f_- = 2 := \langle \phi | 3 \rangle \}$$

$$\{ f = 12 := \langle 2 | 1 \rangle,\ \ f_+ = 14 := \langle 2 | \phi \rangle,\ \ f_- = 8 := \langle \phi | 1 \rangle \}$$

3.3 Variant Measures

Let Δ be variant measure function [25, 35].

$$\Delta = \langle \Delta_\perp, \Delta_+, \Delta_-, \Delta_T \rangle$$

$$\Delta f(x) = \langle \Delta_{\perp} f(x), \Delta_{+} f(x), \Delta_{-} f(x), \Delta_{T} f(x) \rangle$$
$$= \langle \Delta f_{\perp}(x),\ \Delta f_{+}(x),\ \Delta f_{-}(x),\ \Delta f_{T}(x) \rangle$$

$$\Delta f_{\alpha}(x) = \begin{cases} 1, & if \quad f(x) = f_{\alpha}(x), \alpha \in \{\perp, +, -, \mathrm{T}\} \\ & 0, \quad \text{others} \end{cases}$$

For any given n-variable state there is one position in $\Delta f(x)$ to be 1 and the other 3 positions are 0.

For any N bit 0-1 vector X; $X = X_{N-1} \ldots X_{J} \ldots X_{0}$, $0 \le J < N$, $X_{\mathrm{J}} \in \beta_{2}$, $X \in \beta_{2}^{N}$ under n -variable function f, n bit 0-1 output vector Y, $Y = f(X) = \langle f_{+} \mid X \mid f_{-} \rangle$, $Y = Y_{N-1} \ldots Y_{\mathrm{J}} \ldots Y_{0}$, $0 \le \mathrm{J} < N$, $Y_{\mathrm{j}} \in \beta_{2}$, $Y \in \beta_{2}^{N}$.

For the J -th position be $x^{\mathrm{J}} = [\ldots X_{\mathrm{J}} \ldots] \in \beta_{2}^{n}$ to form $Y_{\mathrm{J}} = f(x^{\mathrm{J}}) = \langle f_{+} \mid x^{\mathrm{J}} \mid f_{-} \rangle$, let N bit positions be cyclic linked. Variant measures of $f(X)$ can be decomposed

$$\Delta \langle X : Y \rangle = \Delta f(X) = \sum_{J=0}^{N-1} \Delta f(x^{J}) = \langle N_{\perp}, N_{+}, N_{-}, N_{T} \rangle$$

as a quaternion $\langle N_{\perp}, N_{+}, N_{-}, N_{T} \rangle$

E.g. $N = 10$, given f, $Y = f(X)$.

X	=	0	1	1	0	0	1	1	1	0	0
Y	=	1	0	1	0	1	0	1	0	1	0
$\Delta(X:Y)$	=	+	-	T	$\perp$	+	-	T	-	+	$\perp$

$$\Delta f(X) = \langle N_{\perp}, N_{+}, N_{-}, N_{T} \rangle = \langle 2,3,3,2 \rangle, N = 10$$

Input and output pairs are 0-1 variables with only four combinations. For any given function f, the quantitative relationship of $\{\perp, +, -, T\}$ is determined directly from input/output sequences.

3.4 Measurement Equations

Using variant quaternion, signals are calculated by the following equations. For any N bit 0-1 vector X, function f , under Δ measurement: $\Delta f(X)=\langle N_{\perp},N_{+},N_{-},N_{T}\rangle$, $N=N_{\perp}+N_{+}+N_{-}+N_{T}$ Signal ρ is defined by

$$\rho=\frac{\Delta f(X)}{N}=\langle\rho_{\perp},\rho_{+},\rho_{-},\rho_{T}\rangle$$

$$\rho_{\alpha}=\frac{N_{\alpha}}{N},0\le\rho_{\alpha}\le1,\quad\alpha\in\{\perp,+,-,T\}$$

Using $\{\rho_{+},\rho_{-}\}$, a pair of signals $\{u,v\}$ are formulated:

$$\begin{cases}u=\langle u_0,u_+,u_-,u_1\rangle=\{u_\beta\}\\ v=\langle v_0,v_+,v_-,v_1\rangle=\{v_\beta\}\end{cases}$$

$$\beta\in\{0,+,-,1\}$$

$$\begin{cases}u_0=\rho_-\oplus\rho_+\\ v_0=(1-\rho_-)/2\oplus(1+\rho_+)/2\\ u_+=\rho_+\\ v_+=(1+\rho_+)/2\\ u_-=\rho_-\\ v_-=(1-\rho_-)/2\\ u_1=\rho_-+\rho_+\\ v_1=(1-\rho_-+\rho_+)/2\end{cases}$$

Where $0\le u_\beta,v_\beta\le1$, $\beta\in\{0,+,-,1\}$, $\oplus$: Asynchronous addition, +: Synchronous addition.

Using $\{u,v\}$ signals, each u_β (v_β) determines a fixed position in relevant histogram to make vector X on a position. After completing 2^N data sequences, eight symmetry/anti-symmetry histograms of $\{H(u_\beta\mid f)\}$, $(\{H(v_\beta\mid f)\})$, $\beta\in\{0,+,-,1\}$ are generated.

4 Simulation Results

The simulation provides a series of output results. In this section, two cases are selected: $N=\{12, 13\}$, $n=2$, $j=0$,

$$\{f=3,\ f_+=11,\ f_-=2\} \quad \text{and} \quad \{f=12,\ f_+=14,\ f_-=8\}.$$

These correspond to double path, right path, left path, symmetry and non-symmetry conditions respectively. For convenience of comparison, sample cases are shown in Figures 3(a-c). In Figure 3(a), representation patterns are illustrated. Figure 3(b) represents $f=3$ conditions and Figure 3(c) represents $f=12$ conditions respectively. Eight histograms of $H(u_+ \mid f)=H(u_- \mid f)$ are shown with results represented by symmetric meta-functions in four groups.

5 Analysis of Results

5.1 Visual Distributions

In $H(u_+ \mid f)=H(u_- \mid f)$ conditions, $\{\ H(u_1 \mid f), H(v_1 \mid f)\ \}$ have significant interference patterns different from other conditions. Output results are balanced.

5.2 Particle Statistical Distributions

For all symmetric or non-symmetric cases under $\oplus$ asynchronous addition operations, relevant values meet $0 \le u_0, v_0, u_-, v_-, u_+, v_+ \le 1$.

Checking $\{\ H(u_0 \mid f), H(v_0 \mid f)\ \}$ series, $\{\ H(u_+ \mid f), H(u_- \mid f)\ \}$ and $\{\ H(v_+ \mid f), H(v_- \mid f)\ \}$ satisfy following equation:

$$\begin{cases} H(u_0 \mid f)=H(u_- \mid f)+H(u_+ \mid f) \\ H(v_0 \mid f)=H(v_- \mid f)+H(v_+ \mid f) \end{cases}$$

The equation is satisfied even for different values of N and n.

5.3 Wave Interference Patterns

Different interference properties are observed clearly in $H(u_+ \mid f) = H(u_- \mid f)$ and $H(v_+ \mid f) = H(1-v_- \mid f)$ conditions. Under + synchronous addition operations, relevant values meet $0 \le u_1, v_1, u_-, v_-, u_+, v_+ \le 1$.

An examination of $\{\ H(u_1 \mid f), H(v_1 \mid f)\ \}$ distributions especially for cases in Figure 3(b-c) denoted $\{\ u_1, v_1\ \}$ showed the appearance of particularly significant interferences compared with $\{\ H(u_+ \mid f), H(u_- \mid f)\ \}$ and $\{\ H(v_+ \mid f), H(v_- \mid f)\ \}$. Spectra in different cases illustrate wave interference properties. From listed histogram distributions, the following are all satisfied:

$$\begin{cases} H(u_1 \mid f) \ne H(u_- \mid f) + H(u_+ \mid f) = H(u_0 \mid f) \\ H(v_1 \mid f) \ne H(v_- \mid f) + H(v_+ \mid f) = H(v_0 \mid f) \end{cases}$$

Single and double peaks are shown in interference patterns as classical double slit distributions.

5.4 Quaternion Measures

It is interesting to see the relationship between the variant quaternion and other measures.

In the variant quaternion, $\Delta f(X) = \langle N_\perp, N_+, N_-, N_T \rangle$, $N = N_\perp + N_+ + N_- + N_T$.

In Einstein's two-state system of interaction $\langle N_1, N_2, N_{12}, N_{21} \rangle$ allows the following equations to be established:

$$\begin{cases} N_1 = N_\perp + N_+ \\ N_2 = N_- + N_T \\ N_{12} = N_+ \\ N_{21} = N_- \\ N = N_1 + N_2 \end{cases}$$

From the equations, the measured pair $\{\ N_{21}, N_{12}\ \}$ has a 1-1 correspondence to$\{\ N_{-}, N_{+}\ \}$.

Selecting $+ \to 1$, $- \to 0$, CHSHs $N_{\pm,\mp}(a,b)$ measures meet

$$\begin{cases} N_{+,+}(a,b) \to N_T \\ N_{+,-}(a,b) \to N_{-} \\ N_{-,+}(a,b) \to N_{+} \\ N_{-,-}(a,b) \to N_{\perp} \end{cases}$$

$$(N_{++}, N_{+-}, N_{-+}, N_{--}) \to (N_T, N_{-}, N_{+}, N_{\perp}),$$

Let $N = N_{++} + N_{+-} + N_{-+} + N_{--}$, CHSH quaternion is a permutation of the variant quaternion.

Aspect's quaternion $(N_t, N_r, N_c, N_\omega)$ have following corresponding:

$$\begin{cases} N_t \to N_{-} \\ N_r \to N_{+} \\ N_\omega \to N \end{cases}$$

There is no parameter in the variant quaternion for parameter N_c. It indicates joined action numbers to distinguish single and double paths, corresponding to $\{\ u_0, v_0\ \}$and $\{\ u_1, v_1\ \}$ times. In an actual experiment, this is a significant parameter. In a simulated system, the parameter serves as a control coefficient that separates two types of measured paths $\{\ u_0, v_0\ \}$ and $\{\ u_1, v_1\ \}$ in the integration of comparisons with real experiments.

6 Conclusions

In an analysis of N bit 0-1 vector and its exhaustive sequences for variant measurement, this system simulates double path interference properties through different accurate distributions. Using this model, two groups of parameters $\{\ u_\beta\ \}$ and $\{\ v_\beta\ \}$ describe left path, right path, double paths for particle and

double path for wave with distinguished symmetry and anti-symmetry properties.
Under synchronous conditions, the double path system provides wave-like interference patterns that are not consistent with those that would be anticipated using a classical model.
Compared with the variant quaternion and other quaternion structures, it is helpful to understand the possible properties of the uses and limitations of variant simulation systems.
The complexity of n-variable function space has a size of 2^{2^n}. Whole simulation complexity is determined by $O(2^{2^n} \times 2^N)$ as ultra exponent productions. How to overcome the limitations imposed by such complexity and how best to compare and contrast such simulations with real world experimentation will be key issues in future work.

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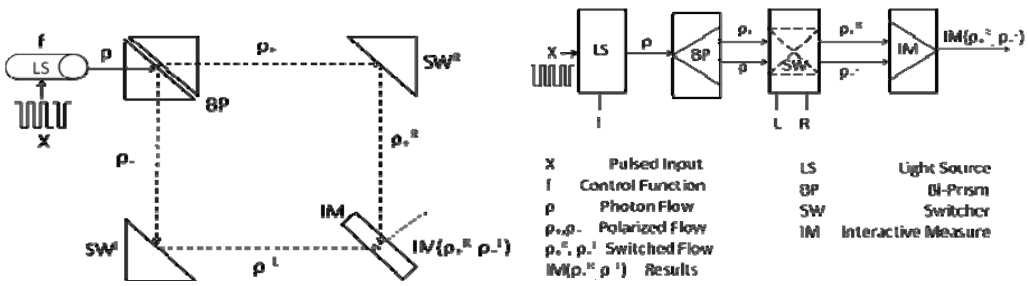


(a) (b)

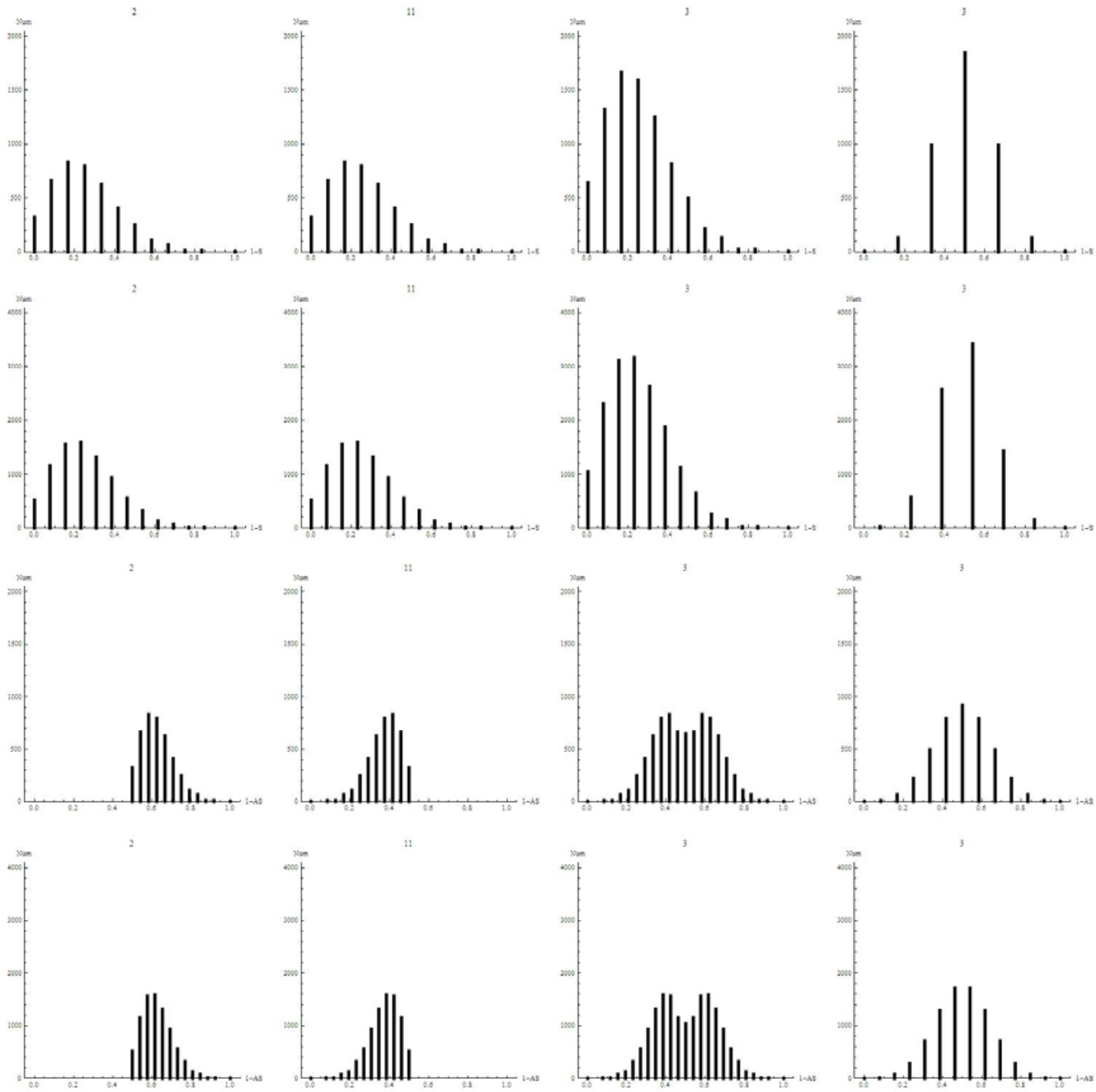
Figure 1: Double Path Model (a) Mach-Zehnder Double Path Model
(b) Description Model

Table 1. Two Variable Logic Functions and Variable Logic Representation (n=2, j=0)

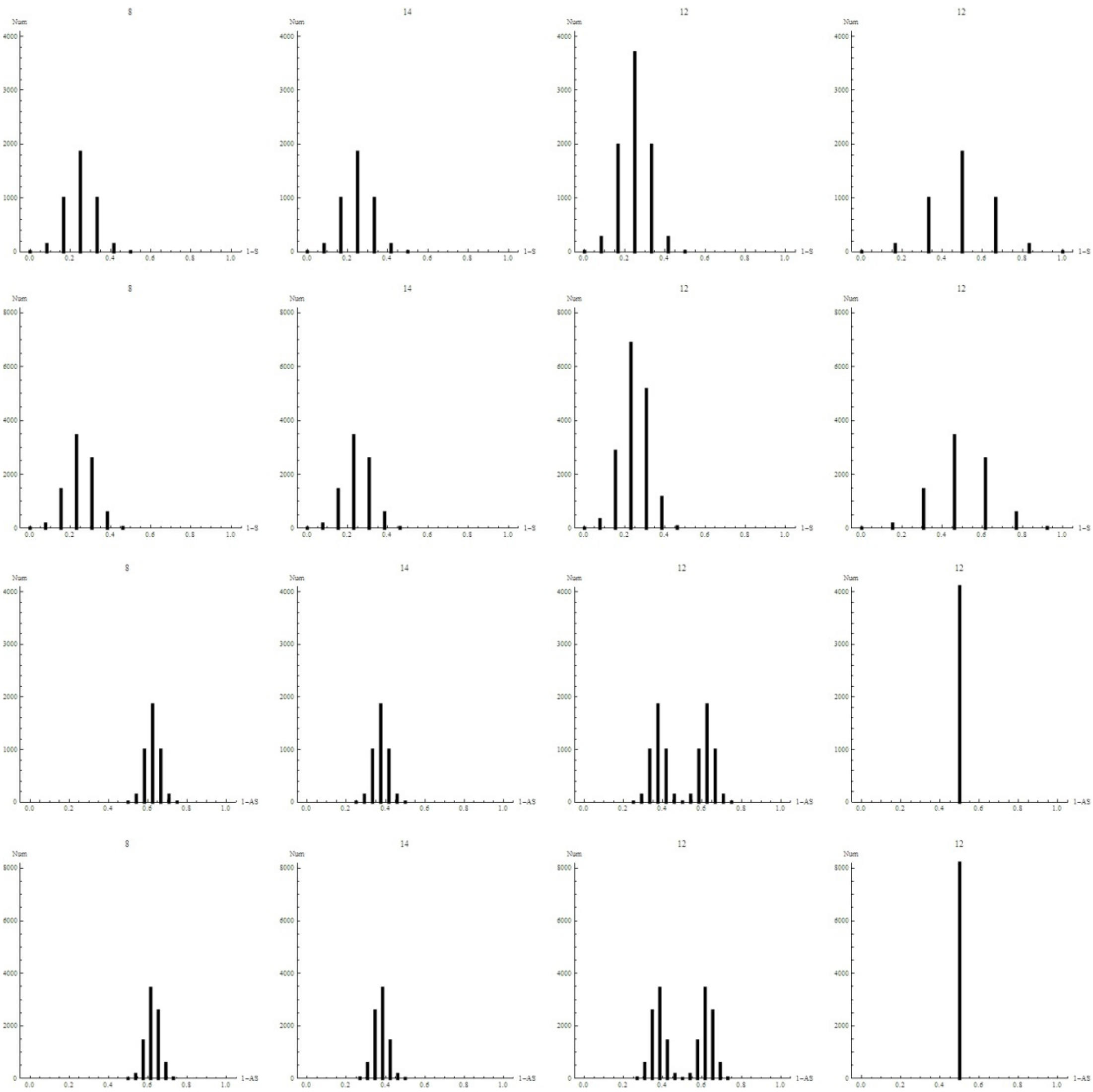
f No.	$f \in S(2)$	3 11	2 10	1 01	0 00	$f_+ \in S_0^0(2)$	3^0 11^0	2^1 10^1	1^0 01^0	0^1 00^1	$f_- \in S_1^0(2)$
0	$\{\varnothing\}$	0	0	0	0	$\langle\varnothing\|$	1	0	1	0	$\|3,1\rangle$
1	$\{0\}$	0	0	0	1	$\langle 0\|$	1	0	1	1	$\|3,1\rangle$
2	$\{1\}$	0	0	1	0	$\langle\varnothing\|$	1	0	0	0	$\|3\rangle$
3	$\{1,0\}$	0	0	1	1	$\langle 0\|$	1	0	0	1	$\|3\rangle$
4	$\{2\}$	0	1	0	0	$\langle 2\|$	1	1	1	0	$\|3,1\rangle$
5	$\{2,0\}$	0	1	0	1	$\langle 2,0\|$	1	1	1	1	$\|3,1\rangle$
6	$\{2,1\}$	0	1	1	0	$\langle 2\|$	1	1	0	0	$\|3\rangle$
7	$\{2,1,0\}$	0	1	1	1	$\langle 2,0\|$	1	1	0	1	$\|3\rangle$
8	$\{3\}$	1	0	0	0	$\langle\varnothing\|$	0	0	1	0	$\|1\rangle$
9	$\{3,0\}$	1	0	0	1	$\langle 0\|$	0	0	1	1	$\|1\rangle$
10	$\{3,1\}$	1	0	1	0	$\langle\varnothing\|$	0	0	0	0	$\|\varnothing\rangle$
11	$\{3,1,0\}$	1	0	1	1	$\langle 0\|$	0	0	0	1	$\|\varnothing\rangle$
12	$\{3,2\}$	1	1	0	0	$\langle 2\|$	0	1	1	0	$\|1\rangle$
13	$\{3,2,0\}$	1	1	0	1	$\langle 2,0\|$	0	1	1	1	$\|1\rangle$
14	$\{3,2,1\}$	1	1	1	0	$\langle 2\|$	0	1	0	0	$\|\varnothing\rangle$
15	$\{3,2,1,0\}$	1	1	1	1	$\langle 2,0\|$	0	1	0	1	$\|\varnothing\rangle$

N	Left Path	Right Path	Double-Particle	Double-Wave	Conditions
12	$H(u_+ \mid f)$	$H(u_- \mid f)$	$H(u_\mathbf{0} \mid f)$	$H(u_\mathbf{1} \mid f)$	Symmetric Meta Distributions $H(u_+ \mid f) = H(u_- \mid f)$
13	$H(u_+ \mid f)$	$H(u_- \mid f)$	$H(u_\mathbf{0} \mid f)$	$H(u_\mathbf{1} \mid f)$	

(a) Statistical Histogram Patterns



(b) $N = \{12,13\}$, $f = 3$, Histograms of Symmetric Meta Distributions



(c) $N = \{12,13\}$, $f = 12$, Histograms of Symmetric Meta Distributions

Figure 3: Results of Symmetric Meta Distributions