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Optimal Treatment of Queueing Model for Highway

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Abstract

We develop some analytic queueing models based on traffic and we model the behavior of traffic flows as a function of some of the most relevant determinants. These analytic models allow for parameterized experiments, which pave the way towards our research objectives: assessing what-if sensitivity analysis for traffic management, congestion control, traffic design and the environmental impact of road traffic. We illustrate our results for a highway.

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1 Introduction

When modeling the environmental impact of road traffic, we can distinguish between both a static and dynamic impact of infrastructures and vehicles on emissions and waste.

On the one hand, roads can be considered as a visual intrusion. In addition, they may cause damage to natural watercourses or threaten the natural habitat of wildlife. Vehicles in turn consume natural resources and impose a strain on the environment at the end of their life cycle.

On the other hand, as they form part of traffic flows, infrastructures and vehicles also have a dynamic impact on the environment. Vehicles in use produce emissions and noise. Toxic fumes escape in the atmosphere when fuel tanks are filled, while driving leads to further emissions ($CO2$, $NO2$ and $SO2$) and dust. Furthermore, an increase in garbage, accidents (physical and material damage) and, occasionally, distortion of infrastructures and nature elements (trees, animals,..,etc.) can be observed. Because traffic flows are a function of both the number of vehicles on the roads and the vehicle speed, as traffic flows occupy a central position in the assessment of road traffic.

The objective of this approach is mainly explorative and explanatory. These descriptive models give an empirical justification of the well-known speed-flow and speed-density diagrams, but are limited in terms of predictive power and the possibility of sensitivity analysis [5,7,8]. An alternative approach is to use speed-flow, speed-density and flow-density diagrams, in which data on traffic flows are collected and are fit into curves [4,6].

Compared to these descriptive models, this paper presents a more operational approach using queueing theory. Queueing theory is almost exclusively used to describe traffic behavior at signalized and unsignalized intersections [1,2,3,4,7,9].

2 Decryption of Queueing Model with traffic flow theory

On of the most important equations in traffic flow theory incorporates the interdependence of traffic flow q, traffic density E and speed s:

$$q = E \cdot s \tag{1}$$

When two of the three variables are known, the third variable can easily be obtained. If traffic count data are available, traffic flows can be assumed as given, which leaves us to calculate either traffic density or speed to complete the formula and use either as input for the appropriate queueing model.

Table 1: Overview of used parameters

Parameter	Description
E	Traffic density (vehicle/km)
C	Maximum traffic density (vehicle/km)
s	Effective speed (km/h)
r	Relative speed
SN	Nominal Speed (km/h)
q	Traffic flow (vehicle/h)
λ	Arrival rate (vehicle/h)
μ	Service rate (vehicle/h)
ρ	Traffic intensity=λ/μ
W	Time in the system (h)

In our model we define C as the maximum traffic density. Roads are divided into segments of equal length $1/C$, which matches the minimal length needed by one vehicle on that particular road. Each road segment is considered as a service station, in which vehicles arrive at rate λ and get served at rate μ (Figure1).

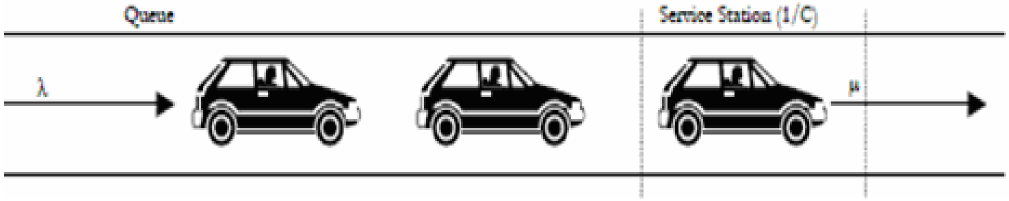

Figure 1: Queuing Representation of traffic flows

We define W as the total time a vehicle spends in the system, which equals the sum of waiting time and service time. The higher the traffic intensity, the higher the time in the system becomes (the exact relation between W and ρ depends upon the queueing model. When W is known, the effective speed can easily be calculated as:

$$s = \frac{1/C}{W} \tag{2}$$

The relative speed r, by definition:

$$r = \frac{s}{SN} = \frac{1/C}{W \cdot SN} \tag{3}$$

Queueing models are often referred to using the Kendall notation, consisting of several symbols - e.g. M/G/1. The first symbol is shorthand for the distribution of inter-arrival times, the second for the distribution of service times and the last one indicates the number of servers in the system.

3 Analysis of M/M/1 MODEL

The inter-arrival times are exponentially distributed (the arrival rate follows a Poisson distribution) with expected inter-arrival time equal to $1/\lambda$ (with λ equal to the product of the traffic density *E* and the nominal speed *SN*). The service time delineates the time needed for a vehicle to pass one road segment and is exponentially distributed with expected service time μ (the service rate follows a Poisson distribution). When a vehicle drives at nominal speed *SN*, service time can be written as: $1/(SN \cdot C)$ and μ equals the product of nominal speed *SN* with the maximum traffic density *C*. Using these formulas for λ and μ, we obtain W as:

$$\mathrm{W} = \frac{1}{\mu - \lambda} = \frac{1}{SN \cdot (C - E)} \qquad (4)$$

Using this expression for W, the effective speed and relative speed are obtained:

$$s = \frac{SN \cdot (C - E)}{C} = SN \cdot (1 - \rho) \qquad r = \frac{s}{SN} = 1 - \rho \qquad (5)$$

with ρ the traffic intensity:

$$\rho = \frac{\lambda}{\mu} = \frac{E}{C} \qquad (6)$$

Substituting for *E* (= *q/s*) in (5) the following expression is obtained:

$$f(s,q) = s^2 \cdot C - s \cdot \ C \ \cdot SN + SN \cdot q = 0 \qquad (7)$$

Maximizing $f(s,q)$ for *s* and substituting this value into (7), $q_{\max}$ can be written

$$q_{\max} = \frac{SN \cdot C}{4} \qquad (8)$$

Traffic density is low; vehicles do not obstruct one another, which lead to higher effective speeds. When more vehicles arrive on the road, the effective speed *s* decreases.

Using equation (1): $q = E \cdot s$ and the above formula for *s*, we can construct the speed-flow and the flow-density diagrams for the M/M/1 model. The speed-flow diagram is the envelope of all possible combinations of the effective speed and traffic flow. There are two speeds for every traffic flow: an upper branch (*s2*) where speed decreases with flow and a lower branch (*s1*) with an increasing speed in terms of flow. An intuitive explanation can be as follows: as the flow moves from *SN* to $q_{\max}$, congestion increases but the flow rises because the decline in speed is offset by the higher. If traffic continues to enter the flow past q_{max}, flow falls because the decline in speed more than offsets the additional vehicle numbers further increasing congestion, [1].

The M/M/1 model is interesting as a base case, but is inadequate to represent real-life traffic flows. In the next two sections we will relax the M/M/1model:

- first, the service times follow a general distribution (M/G/1) and,
- secondly, both arrival and service times follow a general distribution (G/G/1).

As in the M/M/1 model inter-arrival times follow an exponential distribution with expected inter-arrival time $1/\lambda$, λ being the product of traffic density and nominal speed. The service time however is generally distributed with an expected service time of $1/\mu$ and a standard deviation of σ. Expected service rate is μ, which equals the product of nominal speed *SN* with maximum traffic density *C*. Combining Little's theorem and the Pollaczek-Khintchine formula for *L* (defined as the average number of cars in the system) [5,8,7,9] and substituting for λ and μ, we obtain the following formula for the total time in the system W:

$$W = \frac{1}{SN \cdot C} + \frac{\rho^2 + SN^2 \cdot E^2 \cdot \sigma^2}{2 \cdot SN \cdot E \cdot (1-\rho)} \tag{9}$$

Using the above expression for W, effective and relative speed can be calculated in an analog way as in the M/M/1 model:

$$s = \frac{2 \cdot SN \cdot (C - E)}{2 \cdot C + E \cdot (\beta^2 - 1)} = \frac{2 \cdot SN \cdot (1-\rho)}{2 + \rho \cdot (\beta^2 - 1)} \qquad r = \frac{2 \cdot (1-\rho)}{2 + \rho \cdot (\beta^2 - 1)} \tag{10}$$

with β delineating the coefficient of variation of service time (or $\beta = \sigma \cdot SN \cdot C$).

Using these formulas we can construct the speed-flow, speed-density and flow-density diagrams for the M/G/1 model. The exact shape of these curves depends upon the variation coefficient of the service time, β.

Substituting *E* (= *q/s*) in above formula (8) and rewriting, the following expression for the speed-flow diagram is obtained:

$$f(s,q) = 2\cdot C\cdot s^2 + \left\lfloor q\cdot(\beta^2-1) - 2\cdot C\cdot SN \right\rfloor \cdot s + 2\cdot q\cdot SN = 0 \quad (11)$$

Maximizing this equation for *s*, we can calculate the maximum traffic flow ($q_{\max}$):

$$q_{\max} = 2\cdot SN\cdot C\cdot\left[\frac{\sqrt{\beta^2+1}-\sqrt{2}}{\beta-1}\right]^2 \qquad \beta \geq 0$$
$$q_{\max} = \frac{SN\cdot C}{4} \qquad \beta = 1 \quad (12)$$

The value of $q_{\max}$ is a function of the variation parameter β.

Using the above expression for W, effective and relative speed can be calculated in an analog way as in the M/M/1 model. With the G/G/1 model both arrival times and service times follow a general distribution with expected arrival time $1/\lambda$ and standard deviation σ_a, expected service times $1/\mu$ and standard deviation of σ_b, respectively.

Consequently, the shape of the speed-flow-density diagrams will depend not only on the variance of the service times but also on the variance of the inter-arrival times. Combining Little's theorem and [2, 3, 5, 6, 7] formula for *L* and substituting for λ and μ, we obtain the following formulas for the total time in the system W :

$$W = \frac{1}{SN\cdot C} + \frac{\rho^2\cdot(c_\alpha^2+c_s^2)}{2\cdot SN\cdot E\cdot(1-\rho)}\cdot e^{\frac{-2\cdot(1-\rho)\cdot(1-c_\alpha^2)^2}{3\cdot\rho\cdot(c_\alpha^2+c_s^2)}} \quad , \quad c_\alpha^2 \leq 1$$

$$W = \frac{1}{SN\cdot C} + \frac{\rho^2\cdot(c_\alpha^2+c_s^2)}{2\cdot SN\cdot E\cdot(1-\rho)}\cdot e^{\frac{-\cdot(1-\rho)\cdot(c_\alpha^2-1)^2}{(1+\rho)\cdot(c_\alpha^2+10c_s^2)}} \quad , \quad c_\alpha^2 > 1$$

with c_α^2 representing the squared coefficient of variation of inter-arrival times and c_s^2 the squared coefficient of variation of service time.

Using (5) and the above expressions for W, the effective speed formulas become:

$$s = \frac{2 \cdot SN \cdot (1-\rho)}{2 \cdot (1-\rho) + \rho \cdot (c_\alpha^2 + c_s^2) \cdot e^{\frac{-2\cdot(1-\rho)\cdot(1-c_\alpha^2)^2}{3\cdot\rho\cdot(c_\alpha^2+c_s^2)}}} \quad , \quad c_\alpha^2 \leq 1$$

$$s = \frac{2 \cdot SN \cdot (1-\rho)}{2 \cdot (1-\rho) + \rho \cdot (c_\alpha^2 + c_s^2) \cdot e^{\frac{-\cdot(1-\rho)\cdot(c_\alpha^2-1)^2}{(1+\rho)\cdot(c_\alpha^2+10c_s^2)}}} \quad , \quad c_\alpha^2 > 1$$

The exact shape of the diagrams depends not only on the variation coefficient of service times but also on the variation coefficient of inter-arrival times.

4 Main Results

We see that the variance on the arrival rate ($c_a = 1$ and $c_s = 0$) has a larger impact than the variance on the service rate ($c_a = 0$ and $c_s = 1$).
Actions to increase traffic flow should primarily be focused on the arrival rate variance. A similar conclusion can be obtained using the flow- density diagram. Finally, the speed-density diagram is constructed for a given density of 40 vehicles per km: we see that the effective speed ranges from approximately 50 (high variance) to approximately 110 (low variance) km/h.
The results can easily be compared with the constructed speed-flow-density diagrams.

For the case with high variances (c_a and c_s both equal to one), at hour 8.00, 9.00 and 10.00 a.m., the observed traffic flow becomes larger than the maximum possible traffic flow on the highway given these variance parameters. Consequently there are no speeds that can be calculated for these instances.

Table 2: Upper and lower speeds for highway

q_{max}		4350		3551		2983		2165	
		$c_a = c_s = 0.5$		$c_a = 0, c_s = 1$		$c_a = 1, c_s = 0$		$c_a = c_s = 1$	
Hour	Q (vech/h)	S1	S2	S1	S2	S1	S2	S1	S2
1	340	5	120	5	120	5	119	5	117
2	225	3	120	3	120	3	120	3	118
3	178	2	120	2	120	2	120	2	119
4	180	2	120	2	120	2	120	3	119
5	303	4	120	4	120	4	119	4	117
6	787	11	120	11	120	11	116	12	110
7	1826	26	120	28	109	29	106	36	88
8	3180	50	117	60	116	-	-	-	-
9	2612	39	119	45	119	47	95	-	-
10	2235	33	120	36	119	37	100	-	-
11	2109	31	120	34	120	35	102	50	77
12	2016	29	120	32	120	33	103	44	81
13	1978	29	120	31	120	32	104	42	83
14	2095	31	120	33	119	34	102	49	78
15	1911	28	120	30	120	31	105	39	85
16	1892	27	120	30	120	30	105	38	86
17	1987	2 9	120	31	120	32	104	42	82
18	1877	29	120	31	120	32	104	42	83
19	1964	29	120	31	120	32	104	41	83
20	1755	25	120	27	120	27	106	33	90
21	1243	18	120	18	120	18	112	21	102
22	1008	14	120	15	120	15	114	16	106
23	762	11	120	11	120	11	116	12	110
24	562	8	120	8	120	8	117	8	113

5 Conclusion

Using several queueing models, speed is determined, based on different arrival and service processes. The exact shape of the different speed-flow-density diagrams is largely determined by the model parameters. Therefore we believe that a good choice of parameters can help to adequately describe reality. We illustrated this with an example, using the most general models for a highway. Our models can be effectively used to assess the environmental impact of road traffic.

References

[1] C.F. Daganzo, *Fundamentals of Transportation and Traffic Operations*, Elsevier ScienceLtd., Oxford, 1997.

[2] Hamady A. Taha, *Operation Research and Introduction*, 2002, Pearson Education Publishing Ltd, New Delhi, 9th edition 2010.

[3] D. Heidemann, Queue length and delay distributions at traffic signals, Transportation Research-B, **28**B, (1994), 377-389.

[4] D. Heidemann, A queueing theory approach to speed-flow-density relationships, Transportation and Traffic Theory, *Proceedings of the 13th International Symposium on Transportation and Traffic Theory*, Lyon, France, 14-26 July 1996.

[5] Ivo Adan and Jacques Rasing, *Queueing theory*, Eindhoven University of Technology, 2002.

[6] R. Jain and J.M. Smith, Modelling vehicular traffic flow using M/G/C/C state dependent queueing models, *Transportation Science*, **31**, (1997), 324-336.

[7] Moshe Zukerman, *Introduction to Queueing Theory and Stochastic Teletraffic Model*, Teaching Textbook Manuscript, Retrieved from http://www.ee.cityu.edu.hk/~zukerman/classnotes.pdf

[8] Prem Kumar Gupta and D.S. Hira, *Operations Research*, 2003.

[9] U.Narayan Bhat, *An Introduction to Queueing Theory Modeling and Analysis in Applications*, Birkhauser, Boston, 2008.