Journal of Applied Mathematics and Bioinformatics, vol. 9, no. 2, 2019, 1-16
ISSN: 1792-6602(print), 1792- 6939(online)
Scientific Press International Limited

Dynamic analysis of Non-Uniform Rayleigh beam Resting on Bi-Parametric Subgrade under Exponentially Varying Moving Loads

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Abstract

The response of non-uniform Rayleigh beam resting on bi-parametric subgrades and subjected to exponentially varying magnitude moving load is investigated in this paper. The governing equation is fourth order partial differential equation with variable coefficient. In order to solve this problem, the versatile Galerkin's method is used to reduce the governing equation to a second order ordinary differential equation. For the solution of this equation, Laplace transformation and convolution theorem are employed. Numerical results in plotted curves are then presented. The results show that response amplitude of the non-uniform Rayleigh beam decreases as the shear modules (G) increases. Also, the deflection profile of the beam decreases with an increasing values of the foundation modulus (k). Furthermore, as the values of the axial force (N), rotatory inertia (R_0^2), and damping coefficient (ε) increases, the response amplitudes of the beam subjected to exponentially varying magnitude moving load decreases. Finally, it was observed that the non-uniform beam undergoes downward deflection profiles from the origin when the effects of each of the parameters such as shear modules, rotatory inertia and damping coefficient on the beam are considered while upward deflection profiles from the origin when the effects of foundation modulus and axial force are noticeable.

Keywords: Bi-parametric subgrades, non-uniform beam, exponentially varying moving load, damping term.

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Article Info: Received: June 12, 2019. Revised: August 1, 2019.
Published online: September 15, 2019.

1. Introduction

In recent years considerable attention has been given to the response of elastic beams on an elastic foundation which is one of the structural engineering problems of theoretical and practical interest. A large number of studies have been devoted to the subject. In most of these studies, beam problems have largely been restricted to the case when the mass per unit length and moment of inertia of the beam structures are constant. In particular, works on non-uniform beam is still not too common in the literatures. Also, in most of these studies, Winkler foundation model are been considered. Studies in which two parameters foundation models are considered are very scanty.

In the governing equation of a non-uniform beam, the flexural rigidity and mass per unit length of the beam become certain functions of the spatial coordinate x. This renders the exact solution for the dynamical problem impossible or difficult to obtain as the governing partial differential equation now has variable coefficient. Amongst some of the earlier researchers that considered the dynamic analysis of elastic beam under moving load was Pestel [1] who applied Rayleigh-Ritz techniques to reduce the problem defined by a continuous differential equation to an approximate system of discrete differential equations with analytic coefficients. The system was reduced by a finite difference scheme for solution, but no numerical results were presented. Ayre et al [2] similarly used infinite series method to obtain the exact solution for the effect of the ratio of the weight of the load to the weight of a simply supported beam for a constant moving mass load. Furthermore, Casonik et al [3] studied the problem of vibrations of Bernoulli-Euler beams on variable winkler foundation. The load acting on the beam in this problem was static, Kenny [4] also investigated the dynamic response of infinite beams on elastic foundation under the action of moving load of constant speed. He included in the governing equation the effect of viscous damping. In more recent development, some of the other researchers that considered the dynamic response of elastic structures under moving loads include Oni and Awodola [5], Huang and Leissa [6], Muscolino and Palmeri [7], Oni and Omolefe [8], Chang and Liu [9].

In the above-mentioned researched works, only uniform structural members lie on the winkler foundation with foundation stiffness K are considered. However, for practical importance, the cross section of some structural members such as bridge, girders, hull of ships, concrete slabs etc. vary from one point to another along the structural members. Also, Winkler foundation model has shortcomings because of its discontinuous behaviour of the surface displacement beyond the load region which is contrary to observation made in practice. Thus, researchers who considered non-uniform beams resting on non-winkler foundation in their works are: Oni and Jimoh [10], Oni and Jimoh [11], Jimoh and Ajoge [12], Jimoh and Ajoge [13], Jimoh [14].

To the best of authors knowledge, Rayleigh beam moving load problem in which the beam under consideration rest on bi-parametric subgrades and is of non-uniform has not been tackled. The present paper is concerned with the response of a non-uniform Rayleigh elastic beam continuously supported by elastic subgrades and traversed by an exponentially varying magnitude moving loads.

2. The Governing Equation

The governing partial differential equation that described the dynamic behavior of a non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load is given by

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 V(x,t)}{\partial x^2}\right)+\mu(x)\frac{\partial^2 V(x,t)}{\partial x^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}$$

$$-R_0^2\,\frac{\partial}{\partial x}\left(\mu(x)\frac{\partial^2 V(x,t)}{\partial x\partial t^2}\right)+\varepsilon\,\frac{\partial V(x,t)}{\partial t}$$

$$=P(x,t)-P_G(x,t) \tag{1}$$

Where
$\mu(x)$ = variable mass per unit length of the beam
$I(x)$ = variable moment of inertia
N = Axial force
ε = damping coefficient
R_0^2 = Rotatory inertia
E = Young modulus
x = spatial coordinate
t = time coordinate
P = is the applied force (which in this present work is a moving load)
P_G= is the foundation reaction
The relationship between the foundation reaction and the lateral deflection $Y(x,t)$ is given by Kerr [15]

$$P_G(x,t)=-\left(\frac{G\partial^2 V(x,t)}{\partial x^2}-KV(x,t)\right) \tag{2}$$

Where K and G are foundation stiffness and shear modulus respectively.
The associated boundary conditions at the ends $x=0$ and $x=L$ are given by

$$V(0,x) = 0 = \frac{\partial V(L,t)}{\partial x} \tag{3}$$

and the initial conditions are

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \tag{4}$$

For the variable moment of inertia $I(x)$ and the mass per unit length $\mu(x)$ of the beam, we adopt the example in [16] and take $I(x)$ and $\mu(x)$ to be of the form

$$I(x) = I_0\left(1 + \frac{\sin \pi x}{L}\right)^3 \tag{5}$$

$$\mu(x) = V_0\left(1 + \frac{\sin \pi x}{L}\right) \tag{6}$$

Furthermore, the exponentially varying magnitude moving force take the form

$$P(x,t) = Pe^{-dt} f(x - ct) \tag{7}$$

Where
P is the moving force of constant magnitude and $f(*)$ is the dirac-delta function.
By substituting (2), (5), (6) and (7) into (1), we obtain

$$EI_0 \frac{\partial^2}{\partial x^2}\left(\left(1 + \frac{\sin \pi x}{L}\right)^3 \frac{\partial^2 V(x,t)}{\partial x^2}\right) + \mu_0\left(1 + \frac{\sin \pi x}{L}\right)\frac{\partial^2 V(x,t)}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2}$$

$$-R_0^2\mu_0 \frac{\partial}{\partial x}\left(\left(1 + \frac{\sin \pi x}{L}\right)\frac{\partial^3 V(x,t)}{\partial x \partial t^2}\right) + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2}$$

$$= Pe^{-dt} f(x - ct) \tag{8}$$

By simplifying (8), we obtain

$$EI_0\left(\frac{5}{2}+\frac{15\sin\pi x}{4L}-\frac{\sin 3\pi x}{4L}-\frac{3\cos 2\pi x}{2L}\right)\frac{\partial^4 V(x,t)}{\partial x^4}$$

$$+EI_0\left(\frac{9\pi\sin 3\pi x}{4L^3}-\frac{15\sin\pi x}{4L^3}-\frac{6\pi^2\cos 2\pi x}{L^3}\right)\frac{\partial^2 V(x,t)}{\partial x^2}+\mu_0\frac{\partial^2 V(x,t)}{\partial t^2}$$

$$+\mu_0\frac{\sin\pi x}{L}\frac{\partial^2 Y(x,t)}{\partial x^2\partial t^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}-R_0^2\mu_0\left(1+\frac{\sin\pi x}{L}\right)\frac{\partial^2 Y(x,t)}{\partial x^2\partial t^2}$$

$$-R_0^2\frac{\pi\mu_0\cos\pi x}{L}\frac{\partial^3 Y(x,t)}{\partial x\partial t^2}+\varepsilon\frac{\partial V(x,t)}{\partial t}+KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2}$$

$$=Pe^{-dt}f(x-ct) \tag{9}$$

To the best of authors knowledge, a closed form solution to the second order partial differential equation (9) does not exist. Consequently, an approximate analytical solution is desirable to obtain some vital information about the vibrating system.

3. Approximate Analytical Solution of the Mathematical problem

In order to solve the beam problem in equation (9) above, we shall use the versatile technique called Galerkin's method. This solution technique involves solving equation of the form

$$\Gamma(V)-P=0 \tag{10}$$

Where

Γ = the differential operator (linear or non-linear)

V = the structural displacement

P = the transverse load acting on the structure

To this effect, the Galerkin's method requires that the solution of equation (9) takes the form

$$V_k(x,t)=\sum_{k=1}^{n} q_k(t)Q_k(x) \tag{11}$$

Where $Q_k(x)$ is chosen such that the desired boundary conditions are satisfied.

Equation (11) when substituted into equation (9) yields

$$\sum_{k=1}^{n}\left\{\frac{EI_0}{4}\left(10+\frac{15\sin\pi x}{L}-\frac{\sin 3\pi x}{L}-\frac{6\cos 2\pi x}{L}\right)Q_k^{iv}(x)q_k(t)\right.$$
$$+\frac{EI_0}{4L^2}\left(\frac{9\pi\sin 3\pi x}{L}-\frac{15\sin\pi x}{L}-\frac{24\pi^2\cos 2\pi x}{L}\right)Q_k^{ii}(x)q_k(t)$$
$$+\mu_0\left(1+\frac{\sin\pi x}{L}\right)Q_k(x)\ddot{q}_k(t)-N\,Q_k^{ii}(x)q_k(t)$$
$$-R_0^2\mu_0\left(1+\frac{\sin\pi x}{L}\right)Q_k^{ii}(x)\ddot{q}_k(t)-R_0^2\frac{\mu_0\pi}{L}\frac{\cos\pi x}{L}Q_k^{i}(x)\ddot{q}_k(t)$$
$$+\ \varepsilon\,Q_k(x)\dot{q}(t)+\ K\,Q_k(x)q(t)-\ G\,Q_k^{ii}(x)q_k(t)$$
$$\left.-Pe^{-dt}f(x-ct)\right\}\ =\ 0 \tag{12}$$

In order to determine $q_k(t)$, it is required that the expression on the left hand side of equation (13) be orthogonal to the function $Q_m(x)$

$$\int_0^L\sum_{k=1}^{n}\left\{\frac{EI_0}{4}\left(10+\frac{15\sin\pi x}{L}-\frac{\sin 3\pi x}{L}-\frac{6\cos 2\pi x}{L}\right)Q_k^{iv}(x)q_k(t)\right.$$
$$+\frac{EI_0}{4L^2}\left(\frac{9\pi\sin 3\pi x}{L}-\frac{15\sin\pi x}{L}-\frac{24\pi^2\cos 2\pi x}{L}\right)Q_k^{ii}(x)q_k(t)$$
$$+\mu_0\left(1+\frac{\sin\pi x}{L}\right)Q_k(x)\ddot{q}_k(t)-N\,Q_k^{ii}(x)\ q_k(t)$$
$$-R_0^2\mu_0\left(1+\frac{\sin\pi x}{L}\right)Q_k^{ii}(x)\ddot{q}_k(t)-R_0^2\frac{\mu_0\pi}{L}\frac{\cos\pi x}{L}Q_k^{i}(x)\ddot{q}_k(t)$$
$$\left.+\ \varepsilon\,Q_k(x)\dot{q}(t)+\ K\,Q_k(x)q(t)-\ G\,Q_k^{ii}(x)q_k(t)\right\}\ Q_m(x)dx$$
$$=\ \int_0^L Pe^{-dt}f(x-ct)\ \ Q_m(x)dx \tag{13}$$

Since the elastic beam has simple support at *x = 0* and *x = L*, we choose

$$Q_k\,(x)=\frac{\sin k\pi x}{L} \tag{14}$$

$$Q_m\,(x)=\frac{\sin m\pi x}{L} \tag{15}$$

By substituting (14) and (15) into (13), after some rearrangements, and ignoring the summation sign, we obtain

$$
\begin{aligned}
\int_0^L \{ [-R_0^2\mu_0\left(\frac{k\pi}{L}\right)^2 (&\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} + \frac{\sin \pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}) \\
&- R_0^2\mu_0\left(\frac{k\pi}{L}\right)^2 \frac{\cos \pi x}{L}\frac{\cos k\pi x}{L}\frac{\sin m\pi x}{L} \\
&+ \mu_0(\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} + \frac{\sin \pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L})]\, \ddot{q}_k(t) \\
&+ \varepsilon\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}\,\dot{q}_k(t) \\
&+ [\frac{EI_0}{4}\left(\frac{k\pi}{L}\right)^4 \left(\frac{10\sin k\pi x}{L}\frac{\sin m\pi x}{L}\right. \\
&+ 15\frac{\sin \pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} - \frac{\sin 3\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} \\
&\left.- \frac{6\cos 2\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}\right) \\
&- \frac{EI_0}{4L^2}\left(\frac{k\pi}{L}\right)^2 \left(\frac{9\pi^2\sin 3\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}\right. \\
&+ \frac{15\sin \pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} \\
&\left.- \frac{24\pi^2\cos 2\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}\right) \\
&- \left(\frac{k\pi^2}{L}\right)\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} + K\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L} \\
&- G\left(\frac{k\pi^2}{L}\right)\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}]\, q_k(t)\}\, dx \\
&= \int_0^L Pe^{-dt} f(x-ct)\ \frac{\sin m\pi x}{L}\ dx = Pe^{-dt}\frac{\sin m\pi ct}{L}
\end{aligned}
\tag{16}
$$

Equation (16) can be re-written as

$$R_{11}\ddot{q}_k(t) + R_{12}\dot{q}_k(t) + R_{13}q_k(t)\} = Pe^{-dt} sin\ b_0 t \tag{17}$$

Where

$$R_{11} = a_1(I_1 + I_2) - a_1 I_3 + \mu_0(I_1 + I_2) = (a_1 + V_0)(b_1 + b_2) - a_1 b_3 \tag{18}$$

$$R_{12} = \varepsilon\, I_1 \tag{19}$$

$$R_{13} = a_2\left(\frac{5}{2}I_1 + \frac{15}{4}I_2 - \frac{1}{4}I_4 - \frac{3}{2}I_5\right) - a_3\left(\frac{9\pi^2}{4L^2}I_4 - \frac{15}{4L^2}I_2 + \frac{6\pi^2}{L^2}I_5\right)$$

$$+(a_4 - a_5 + k)I_1 \qquad (20)$$

$$a_1 = -R_0^2\mu_0\left(\frac{k\pi}{L}\right)^2, a_2 = EI_0\left(\frac{k\pi}{L}\right)^4. a_3 = EI_0\left(\frac{k\pi}{L}\right)^2, a_4 = N\left(\frac{k\pi}{L}\right)^2, a_5$$

$$= G\left(\frac{k\pi}{L}\right)^2, b_0 = \frac{m\pi c}{L} \qquad (21)$$

$$I_1 = \int_0^L \frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}dx \qquad (22a)$$

$$I_2 = \int_0^L \frac{\sin \pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}dx \qquad (22b)$$

$$I_3 = \int_0^L \frac{\cos\ \pi x}{L}\frac{\cos\ k\pi x}{L}\frac{\sin m\pi x}{L}dx \qquad (22c)$$

$$I_4 = \int_0^L \frac{\sin 3\,\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}dx \qquad (22d)$$

$$I_5 = \int_0^L \frac{\cos 2\,\pi x}{L}\frac{\sin k\pi x}{L}\frac{\sin m\pi x}{L}dx \qquad (22e)$$

Evaluating the integral (22a – 22e), we have

$$I_1 = \frac{L}{2}\left[\frac{\sin(k-m)}{k-m} - \frac{\sin(k+m)}{k+m}\right] \qquad (23a)$$

$$I_2 = \frac{L}{4}\left[\frac{\cos(\pi+k+m)-1}{(\pi+k+m)} + \frac{\cos(\pi-k-m)-1}{(\pi-k-m)} + \frac{1-\cos(\pi+k-m)}{(\pi+k-m)} + \frac{1-\cos(\pi-k+m)}{(\pi-k+m)}\right] \qquad (23b)$$

$$I_3 = \frac{L}{4}\left[\frac{1-\cos(\pi+k+m)}{(\pi+k+m)} + \frac{\cos(\pi-k-m)-1}{(\pi-k-m)} + \frac{\cos(\pi+k-m)-1}{(\pi+k-m)} + \frac{1-\cos(\pi-k+m)}{(\pi-k+m)}\right] \qquad (23c)$$

$$I_4 = \frac{L}{4}\left[\frac{\cos(3\pi+k+m)-1}{(3\pi+k+m)} + \frac{\cos(3\pi-k-m)-1}{(3\pi-k-m)} + \frac{1-\cos(3\pi+k-m)}{(3\pi+k-m)} + \frac{1-\cos(3\pi-k+m)}{(3\pi-k+m)}\right] \qquad (23d)$$

$$I_5 = \frac{L}{4}\left[\frac{\sin(2\pi+k-m)}{(2\pi+k-m)} + \frac{\sin(2\pi-k+m)}{(2\pi-k+m)} - \frac{\sin(2\pi+k+m)}{(2\pi+k+m)} - \frac{\sin(2\pi-k-m)}{(2\pi-k-m)}\right] \qquad (23e)$$

In what follows we subject the ordinary differential equation (17) to a Laplace transformation defined as

$$Le\left(q(t)\right) = q(s) = \int_0^\infty q(t)e^{-st}\, dt \qquad (24)$$

By using the transformation (24) on equation (17) in conjunction with the initial conditions (4) upon simplification, we obtain

$$q_k(s) + B_{11}q_k(s) + B_{22}q_k(s) = B_{33}\left(\frac{b_0}{(b_0+d)^2 + {b_0}^2}\right) \qquad (25)$$

Where

$$B_{11} = \frac{B_{11}}{R_{11}}, \qquad B_{22} = \frac{R_{13}}{R_{11}}, \qquad B_{22} = \frac{P}{R_{11}} \qquad (26)$$

Further simplification of (25) to obtain

$$q_k(s) = \frac{B_{33}}{c_1 - c_2}\left[\left(\frac{1}{s-c_1}\right)\left(\frac{b_0}{(b_0+d)^2+{b_0}^2}\right) - \left(\frac{1}{s-c_2}\right)\left(\frac{b_0}{(b_0+d)^2+{b_0}^2}\right)\right] \qquad (27)$$

Where

$$c_1 = \frac{-B_{11}{}^2}{2} + \frac{\sqrt{B_{11}{}^2 - 4B_{22}}}{2}, c_1 = \frac{-B_{11}{}^2}{2} - \frac{\sqrt{B_{11}{}^2 - 4B_{22}}}{2} \quad (28)$$

We adopt the following representation in order to obtain the Laplace inversion of

$$Q_1(s) = \frac{1}{s-c_1},\ H(s) = \frac{b_0}{s^2+b_0{}^2},\ Q_2(s) = \frac{1}{s-c_2} \quad (29)$$

Substituting (29) into (27) to obtain

$$q_k(s) = \frac{B_{33}}{c_1-c_2}\big(Q_1(s)H(s) + Q_2(s)H(s)\big) \quad (30)$$

We apply convolution theorem defined as

$$(Q_i * H)(t) = \int_0^t Q_i\,(t-u)\,H(u)du,\ \ i = 1,2\ ... \quad (31)$$

In order to obtain the Laplace inversion of (30) as

$$q_k(t) = \Delta\,(e^{c_1 t} J_1 - e^{c_2 t} J_2) \quad (32)$$

Where

$$\Delta = \frac{B_{33}}{c_1 - c_2}$$

$$J_1 = \int_0^t e^{-(c_1+d)u} \sin b_0 u\, du \quad (33)$$

$$J_2 = \int_0^t e^{-(c_2+d)u} \sin b_0 u\, du \quad (34)$$

By using integration by part, we evaluate integral (33) and (34) to obtain

$$J_1 = \frac{b_0}{b_0{}^2 + (c_1+d)^2}\left(1 - e^{-(c_1+d)t} \cos b_0 t\right) - \frac{(c_1+d)}{b_0{}^2 + (c_1+d)^2} e^{-(c_1+d)t} \sin b_0 t \quad (35)$$

$$J_2 = \frac{b_0}{{b_0}^2 + (c_2 + d)^2}\left(1 - e^{-(c_2+d)t}\cos b_0 t\right) - \frac{(c_2 + d)}{{b_0}^2 + (c_2 + d)^2} e^{-(c_2+d)t}\sin b_0 t \tag{36}$$

Substituting (35) and (36) into (32) and simplified to obtain

$$q_k(t) = \frac{\Delta b_0}{{b_0}^2 + (c_1 + d)^2}\left(e^{c_1 t} - e^{-dt}\cos b_0 t\right) - \frac{\Delta (c_1 + d) e^{-dt}\sin b_0 t}{{b_0}^2 + (c_1 + d)^2} - \frac{\Delta b_0}{{b_0}^2 + (c_2 + d)^2}\left(e^{c_2 t} - e^{-dt}\cos b_0 t\right) + \frac{\Delta (c_2 + d) e^{-dt}\sin b_0 t}{{b_0}^2 + (c_2 + d)^2} \tag{37}$$

Substituting (37) into (11) to obtain

$$V_k(x,t) = \sum_{k=1}^{\infty}\left[\frac{\Delta b_0}{{b_0}^2 + (c_1 + d)^2}\left(e^{c_1 t} - e^{-dt}\cos b_0 t\right) - \frac{\Delta (c_1 + d) e^{-dt}\sin b_0 t}{{b_0}^2 + (c_1 + d)^2} - \frac{\Delta b_0}{{b_0}^2 + (c_2 + d)^2}\left(e^{c_2 t} - e^{-dt}\cos b_0 t\right) + \frac{\Delta (c_2 + d) e^{-dt}\sin b_0 t}{{b_0}^2 + (c_2 + d)^2}\right]\frac{\sin k\pi x}{L} \tag{38}$$

Equation (38) represents the response amplitude to exponentially varying magnitude moving load of non-uniform Rayleigh beam resting on bi-parametric subgrades.

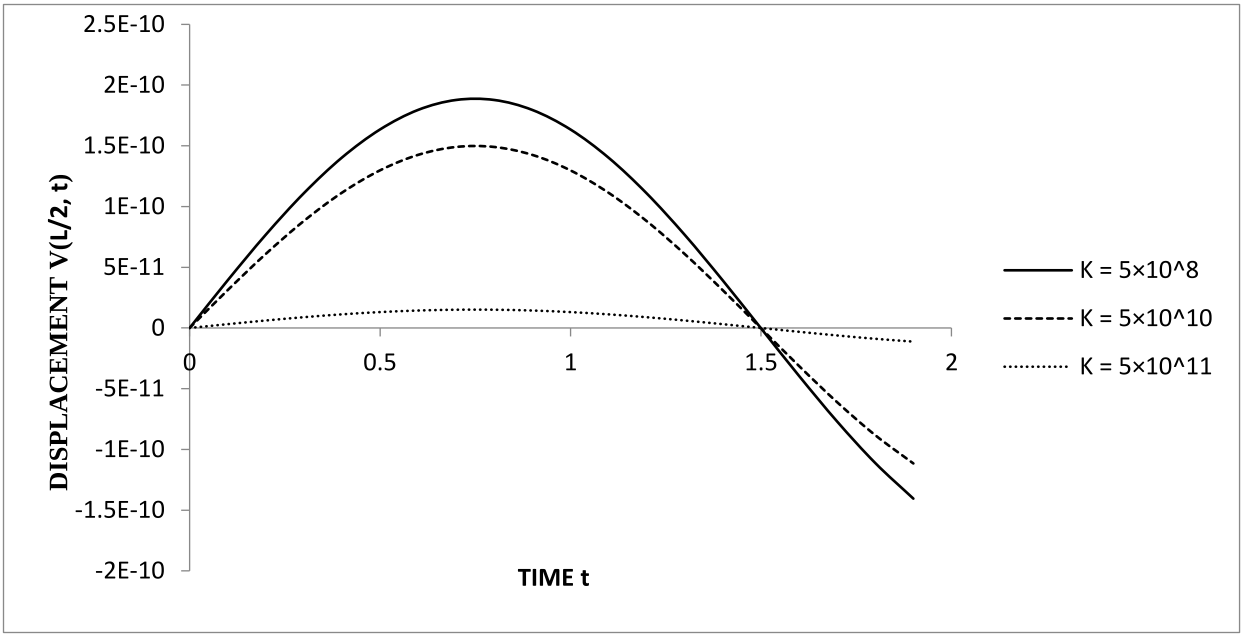

Figure 1: Deflection profile of non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load for various values of foundation modulus (K).

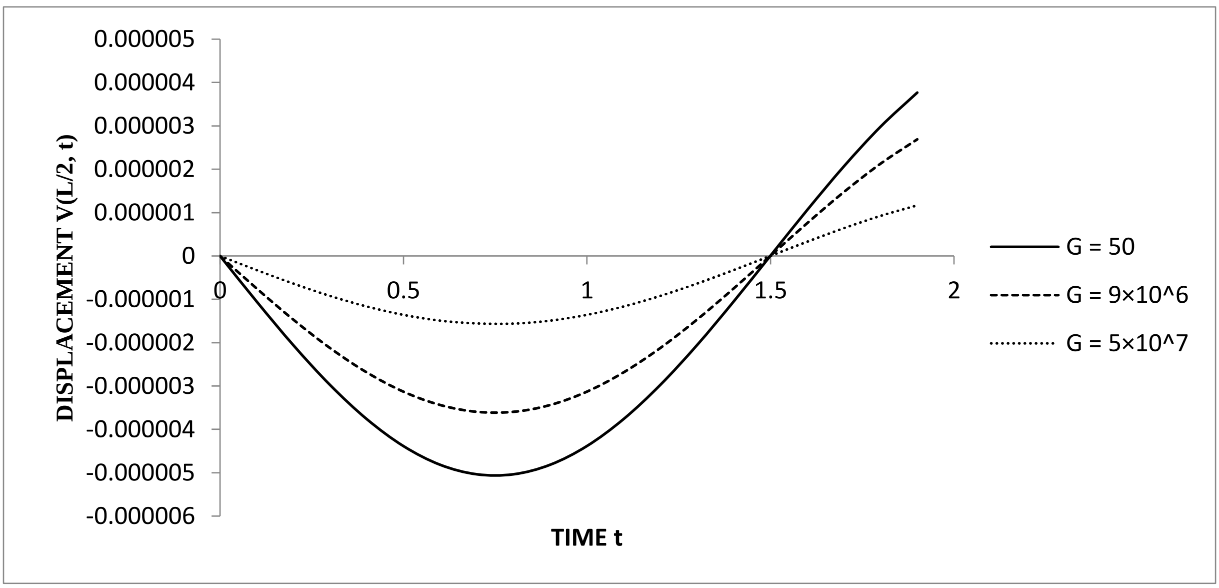

Figure 2: Deflection profile of non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load for various values of shear modulus (G).

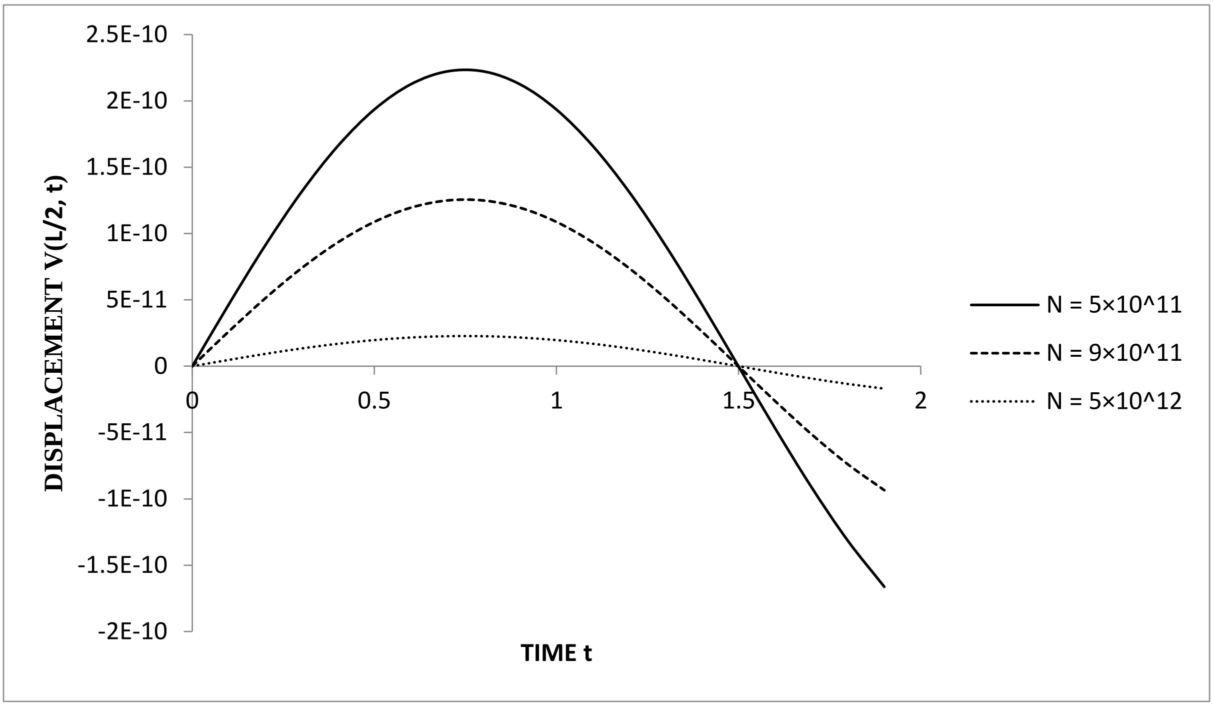

Figure 3: Deflection profile of non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load for various values of axial force (N).

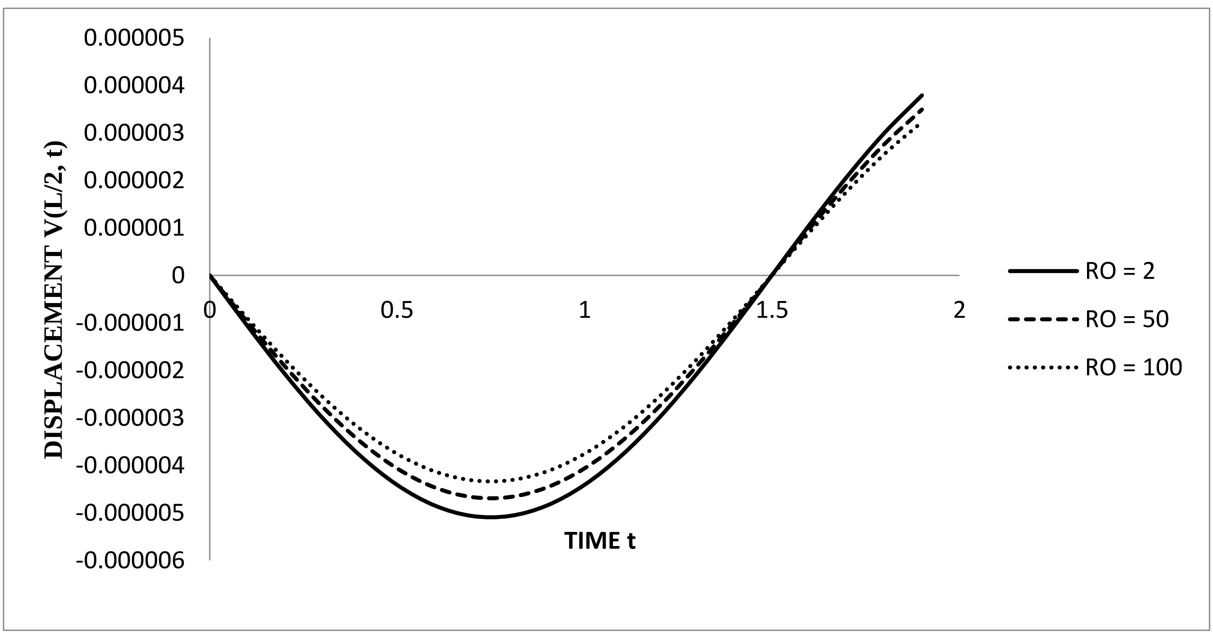

Figure 4: Deflection profile of non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load for various values of rotary Inertia (R0).

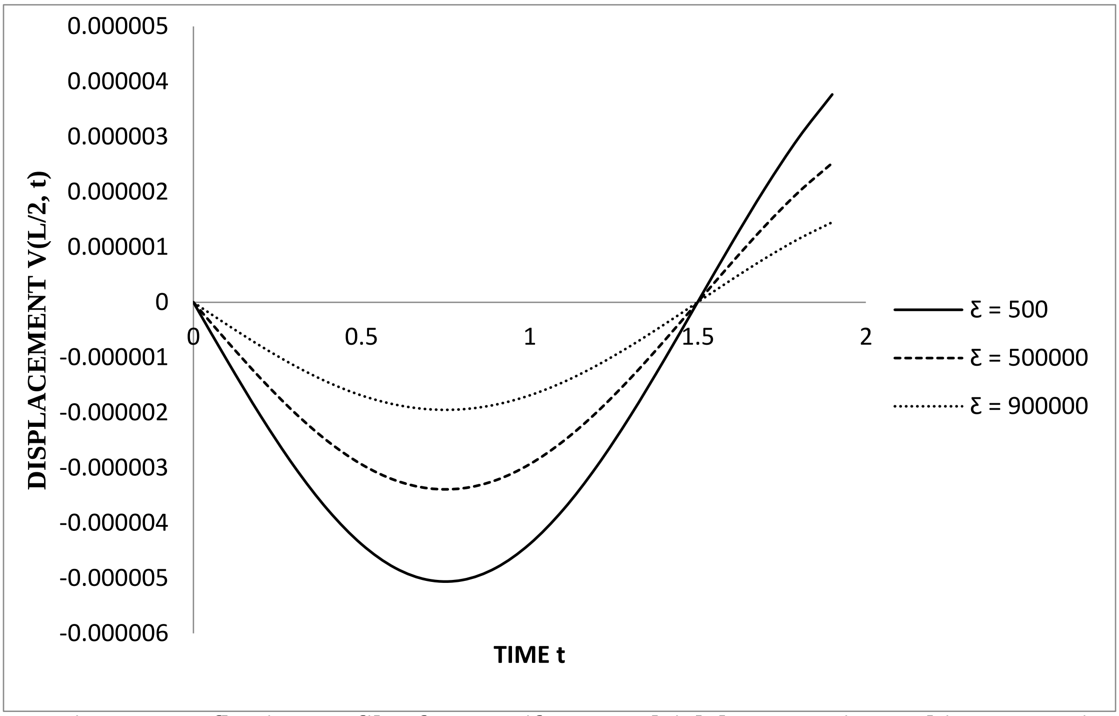

Figure 5: Deflection profile of non-uniform Rayleigh beam resting on bi-parametric subgrades under exponentially varying magnitude moving load for various values of damping coefficient (ɛ).

4. Numerical Analysis and Discussion of Results

In order to illustrate the foregoing analysis, the non-uniform Rayleigh beam of length 12.19 *m*, velocity of the moving load is taken to be 8.128 *m/s* are considered. Other values used in the analysis are, modulus of elasticity $E = 2.10924 \times 10^8\ N/m^2$. The constant moment of inertia $I_0 = 2.876988 \times 10^{-3}\ m$ and constant mass per unit length of the beam $\mu_0 = 3401.563\ kg/m$. The values for the foundation modulus (K) varies between $5\times10^8\ N/m^3$ and $5\times10^{11}\ N/m^3$ while that of the shear modulus (G) varies between 50 N/m^3 and $5\times10^7\ N/m^3$, values of the axial force (N) is between $5\times10^{11}\ N$ and $5\times10^{12}\ N$, values of the rotator inertia (R_0) is between 0 and 100, and finally the values of the damping coefficient (ε) is between 500 and 900 000.

Figure 1 display the deflection profile of non-uniform Rayleigh beam under exponentially varying moving load. From the figure, it was observed that as the values of the foundation modulus (K) increases, the deflection profile of the beam decreases upward from the origin. Figure 2 also show that an increase in shear modulus (G) will lead to downward decrease in deflection profile of the beam from the origin. Similarly, figures 3, 4 and 5 shows that an increase in axial force (N), rotatory inertia R_0 and damping coefficient will in each case reduce the response amplitude of the beam. Finally, it was observed that the shear modulus gave more noticeable effect compared to that of the foundation modulus as can be seen in figure 1 and 2 respectively.

5. Conclusion

The problem of the dynamic response to exponentially varying magnitude moving load of non-uniform Rayleigh beam resting on bi-parametric subgrades is investigated in this paper. The analytical approximate technique is based on Galerkin's method, Laplace transformation and Convolution theorem. Analytical solution and numerical results are presented in graphs as shown above. From the figures, increases values of foundation modulus, shear modulus, axial force, rotator inertia, and damping coefficient lead to decreases in the response amplitude of the non-uniform beam. Furthermore, it was also observed that smaller values of the shear modulus are required for noticeable deflection compared to that of foundation modulus. Finally, it was observed that rotator inertia has more influence on the non-uniform beam when compared to other structural parameters.

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