Journal of Applied Mathematics & Bioinformatics, vol.8, no.2, 2018, 17-28
ISSN: 1792-6602 (print), 1792-6939 (online)
Scienpress Ltd, 2018

Fast control of a gun-turret using shortcuts to adiabaticity

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Abstract

In this article we use the inverse engineering method of shortcuts to adiabaticity, extensively used for the control of various quantum systems, to design fast and smooth controls (torques) which can drive a gun-turret system to a desired final orientation. The obtained controls are less sensitive to small errors in the initial orientation than the minimum-time controls and easier to implement since they do not contain discontinuities, while they have a moderately longer duration. The presented methodology is not restricted to the specific example under consideration, but it can also be used for the control of other mechanical systems.

Mathematics Subject Classification(2010): 70Q05; 70E60; 93C15
Keywords: Control of mechanical systems; Gun-turret; Shortcuts to adiabaticity

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Article Info: *Received* : January 7, 2018. *Revised* : March 2, 2018.
Published online : September 15, 2018

1 Introduction

The method of *shortcuts to adiabaticity* [?] provides an efficient way to design fast but effectively adiabatic transitions for classical and quantum systems [?]. The main idea behind the method is that the system of interest does not follow the adiabatic path at each moment, thus the evolution can be faster, but the final state is the same with that obtained by a slow adiabatic process. Shortcuts to adiabaticity is actually an inverse engineering method, where the path of the system state variables is prescribed first, usually with polynomial interpolation between the initial and the desired final state, and it is subsequently used to derive the controls which can implement this transition.

This general and very useful method has found a wide spectrum of applications. These include the cooling of trapped atoms [?] and Bose-Einstein condensates [?], manipulation of two- and three-level quantum systems [?], quantum computation [?], fast cooling of mechanical resonators [?], design of optical [?] and Glauber-Fock photonic lattices [?], fast transport of trapped ions [?], efficient scaling of quantum heat engines [?], design of optical waveguides [?], cutting a spin chain [?], and even the fast relaxation towards equilibrium of a brownian particle [?], to name a few.

In the present article we use this inverse engineering approach to find smooth controls which can quickly drive a gun-turret system from some initial to a desired final orientation, under kinematic constraints stemming from a popular real world system, the Oto Melara 76 mm gun [?], which is currently installed in most ships of the Greek navy. For comparison reasons, we also find the minimum-time optimal control strategy under the same constraints. The torques obtained with the inverse engineering method are smooth, have smaller amplitude and are less sensitive to errors in the initial conditions compared to the discontinuous minimum-time controls. The price paid for these advantageous characteristics is, as expected, a moderately longer duration. The method is not restricted to the particular example studied here but is expected to find applications in the area of control of mechanical systems.

2 The gun-turret as a mechanical system with two degrees of freedom

In a simplified description, we can consider the gun-turret as a mechanical system with two degrees of freedom which describe the orientation of the gun, the azimuth angle ϕ and the elevation angle θ, see Fig. 1.

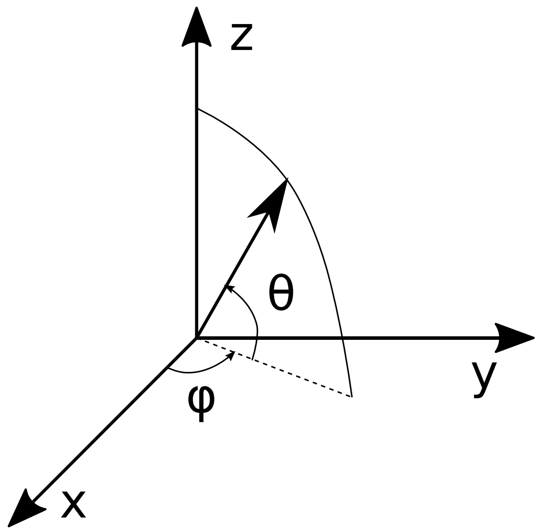


Figure 1: The orientation of the gun is determined by the azimuth angle ϕ and elevation angle θ

Let I_1 denote the turret moment of inertia around z-axis, $I_2 = \frac{1}{3}ml^3$ the gun moment of inertia around a horizontal axis perpendicular to the elevation plane, m the mass and l the length of the gun. The Lagrangian of the system is

$$L = \frac{1}{2}I_1\dot{\phi}^2 + \frac{1}{2}I_2(\dot{\theta}^2 + \cos^2\theta\dot{\phi}^2) - \frac{1}{2}mgl\sin\theta, \tag{1}$$

where the first term expresses the kinetic energy of the turret, the second term the kinetic energy of the gun due to rotations on the θ-plane and around the z-axis, while the last one corresponds to the potential energy of the gun. Note that the gun is treated as a uniform rod.

Using the Euler-Lagrange equations

$$M_\phi = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}}\right) - \frac{\partial L}{\partial\phi}, \quad M_\theta = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta},$$

we obtain the following expressions for the torque M_ϕ around the vertical axis and the torque M_θ around the horizontal axis perpendicular to the elevation plane at each moment

$$\begin{aligned} M_\phi &= I_2[(r+\cos^2\theta)\ddot{\phi} - \sin(2\theta)\dot{\phi}\dot{\theta}], \\ M_\theta &= I_2\ddot{\theta} + \frac{1}{2}I_2\sin(2\theta)\dot{\phi}^2 + \frac{1}{2}mgl\cos\theta, \end{aligned}$$

where

$$r = \frac{I_1}{I_2} \tag{2}$$

is the ratio of the moments of inertia. If we normalize the torques with $mgl/2$, the maximum torque of the gun weight, $M'_\phi = \frac{2M_\phi}{mgl}, M'_\theta = \frac{2M_\theta}{mgl}$, and the time with $\omega = \sqrt{\frac{mgl}{2I_2}} = \sqrt{\frac{3g}{2l}}$, the angular frequency of the gun as a physical pendulum, $t' = \omega t$, but for simplicity we use the previous symbols M_ϕ, M_θ, t for M'_ϕ, M'_θ, t', we obtain the equations

$$M_\phi = (r+\cos^2\theta)\ddot{\phi} - \sin(2\theta)\dot{\phi}\dot{\theta}, \tag{3}$$

$$M_\theta = \ddot{\theta} + \frac{1}{2}\sin(2\theta)\dot{\phi}^2 + \cos\theta. \tag{4}$$

3 Design of fast and smooth controls using shortcuts to adiabaticity

According to the inverse engineering method of shortcuts to adiabaticity, the paths of the state variables $\theta(t), \phi(t)$ connecting the initial states

$$\phi(0) = \phi_0, \quad \theta(0) = \theta_0 \tag{5}$$

to the desired final states at $t = T$

$$\phi(T) = \phi_T, \quad \theta(T) = \theta_T \tag{6}$$

are prescribed first, and then system equations (3), (4) are used to find the corresponding control torques $M_\phi(t), M_\theta(t)$. Note that the desired final orientation (6) can be provided for example from a dedicated system estimating the location of the target. In order to obtain smooth states and controls, the following boundary conditions are imposed on the first and second time derivatives of the angles

$$\dot{\theta}(0) = \dot{\theta}(T) = \dot{\phi}(0) = \dot{\phi}(T) = 0, \tag{7}$$
$$\ddot{\theta}(0) = \ddot{\theta}(T) = \ddot{\phi}(0) = \ddot{\phi}(T) = 0. \tag{8}$$

If we fix the final time T to a desired short value and define the normalized time

$$s = \frac{t}{T}, \quad 0 \leq s \leq 1,$$

then the following polynomial satisfies the boundary conditions [?] for both angles ϕ and θ, so we use the symbol ψ for both, while ψ_0, ψ_T denote the initial and final values of the corresponding angle (θ or ϕ)

$$\psi(s) = \psi_0 + (\psi_T - \psi_0)(10s^3 - 15s^4 + 6s^5), \tag{10}$$
$$\dot{\psi} = \frac{d\psi}{dt} = \frac{30(\psi_T - \psi_0)}{T} s^2(1-s)^2, \tag{11}$$
$$\ddot{\psi} = \frac{d^2\psi}{dt^2} = \frac{60(\psi_T - \psi_0)}{T^2} s(1-s)(1-2s), \tag{12}$$

where we have also calculated the derivatives of ψ with respect to time t, since we will need then in finding the torques from system equations.

In theory, the duration T can be made arbitrarily small. In practise, there are always realistic constraints which limit this value to a finite time. For example, from the technical specifications of the Oto Melara 76 mm gun we find the following kinematic constraints for the angular velocities

$$|\dot{\phi}| \leq \frac{60^\circ}{s}, \quad |\dot{\theta}| \leq \frac{35^\circ}{s} \tag{13}$$

and the angular accelerations

$$|\ddot{\phi}|, |\ddot{\theta}| \leq \frac{72^\circ}{s^2}. \tag{14}$$

From the expressions (11), (12) for the time derivatives of the angle path (10) we can easily obtain the following maximum values for the angular velocities and accelerations, which depend on the difference between the initial and final angles and the duration T

$$|\dot{\phi}| \leq \frac{15(\phi_T - \phi_0)}{8T}, \qquad |\dot{\theta}| \leq \frac{15(\theta_T - \theta_0)}{8T}, \tag{15}$$
$$|\ddot{\phi}| \leq \frac{10(\phi_T - \phi_0)}{\sqrt{3}T^2}, \qquad |\ddot{\theta}| \leq \frac{10(\theta_T - \theta_0)}{\sqrt{3}T^2}. \tag{16}$$

From (15), (16) and for fixed initial and final angle values, we can find the minimum duration T such that the kinematic constraints (13), (14) are satisfied.

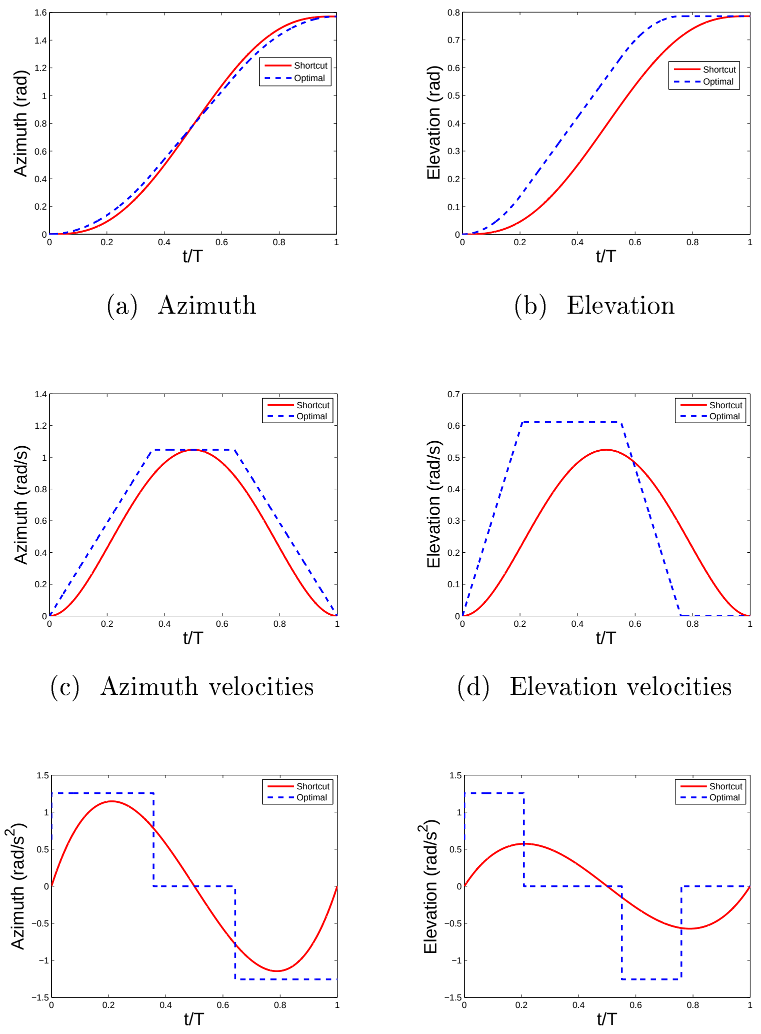
4 Specific example and comparison with the minimum-time optimal control strategy

We now demonstrate the presented methodology with a specific example. For initial and final angles

$$\phi_0 = \theta_0 = 0^\circ, \quad \phi_T = 90^\circ, \quad \theta_T = 45^\circ \tag{17}$$

we find that the minimum duration satisfying the constraints (15), (16) is $T = 2.81\,s$. The effective constraint for this example is the bound for $\dot{\phi}$. Having specified the initial and final angles and determined the duration T, the time dependence of $\phi(t), \theta(t)$ is prescribed. In order to use equations (3), (4) for the torques, we further need the ratio r of the moments of inertia (2). From the technical specifications of the Oto Melara 76 mm gun we find the mass $m = 765\,kg$ and the length $l = 4.724\,m$ of the gun, while the turret mass is $M = 7439\,kg$ and we can take its average radius to be about $R = 1.5\,m$. By assuming the turret to be a homogeneous cylinder we have $I_1 = \frac{1}{2}MR^2$, while by considering the gun as a uniform cylindrical rod we have $I_2 = \frac{1}{3}ml^2$. The corresponding value of the ratio is $r = \frac{I_1}{I_2} = 1.47$. We would like to point out that our analysis does not rely on these assumptions and it can be carried out using the real values of I_1, I_2 which can be obtained by measurement. We make these assumptions in order to obtain some realistic estimates of the moments of inertia for the example that we present. In Fig. 2 we plot with red solid line the azimuth and elevation angles, velocities and accelerations derived from the shortcuts to adiabaticity equations (10), (11) and (12), for the boundary conditions given in (17), corresponding to a minimum duration $T = 2.81\,s$. Observe that only the azimuth velocity attains at some point the maximum allowed value given in (13), and thus is the limiting factor on how fast can be obtained the final orientation.

For comparison reasons, we also discuss the minimum-time optimal control strategy under the kinematic constraints (13) and (14). For this kind of angu-



(a) Azimuth (b) Elevation

(c) Azimuth velocities (d) Elevation velocities

(e) Azimuth accelerations (f) Elevation accelerations

Figure 2: (Color online) Azimuth and elevation angles (a, b), velocities (c, d) and accelerations (e, f), for the shortcut (red solid line) and the minimum-time strategies (blue dashed line), for the parameter values given in the text. For the shortcut strategy $T = 2.81\,s$, while for the minimum-time strategy $T = 2.33\,s$.

lar rate constraints and with unbounded controls (torques), the minimum-time strategy is relatively simple. First note that each angle (azimuth, elevation) can be controlled independently by the corresponding torque. The minimum-time strategy for each angle is to move with the maximum angular acceleration

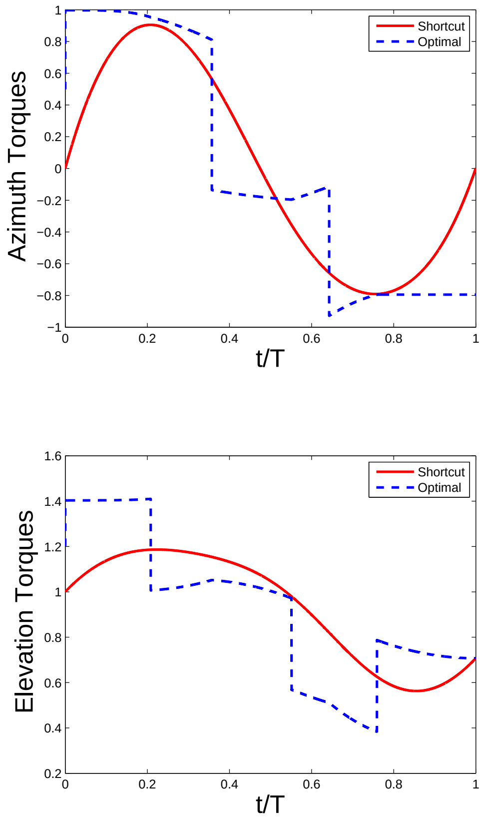


Figure 3: (Color online) Azimuth (a) and elevation (b) torques for the shortcut (red solid line) and the minimum-time (blue dashed line) strategies.

until the maximum angular velocity is reached, then maintain this maximum angular velocity for some time, and finally apply the maximum angular deceleration until the angular velocity vanishes and the desired final angle is reached. The minimum time is determined by the maximum time needed to reach the final value of both angles. For the angle where the final value is reached first, the applied control maintains this value until the final value of the other angle is also reached. In terms of angular accelerations of the two angles, the minimum-time strategy is bang-off-bang for the “slow” one and bang-off-bang-off for the “fast” one. Note that if the difference between the initial and the final angles is lower than a certain threshold, then the intermediate off pulse (zero angular acceleration-constant angular velocity) is omitted. In Fig. 2 we plot with blue dashed line the azimuth and elevation angles, ve-

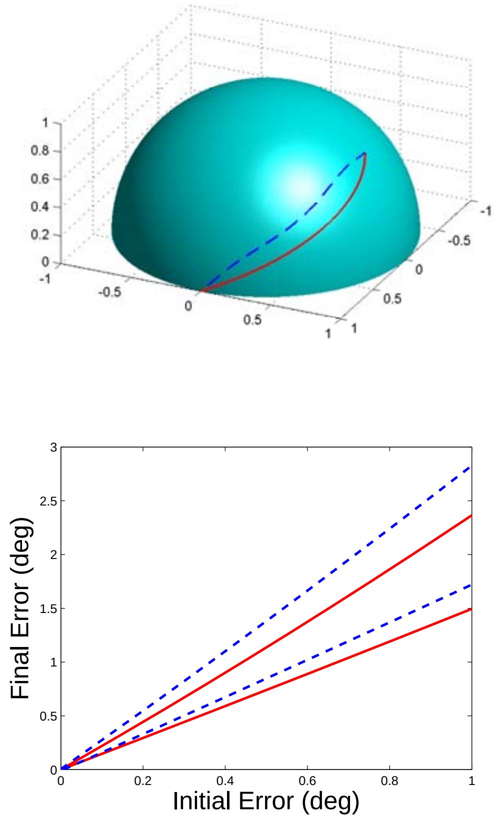


Figure 4: (Color online) (a) Shortcut (red solid line) and minimum-time (blue dashed line) trajectories. (b) Azimuth (two upper lines) and elevation (two lower lines) final error for the shortcut (red solid line) and the minimum-time (blue dashed line) strategies, as a function of a small initial error which is taken for simplicity to be the same in both angles.

locities and accelerations corresponding to the minimum-time strategy, for the same parameter values as before. Note that the duration corresponding to the minimum-time strategy is $T = 2.33\,s$, about half a second less than before. Observe the trapezoidal shape of the angular velocity for the “slow” azimuth angle, while for the “fast” elevation angle a zero tail is appended so the obtained final value is maintained until the final value of the slow angle is also reached. Also observe the bang-off-bang shape of the azimuth angular acceleration, and the bang-off-bang-off profile of the elevation angular acceleration. If the difference between the initial and final angles is smaller than a specific threshold, the intermediate off pulse is absent and the slow angle acceleration has the bang-bang form.

In Fig. 3 we plot the corresponding azimuth and elevation torques for the shortcut (red solid line) and the minimum-time (blue dashed line) strategies. Observe that the shortcut torques are smooth functions of time while the minimum-time torques have discontinuities, due to the discontinuities in the optimal accelerations. Another advantage of the shortcut torques is that their extremal values (minimum and maximum) are smaller than the corresponding values of the minimum-time torques. The price paid for these advantages is of course the extra time needed to reach the target orientation with the shortcut strategy, which is about $0.5\,s$.

In Fig. 4(a) we plot the trajectory of the gun tip for the shortcut (red solid line) and the minimum-time (blue dashed line) strategies. In Fig. 4(b) we make an elementary sensitivity analysis. The upper two lines represent the azimuth final error for the two strategies while the lower two lines the elevation final error, corresponding to a small error in the initial conditions (horizontal axis), taken for simplicity to be the same for both angles. Observe that the shortcut strategy is less sensitive to small perturbations in the initial conditions for the angles.

5 Conclusion

In the present work we employed a modern inverse engineering method, which is currently widely used to control quantum dynamics, in order to find smooth controls (torques) which can drive a gun-turret system from some initial to a desired target orientation, under realistic kinematic constraints. The derived controls are less sensitive to errors in the initial conditions when compared to the minimum-time controls and easier to implement since they do not have discontinuities, while they possess a moderately larger duration. The above analysis can be carried out even in the more complicated case where the servomechanisms driving the gun-turret system are incorporated in the description. Finally, we would like to point out that the presented methodology, originated from Physics, can be of great interest for the Mechanical and Control Engineering communities.

References

[1] M. Beau, J. Jaramillo, and A. del Campo, *Scaling-up quantum heat engines efficiently via shortcuts to adiabaticity*, Entropy **18** (2016), 168.

[2] X. Chen, I. Lizuain, A. Ruschhaupt, D. Guéry-Odelin, and J. G. Muga, *Shortcut to adiabatic passage in two- and three-level atoms*, Physical Review Letters **105** (2010), 123003.

[3] X. Chen, A. Ruschhaupt, S. Schmidt, A. del Campo, D. Guéry-Odelin, and J. G. Muga, *Fast optimal frictionless atom cooling in harmonic traps: Shortcut to adiabaticity*, Physical Review Letters **104** (2010), 063002.

[4] S. Deffner, C. Jarzynski, and A. del Campo, *Classical and quantum shortcuts to adiabaticity for scale-invariant driving*, Physical Review X **4** (2014), 021013.

[5] Y. Li, L.-A. Wu, and Z. D. Wang, *Fast ground-state cooling of mechanical resonators with time-dependent optical cavities*, Physical Review A **83** (2011), 043804.

[6] X.-J. Lu, J. G. Muga, X. Chen, U. G. Poschinger, F. Schmidt-Kaler, and A. Ruschhaupt, *Fast shuttling of a trapped ion in the presence of noise*, Physical Review A **89** (2014), 063414.

[7] I. A. Martínez, A. Petrosyan, , D. Guéry-Odelin, E. Trizac, and S. Ciliberto, *Engineered swift equilibration of a brownian particle*, Nature Physics **12** (2016), 021013.

[8] S. Martínez-Garaot, J. G. Muga, and S.-Y. Tseng, *Shortcuts to adiabaticity in optical waveguides using fast quasiadiabatic dynamics*, Optics Express **25** (2017), 159–167.

[9] F.-H. Ren, Z.-M. Wang, and Y.-J. Gu, *Shortcuts to adiabaticity in cutting a spin chain*, Physics Letters A **381** (2017), 70–75.

[10] M. S. Sarandy, E. I. Duzzioni, and R. M. Serra, *Quantum computation in continuous time using dynamic invariants*, Physics Letters A **375** (2011), 3343–3347.

[11] J.-F. Schaff, X.-L. Song, P. Capuzzi, P. Vignolo, and G. Labeyrie, *Shortcut to adiabaticity for an interacting bose-einstein condensate*, EPL (Europhysics Letters) **93** (2011), 23001.

[12] D. Stefanatos, *Design of a photonic lattice using shortcuts to adiabaticity*, Physical Review A **90** (2014), 023811.

[13] E. Torrontegui, S. Ibáñez, S. Martínez-Garaot, M. Modugno, A. del Campo, D. Guéry-Odelin, A. Ruschhaupt, X. Chen, and J. G. Muga, *Shortcuts to Adiabaticity*, Advances in Atomic Molecular and Optical Physics **62** (2013), 117–169.

[14] Wikipedia, *Oto melara 76 mm — Wikipedia, the free encyclopedia*, 2017, [Online; accessed 22-July-2017].

[15] C. Yuce, *Fast frictionless expansion of an optical lattice*, Physics Letters A **376** (2012), 1717–1720.