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Cooperation of Autonomous Mobile Robots for Surveillance Missions Based on Hyperchaos Synchronization

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Abstract

In this paper, a novel collaboration scheme of autonomous mobile robots, which is based on chaos synchronization with application to complete and fast coverage of the whole workspace, is presented. Each one of the mobile robots is controlled by the simplest four-dimensional hyperchaotic Lorenz-type system, producing an unpredictable trajectory. When multiple robots are employed for faster and more complete

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coverage, effective cooperation can be achieved by synchronizing these chaotic motions. For this reason, the nonlinear open loop controller to target the synchronization state of chaotic oscillators is adopted. Computer simulation illustrates that the proposed synchronized chaotic robots improve the efficiency of the non-synchronized robots in finishing faster the same coverage work area.

Mathematics Subject Classification: 70E60
Keywords: Autonomous mobile robot; cooperative mission; hyperchaos; synchronization; nonlinear open loop controller; coverage rate

1 Introduction

In the last two decades the design and development of autonomous mobile robots has become a very interesting research subject because of their civil or military applications in difficult or dangerous tasks for humans. This class of robots has the advantage that they can perform a task without continuous human supervision. Transportation [1], search and rescue of human victims on disaster places [2], map buildings [3], complete coverage of a terrain [4], terrain exploration for searching for explosives [5] and surveillance of terrains [6] are some of the tasks, where autonomous mobile robots can be used with very satisfactory results.

In the design and development of autonomous mobile robots the fundamental issues are the locomotion, the sensing, the localization and the navigation. However, the most challenging of all these issues is the choice of the navigation strategy, which can be defined as the set of various techniques that allow the mobile robot to autonomously decide where to move in the workspace in order to accomplish a given task.

Covering all the possible classes of problems involving navigation strategies, such as the exploration of an initially unknown workspace in order to discover explosives or the patrolling of a known environment, the key-point is the robot's motion planning for the complete and fast workspace scanning. For accomplishing these requirements, a mobile robot which is capable of crossing every region, covering systematically the entire workspace, should be designed.

For achieving the aforementioned requirements Nakamura and Sekiguchi proposed in 2001 a navigation strategy, which is based on chaotic systems [7]. In that work, the chaotic behavior of a dynamical system is imparted to the mobile robot's motion control. Since then, a great number of relative research works in the field of autonomous mobile robots has been presented, because the chaotic motion guarantees the scanning of the whole workspace without a terrain map or motion plan [8-14].

Furthermore a very interesting task, especially the last few years, is the design and implementation of collaborative schemes of autonomous mobile robots, for accomplishing faster and with better results various missions, not only in military operations but also for use in industry.

This work presents a novel strategy of collaborative autonomous mobile robots, which is based on chaos synchronization scheme with application to complete and fast coverage of the whole workspace. Each one of the mobile robots is controlled by a nonlinear system, having different initial conditions, producing an unpredictable trajectory. For increasing the desired unpredictability of robot's motion, the simplest four-dimensional hyperchaotic Lorenz-type system is chosen. So, by synchronizing the hyperchaotic motions of the collaborative autonomous robots the faster and more complete coverage can be achieved. For this reason, the nonlinear open loop controller to target the synchronization state chaotic oscillators is adopted. Computer simulation illustrates that the proposed synchronized mobile robots improve the efficiency of the non-synchronized robots in finishing faster the same coverage work area.

This paper is organized as follows. In Section 2 the definition of chaotic systems as well as the case of chaotic synchronization and the description of the four-dimensional modified Lorenz hyperchaotic system, which has been used, is given in details. Section 3 presents the recently new proposed nonlinear open loop control scheme, which is the basic component of this work. The adopted model of the collaborative mobile robots scheme and the simulation results of this scheme are described in Section 4. Finally, Section 5 includes the conclusion remarks and some thoughts for future work.

2 The Proposed Hyperchaotic System

It is known that nonlinear systems have a very rich dynamic behavior, showing a variety of dynamical phenomena. Especially, the chaotic behavior is the reason for which nonlinear systems have been used in many other engineering fields such as communications, control, cryptography, random bits generators and neuronal networks [15-21]. A nonlinear dynamical system, in order to be considered as chaotic, must fulfil three basic conditions [22]. Its periodic orbits must be dense, it must be also topologically mixing and it must be very sensitive on initial conditions.

From the aforementioned features of chaotic systems, the most important is the sensitivity on initial conditions. This means, that a small variation on a system's initial conditions will produce a totally different chaotic trajectory. So, this is the basic feature, which contributes to the desired robot's unpredictable trajectory.

One of the most challenging research task in the field of nonlinear systems is the synchronization between coupled chaotic systems, due to the fact that is a rich and multi-disciplinary phenomenon with broad range applications, such as in a variety of complex physical, chemical, and biological systems as well as in secure communications, cryptography and broadband communication systems [23-29]. In more details, synchronization of chaos is a process, where two or more chaotic systems adjust a given property of their dynamics motion to a common behavior, such as identical trajectories or phase locking, due to coupling or forcing. Because of the exponential divergence of the nearby trajectories of a chaotic system, having two chaotic systems being synchronized, might be a surprise. However, today the synchronization of coupled chaotic oscillators is a phenomenon well established experimentally and reasonably well understood theoretically.

The history of chaotic synchronization's theory began with the study of the interaction between coupled chaotic systems in the 1980's and early 1990's [30-32]. Since then, a wide range of research activity based on synchronization of nonlinear systems has risen and a variety of synchronization's types depending on the nature of the interacting systems and of the coupling schemes has been presented. In particular, the phenomenon of complete synchronization is the most studied type of synchronization. In this case, two coupled chaotic systems are leaded to a perfect coincidence of their chaotic trajectories i.e.,

$$x_1(t) = x_2(t), t \to \infty \tag{1}$$

Recently, a great interest for dynamical systems with hidden attractors has been raised. The term "hidden attractor" refers to the fact that in this class of systems the attractor is not associated with an unstable equilibrium and thus often goes undiscovered because they may occur in a small region of parameter space and with a small basin of attraction in the space of initial conditions [33-38]. Furthermore, systems with hidden attractors have received attention due to their practical and theoretical importance in other scientific branches, such as in mechanics (unexpected responses to perturbations in a structure like a bridge or in an airplane wing) [39, 40].

In this work, the simplest four-dimensional hyperchaotic Lorenz-type system [39], which belongs to the aforementioned class of dynamical systems, is used. This system, which is an extension of a modified Lorenz system, having only two independent parameters (c, d), is described by the following set of differential equations:

$$\frac{dx}{dt} = y - x \tag{2}$$

$$\frac{dy}{dt} = -xz + u \tag{3}$$

$$\frac{dz}{dt} = xy - c \tag{4}$$

$$\frac{du}{dt} = -dy \tag{5}$$

This system has many interesting properties not found in other proposed systems, such as [39]:

(i) It has very few terms, only seven with two quadratic nonlinearities, and two parameters.

(ii) All its attractors are hidden.

(iii) It exhibits hyperchaos over a large region of parameter space.

(iv) Its Jacobian matrix has rank less than four everywhere in the space of the parameters.

(v) It exhibits a quasiperiodic route to chaos with an attracting torus for some choice of parameters.

(vi) It has regions in which the torus coexists with either a symmetric pair of strange attractors or a symmetric pair of limit cycles and other regions where three limit cycles coexist.

(vii) The basins of attraction have an intricate fractal structure.

(viii) There is a series of Arnold tongues within the quasiperiodic region where the two fundamental oscillations mode-lock and form limit cycles of various periodicities.

The proposed system presents limit cycles, quasiperiodicity, chaos, and hyperchaos, which can make its control difficult in practical applications where a particular dynamic behavior is desired. However, the very interesting feature of the specific system is the existence of hyperchaotic state for a range of $d \in [0.39,\ 0.49]$. In Figure 1 the phase portrait of z versus x, for $c = 2.7$ and $d = 0.44$, for which the system has two positive Lyapunov exponents, ($LE_1 = 0.12806$, $LE_2 = 0.01161$, $LE_3 = 0$ and $LE_4 = -1.58236$), which is an indication of hyperchaos, is depicted. For the chosen set of system's parameters, it is proved that it is also a dissipative system, because $LE_1 + LE_2 + LE_3 + LE_4 = -1.44269 < 0$. Furthermore, the Kaplan-Yorke dimension of the 4-D hyperchaotic Lorenz system is found as:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.0883 \tag{6}$$

3 The Nonlinear Open Loop Control Scheme

As it is known, two identical coupled chaotic systems can be described by the following system of differential equations:

$$\frac{dx}{dt} = f(x) + U_x \tag{7}$$

$$\frac{dy}{dt} = f(y) + U_y \tag{8}$$

where $f(x), f(y) \in R^n$ are the flows of the coupled systems. The coupling of the systems is defined by the Nonlinear Open Loop Controllers (NOLCs), U_x and U_y [41]. The error function is given by $e = \beta y - \alpha x$, where α and β are constants. If one applies the Lyapunov Function Stability (LFS) technique,

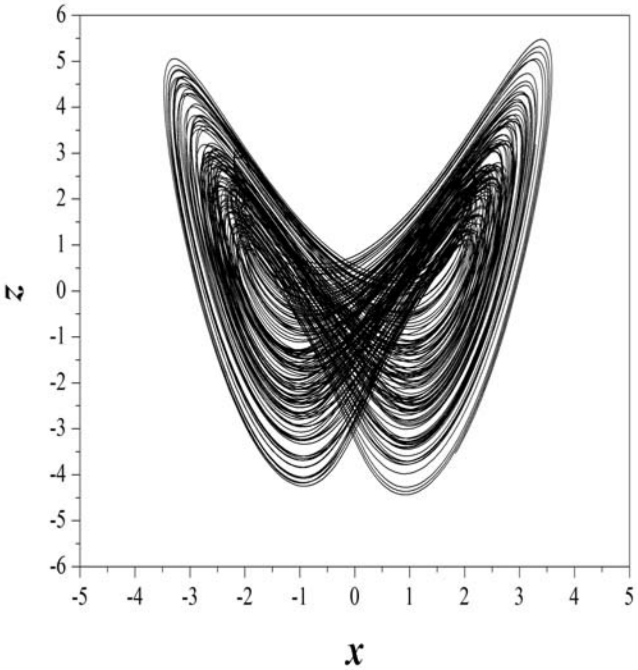

Figure 1: Phase portrait of the hyperchaotic attractor in $x - z$ plane, for $c = 2.7$ and $d = 0.44$.

a stable synchronization state will be realized when the error function of the coupled system follows the limit

$$\lim_{n \to \infty} \|e(t)\| \longrightarrow \infty \tag{9}$$

so that $ax = by$. As it is mentioned, the design process of the coupling scheme, is based on the Lyapunov function

$$V(e) = \frac{1}{2} e^T e \tag{10}$$

where T denotes transpose of a matrix and $V(e)$ is a positive definite function.

For known system's parameters and with the appropriate choice of the controllers U_x and U_y, the coupled system has $dV(e)/dt < 0$. This ensures the asymptotic global stability of synchronization and thereby realizes any desired synchronization state.

By using the appropriate NOLCs functions U_x, U_y and error function's parameters α, β, a unidirectional or bidirectional (mutual) coupling scheme can be implemented. In more details, for ($U_x = 0$, $\beta = 1$) or ($U_y = 0$, $\alpha = 1$), a unidirectional coupling scheme is realized, while for U_x, $U_y \neq 0$ and α, $\beta \neq 0$, a bidirectional coupling scheme is realized, respectively. The signs of α, β play a crucial role to the type of synchronization (complete synchronization or antisynchronization). On the other hand, the ratio of α and β decides the amplification or attenuation of one oscillator relative to another one.

In this work, the bidirectional coupling scheme of two coupled systems of Eqs. (2)-(5), which are described by the following systems' equations (11)-(14), (15)-(18), is adopted.

Coupled Hyperchaotic System-1:

$$\frac{dx_1}{dt} = x_2 - x_1 + U_{x1} \tag{11}$$
$$\frac{dx_2}{dt} = -x_1x_3 + x_4 + U_{x2} \tag{12}$$
$$\frac{dx_3}{dt} = x_1x_2 - c + U_{x3} \tag{13}$$
$$\frac{dx_4}{dt} = -dx_2 + U_{x4} \tag{14}$$

Coupled Hyperchaotic System-2:

$$\frac{dy_1}{dt} = y_2 - y_1 + U_{y1} \tag{15}$$
$$\frac{dy_2}{dt} = -y_1y_3 + y_4 + U_{y2} \tag{16}$$
$$\frac{dy_3}{dt} = y_1y_2 - c + U_{y3} \tag{17}$$
$$\frac{dy_4}{dt} = -dy_2 + U_{y4} \tag{18}$$

where $U_x = [U_{x1}, U_{x2}, U_{x3}, U_{x4}]^T$ and $U_y = [U_{y1}, U_{y2}, U_{y3}, U_{y4}]^T$ are the NOLCs functions. The error function is defined by $e = \beta y - \alpha x$, with $e = [e_1, e_2, e_3, e_4]^T$, $x = [x_1, x_2, x_3, x_4]^T$ and $y = [y_1, y_2, y_3, y_4]^T$. So, the errors dynamics, by taking the difference of Eqs. (11)-(14) and (15)-(18), are written as:

$$\frac{de_1}{dt} = e_2 - e_1 + \beta U_{y1} - \alpha U_{x1} \tag{19}$$
$$\frac{de_2}{dt} = \alpha x_1x_3 - \beta y_1y_3 + e_4 + \beta U_{y2} - \alpha U_{x2} \tag{20}$$
$$\frac{de_3}{dt} = -\alpha x_1x_2 + \beta y_1y_2 - c(\beta - \alpha) + \beta U_{y3} - \alpha U_{x3} \tag{21}$$
$$\frac{de_4}{dt} = -de_2 + \beta U_{y4} - \alpha U_{x4} \tag{22}$$

For stable synchronization $e \to 0$ with $t \to \infty$. By substituting the conditions in Eqs. (19)-(22) and taking the time derivative of Lyapunov function

$$\begin{aligned}\frac{dV(e)}{dt} &= e_1\frac{de_1}{dt} + e_2\frac{de_2}{dt} + e_3\frac{de_3}{dt} + e_4\frac{de_4}{dt} = \\ & e_1(e_2 - e_1 + \beta U_{y1} - \alpha U_{x1}) + e_2(\alpha x_1 x_3 - \beta y_1 y_3 + e_4 + \beta U_{y2} - \\ & -\alpha U_{x2}) + e_3\left[-\alpha x_1 x_2 + \beta y_1 y_2 - c(\beta - \alpha) + \beta U_{y1} - \alpha U_{x1}\right] + \\ & + e_4(-de_2 + \beta U_{y1} - \alpha U_{x1}) \qquad (23)\end{aligned}$$

we consider the following NOLC controllers:

$$U_{x1} = \frac{1}{2\alpha}e_2 \qquad (24)$$

$$U_{x2} = \frac{1}{\alpha}(\alpha x_1 x_3 + e_2 + e_4) \qquad (25)$$

$$U_{x3} = \frac{1}{\alpha}(-\alpha x_1 x_2 + e_3) \qquad (26)$$

$$U_{x4} = \frac{1}{\alpha}(-\frac{d}{2}e_2 + e_4) \qquad (27)$$

and

$$U_{y1} = \frac{1}{2\beta}e_2 \qquad (28)$$

$$U_{y2} = \frac{1}{\beta}(\beta y_1 y_3) \qquad (29)$$

$$U_{y3} = \frac{1}{\beta}(-\beta y_1 y_2 + c(\beta - \alpha)) \qquad (30)$$

$$U_{y4} = \frac{1}{2\beta}(de_2) \qquad (31)$$

such that

$$\frac{dV(e)}{dt} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \qquad (32)$$

In Figure 2 the phase portraits of x_2 versus x_1 and y_2 versus y_1, in the case of bidirectionally coupled hyperchaotic systems with the proposed scheme, are depicted. Also, in this work, the parameters of the error functions are chosen as $\alpha = 1$ and $\beta = 3$, while the systems' parameters are $c = 2.7$, $d = 0.44$. As it is concluded, the hyperchaotic attractor of the first system is enlarged three times, (with black color in Figure 2), in regard to the hyperchaotic attractor of the second system (with red color in Figure 2). The y_1 versus x_1 plot in Figure 3 confirms that the coupled system is in complete synchronization state independently of the values of the error's parameters α, β. So, the NOLC scheme works well in this case, even if the hyperchaotic attractor of the first system has been amplified in regard to the second one.

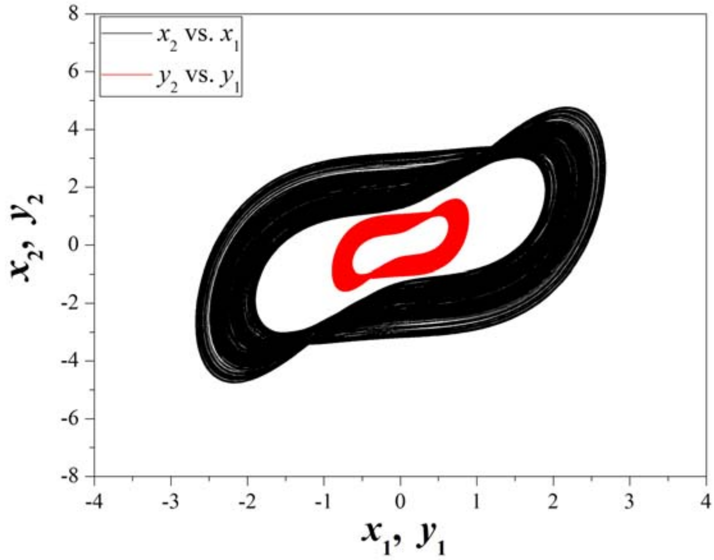

Figure 2: Phase portraits of x_2 versus x_1 (black color) and y_2 versus y_1 (red color), in the case of bidirectionally coupled hyperchaotic systems, for $c = 2.7$, $d = 0.44$ and $\alpha = 1$, $\beta = 3$.

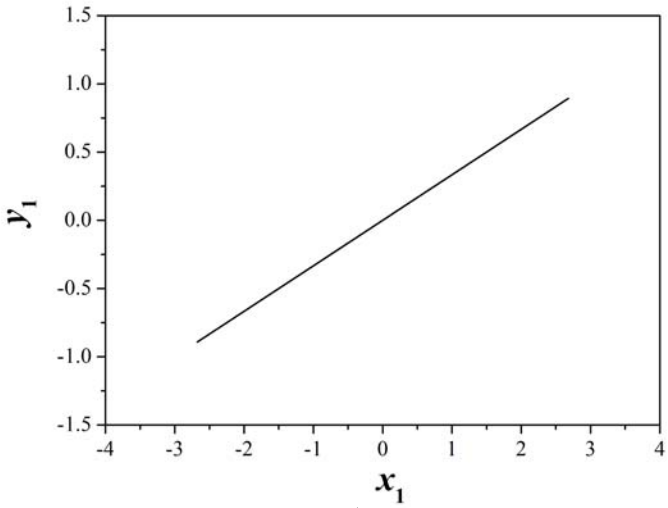

Figure 3: Phase portrait of y_1 versus x_1), in the case of bidirectionally coupled hyperchaotic systems, for $c = 2.7$, $d = 0.44$ and $\alpha = 1$, $\beta = 3$. Complete synchronization state has been achieved.

4 The Model of the Autonomous Mobile Robot

Many works on kinematic control of chaotic mobile robots is based on a typical differential motion with two degrees of freedom, composed by two active, parallel and independent wheels and a third passive wheel [10]. The active wheels are independently controlled on velocity and rotation sense. The above mentioned mechanism has been used to the kinematic control of the robot of this work. So, the proposed mobile robot's motion is described by the linear velocity $V(t)$ [m/s], the angle $\theta(t)$ [rad] describing the orientation of the robot, and the angular velocity $\omega(t)$ [rad/s]. The linear velocity provides a linear motion of the medium point of the wheels axis, while the direction velocity provides a rotational motion of the robot's over the same point. So, the robot's motion control is described by the following system.

$$\frac{dX}{dt} = cos\theta(t)V(t) \quad (33)$$

$$\frac{dY}{dt} = sin\theta(t)V(t) \quad (34)$$

$$\frac{d\theta}{dt} = \omega(t) \quad (35)$$

where, $X(t), Y(t)$ is the robot's position on the plane and $\theta(t)$ is the robot's orientation.

Furthermore, in many cases, robots move in spaces with boundaries like walls or obstacles. So, many robots have sensors, like sonar or infrared devices, which provide the capability to detect the presence of obstacles or even more the recognition of the searched objects or intruders. In this work, for a better understanding of the colaborative mobile robots control scheme we assume that the robots work in a smooth state space without any sensor. Also, in this work for integrating the coupled dynamical system into the system of two collaborative autonomous mobile robots, the following strategy is used. The parameters x_3 and y_3 are adopted as the angular position θ of each robot. Also, by adding into the dynamical systems (11)-(14) and (15)-(19) the two following equations (36) and (37), which correspond to mobile robots motion, two six-dimension systems are created.

$$\frac{dX}{dt} = Vcos(n\theta(t)) \quad (36)$$

$$\frac{dY}{dt} = Vsin(n\theta(t)) \quad (37)$$

In previous equations, n is a factor of normalization, so that the parameters x_3 and y_3, of the coupled system, have the same magnitude.

To test the proposed control strategy of the chaotic mobile robot we used the known coverage rate (C), which represents the effectiveness, as the amount of the total surface covered by the robot running the algorithm. The coverage rate (C) is given by the following equation:

$$C = \frac{1}{M}\sum_{i=1}^{M} I(i) \tag{38}$$

where, $I(i)$ for $i = 1, ..., M$, is the coverage situation for each cell in which the terrain has been divided [42]. This is defined by the following equation.

$$I(i) = \begin{cases} 1, \text{ when the cell} i \text{is covered} \\ \\ 0, \text{ when the cell} i \text{ is not covered} \end{cases} \tag{39}$$

The robot's workplace is supposed to be a square terrain with dimensions $M = 40 \times 40 = 1600$ cells.

Searching for sets of optimal parameters for the dynamical systems for generating the best possible patterns is very time-consuming task. So, the initial conditions of the coupled systems and the factor of normalization n, are chosen as: $(x_1, x_2, x_3, x_4)_0$ = (0.55, -0.493, -0.08, 0.5), $(y_1, y_2, y_3, y_4)_0$ = (-0.4, -0.05, 1.45, -0.3) and $n = 5$. Also, the robots' initial position and velocity of the two mobile robots are chosen to be: (X_1, Y_1) = (19, 19), (X_2, Y_2) = (-19, -19) and $V = 5$ [n.u.]. Duration for run-time of the simulation is 500 [n.u.].

In Figure 4 the trajectories of the collaborative autonomous robots controlled by coupled hyperchaotic systems, by using the proposed control scheme, is depicted. In more details, with black color is the trajectory of the first robot while with red color is the trajectory of the second one. Also, in Figure 5 the trajectories of the autonomous mobile robots controlled by uncoupled hyperchaotic systems is shown. From the comparison of these two robots' motion control schemes (coupled and uncoupled systems), one can see that the first robot follows exactly the same trajectory in the two cases while the second robot has more dense trajectory in the case of coupled hyperchaotic systems. Furthermore, the coverage rate, by using the collaborative autonomous robots controlled by coupled hyperchaotic systems with the proposed control scheme, is calculated to be $C = 40\%$, in contrary to the uncoupled case in which the coverage rate is only $C = 35\%$.

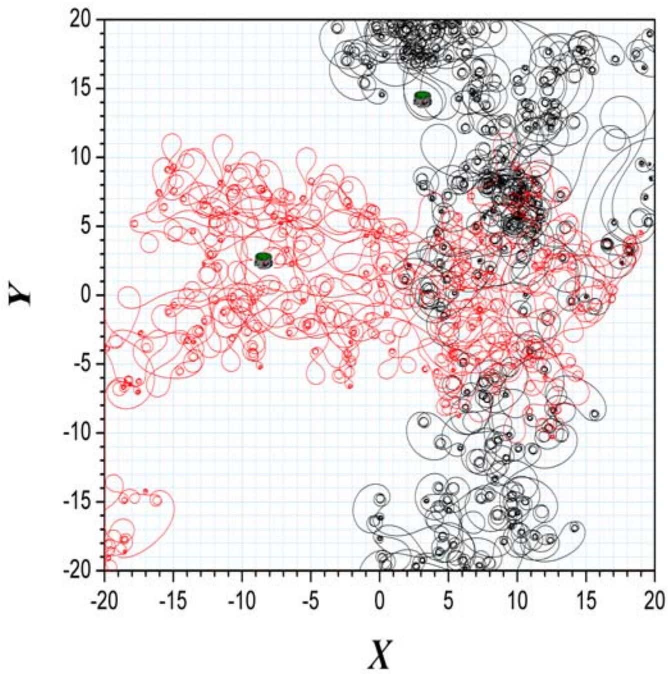

Figure 4: Trajectories of the collaborative autonomous robots controlled by coupled hyperchaotic systems, with (X_1, Y_1) = (5, 5), (X_2, Y_2) = (-10, -19), while the duration is 500 [n.u.].

Finally, by choosing the robots' initial positions (X_1, Y_1) = (5, 5), (X_2, Y_2) = (-10, -19), for which the system of collaborative robots presents the best workspace's coverage rate, we run the simulation for bigger duration (2500 [n.u.]), until the mobile robots covers almost the whole terrain $C = 98.31\%$. From the simulation process of each autonomous mobile robot we can see that the aforementioned result of the coverage rate is greater than the coverage rate of each robot, which is $C = 83.13\%$ for the first and $C = 55.25\%$ for the second one.

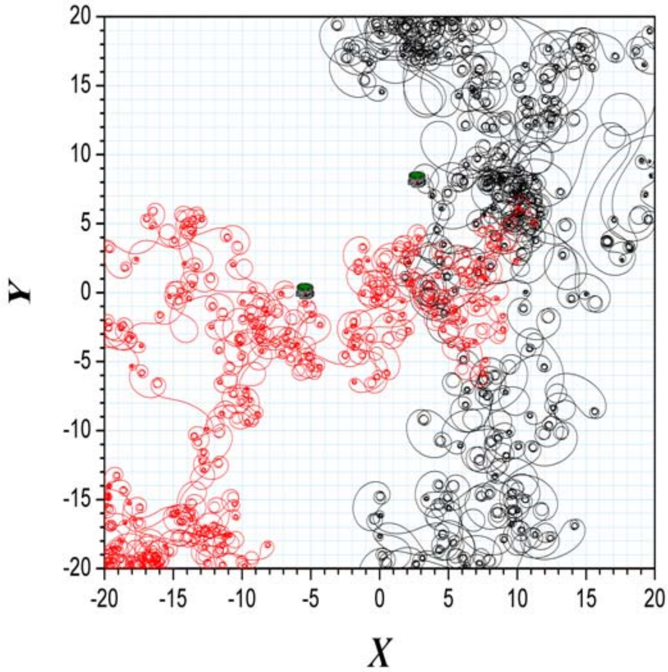

Figure 5: Trajectories of the autonomous robots controlled by uncoupled hyperchaotic systems, with (X_1, Y_1) = (5, 5), (X_2, Y_2) = (-10, -19), while the duration is 500 [n.u.].

5 Conclusion

In this work, a novel method for the collaboration work of two autonomous mobile robots, which was based on chaos synchronization by using the Nonlinear Open Loop Control method, was presented. Each one of the mobile robots was controlled by an hyperchaotic system, producing an unpredictable trajectory. Also, by using this method, the synchronization state with amplification of system's hyperchaotic oscillators was adopted in the robots' control unit.

The proposed approach was followed in order to generate the most unpredictable robots' trajectories, as well as trajectories with the higher coverage rate of the specific terrain. Also, it is concluded that the proposed approach neither requires a map of the workspace nor plans a path through it. Finally, the proposed scheme of multiple robots had as a consequence the faster and more complete coverage of the workspace.

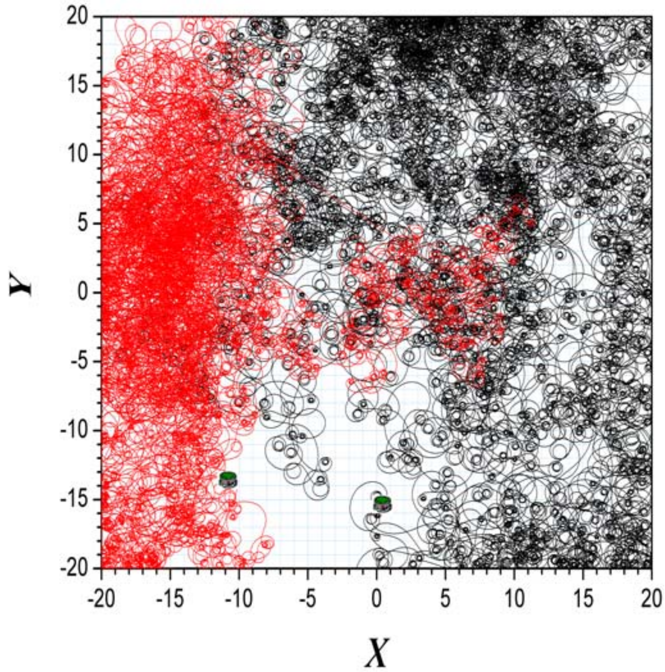

Figure 6: Trajectories of the collaborative autonomous robots controlled by coupled hyperchaotic systems, with (X_1, Y_1) = (5, 5), (X_2, Y_2) = (-10, -19), while the duration is 2500 [n.u.].

As a future work we plan to use more than two autonomous mobile robots, which will be driven by coupled, with Nonlinear Open Loop Controllers, hyperchaotic systems. Also, a very interesting concept is the experimental verification of the simulation results.

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