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Predictive Dynamic Linear Models for External Reserves-Economic Growth Nexus A Case Study

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Abstract

Over the years, there has been a debate whether there is a need to beef up the level of foreign reserves or trim them down, and this debate is becoming more interesting especially in developing countries. It is usual for countries in the world to hold external reserves in order to have a favourable level of exchange rate especially with a view of stabilizing and establishing a robust economy. Most previous studies had concentrated on modeling External Reserves-Economic Growth nexus with classical time series and econometric models with static parameters.

In this paper, we propose a Bayesian time-varying parameter dynamic linear model for econometric modeling of external reserves-economic growth nexus using the Nigerian economy as a case study. We assess the predictive performance of external reserves on economic growth in comparison with some selected macroeconomic variables. Our empirical findings reveal that external reserve has the least Mean Squared Prediction Error (MSPE) among the several one-regressor models considered

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over the years, while the model involving the combination of external reserves and capital expenditure has the least MSPE among the two regressor models considered in our econometric analysis. The economic implications of these results were discussed and used to make policy recommendations.

Mathematics Subject Classification: 62P10
Keywords: Dynamic Model; Bayesian Inference; External Reserves; MCMC.

1 Introduction

External Reserve is a major economic indicator that has been variously described as International Reserves (IR), Foreign Reserves (FR) or Foreign Exchange Reserves (FER). While there are several definitions of external reserves, the most widely accepted is the one proposed by the International Monetary Fund (IMF) in its balance of payments manual and guidelines on foreign exchange reserve management which defined external reserves as consisting of official public sector foreign assets that are readily available to, and controlled by the monetary authorities, for direct financing of payment imbalances through intervention in the exchange markets [1]. There has been a debate among researchers on whether there is a need to beef up the level of foreign reserves or trim them down, and this debate is becoming more interesting especially in the context of a developing economy like Nigeria. Some researchers are of the opinion that keeping scarce resources in reserve when there is a series of burning issues to be attended to domestically may not be a very wise decision [2].

However, some other researchers have argued that the foreign reserve position determines the rating in the global competitive market of a country and will make the country appear financially responsible and creditworthy. The level of external reserves has remained an important parameter in gauging the ability of economies to absorb external shocks [3]. [4, 5] opined that external reserves has, in recent times, played a significant role in growing the

Nigerian economy by increasing the level of money supply and therefore impacting positively on the level of economic activities as more funds became available for investment in productive activities, employment was in turn generated and output was increased. Over the years, Nigeria has taken numerous policy initiatives and measures in the management of her external reserves . The phenomenal rise in the level of Nigerian external reserves, especially since the beginning of 2004 has generated a lot of interest and debate among policy makers and members of the public on how reserves should be managed [6]. Since the early 1970s, Nigerian economy has persistently depended on oil as the main source of foreign exchange earnings with the attendant cycles of economic booms and bursts. Therefore, we are motivated to investigate the predictive contribution of external reserves to economic growth over the years using a specified time-varying parameter Bayesian Dynamic linear model.

To achieve this objective, we consider a case when the observational variance in the Bayesian dynamic linear model of [7] is constant and the evolution variance is represented as a fraction of the filtering variance of the measurement equation. The rest of this paper is organized as follows: In section 2, we present a brief review of literature on application of dynamic linear model in econometric time series analysis. Section 3 borders on our model specification and Markov chain Monte Carlo (MCMC) approach, while section 4 is on empirical analyses and discussion of results. Finally, section 5 is the conclusion of the paper.

2 Brief Review of Application of Dynamic Linear Models (DLM) in Econometric Time Series Analysis

Early applications of dynamic linear models to economic time series data include the works of [8] who modeled the unobserved ex-ante real interest rate as a state variable that follows an AR(1) process. [9] used an unobserved-components model to decompose quarterly real Gross National Product (GNP) data into the two independent components of a stochastic trend component and a cyclical component. Another important contribution is the work of [10] who defined an unobserved variable, which represents the state of the business cycle, to measure the common element of co-movements in various macroeconomic variables. The dynamic linear model with state space approach offers attractive features with respect to their generality, flexibility and transparency. More detailed treatments of dynamic linear models in state space form are given by [11], [12] and [13], among others. Recently, [14] published one of the most succesful methods for analyzing dynamic linear models in the journal of statistical software.

Over the past two decades dynamic time series models have become a standard econometric tool for measuring co-movement in macroeconomic time series data. The popularity of these models have increased as methods have been developed to perform factor analysis on large datasets, such as the time-domain approach of [15] and the frequency-domain approach of [16, 17]. Dynamic time series regression can in very general terms be formulated using state space representation of the observations and the state of the system. AutoRegressive (AR) models [18, 19] which were formulated several years ago falls into the class of dynamic time series regressions to model all stationary time series as long as the appropriate order of the number of AR terms, and the number of Moving Average (MA) terms, were specified .

Time series and econometric literature in the 1970's were dominated by time-domain analysis techniques advocated by [20] due to so many reasons. The main reason perhaps was that Box and Jenkins provided a complete methodology that resolved many practical issues like non-stationarity, forecasting and optimal control, and did so in a way that was easy for the analyst to implement. Box and Jenkins provided a way around the problem of non-stationarity by means of a methodology focused on differencing the data.

Despite the success of this approach in the 1970's , some researchers chose to work within a structural time series framework. For instance, [21] were successful in formulating the linear Gaussian Markovian state-space model within a Bayesian context. Working with the Kalman filter, they were able to specify their form of dynamic linear models based on time-varying parameters in order to account for nonstationarity. Moreover, the Bayesian approach allows one to specify prior distributions on not only parameters, but also the initial conditions, facilitating convergence of the Kalman gain matrix. The works of

[22], [23, 7] and [24] provide Bayesian alternatives to the classical Box and Jenkins approach. In this paper, we apply a variant of the time-varying parameter dynamic linear model of [11, 25, 7] to the econometric modeling of external reserves-economic growth nexus with the aim of assessing the predictive performance of external reserve in the presence of some selected economic variables using Nigeria as a case study.

3 Model Specification and Econometric Methodology

Our model specification takes the following form.

$$y_t = X_t\theta_t + v_t \qquad v_t \sim N(0, V) \tag{1}$$

$$\theta_{t+1} = G_t\theta_t + w_t \qquad w_t \sim N_p(0, W_t) \tag{2}$$

$$\theta_0 \sim N_p(m_0, C_0)$$

Equation (1) is known as the observation equation while equation (2) is the evolution equation. G_t is a known matrix of order $p * p$ that determines how the observation and state equations evolve in time (see [25]). We assume that all v_t's are independent from the w_t's. Since each parameter at time t only depends on results from time $t-1$, the state parameters are time-varying and constitute a Markov chain. By explicitly allowing for variability in the state regression parameters, we let the system properties change in time in the spirit of [26, 27].

In our model, the response y_t is the annual GDP of Nigeria from 1960 to 2009. The matrix X consists of economic indicators measured concurrently with the GDP and includes a column of 1's representing a dynamic intercept term. θ_t are time-varying regression coefficients which model the relationship between the regressors and the response at each time t.

3.1 Bayesian Estimation of the Model Parameters

Parameters of interest which are to be estimated are the matrix θ, the error variances V and W_t, and the one-step-ahead forecasts f_t. Since normality is assumed, we estimate θ and f_t by using the Kalman filter [28]. For the Kalman filter to run, it is necessary to estimate V and W_t. V is assumed to be distributed inverse-gamma a priori and is estimated using Gibbs sampling . This requires us to draw samples from $V|\theta$ as well as from $\theta|V$. The latter draw is performed using the Forward Filtering Backwards Sampling (FFBS) algorithm [29]. This algorithm allows for the implementation of Markov chain Monte Carlo (MCMC) approach to dynamic linear models. The forward filtering step is the standard normal linear analysis to give $P(\theta_t|D_t)$ at each t for $t = 1, ..., n$. Here, we propose the use of discount factors (δ) to estimate W_t in the spirit of [24] .

The Markovian state - space model considered is one which consists of an R^p-valued time series θ_t:$t = 0, 1, 2, ..., T$ and an R^k-valued time series y_t:$t = 1, 2, ..., T$ which satisfies the following assumptions:

- θ_t is a Markov chain.
- Conditional on θ_t, the y_t's are independent and y_t depends on θ_t only.

Due to the Markovian structure of the time varying parameter θ_t, it is estimated by computing the predictive and filtering distributions of θ_t recursively starting from the prior $\theta_0 \sim N(m_0, C_0)$. Consider a vector of unknown regression model slope parameters $\theta_t = (\theta_1, ..., \theta_p)$,the Gibbs sampling algorithm employed proceeds by sampling recursively the conditional posterior distribution where the most recent values of the conditioning parameters are used.

Assume that the observed response is represented by $y = (y_1, y_2, ..., y_T)$ where T denotes the size of the series. Following the Bayesian paradigm, the specification of the model is complete only after specifying the prior distribution of all the unknown quantities of interest in the model. We assign a distribution to θ_t at time t=0, conditional on all the information available before any observation is made. Let D_0 be the set containing all this information, then the prior distribution is $\theta_0|D_0 \sim N(m_0, C_0)$ where m_0 and C_0 are known vector and matrix respectively. Next, an update is made for θ_1 and D_0 which is also normally distributed. Based on this update, the one step-ahead forecast follows from the conditional distribution $y|\theta_0, D_0$. Once the value of y_1 at time $t = 1$ is known, the posterior distribution of θ_1 is obtained recognizing that the information available at time $t = 1$ is $D_1 = y_1, D_0$. The inference is made in this recursive fashion for every time t. The Kalman filter was used to calculate the mean and variance of the unobserved state θ_t, given the observations y_t. It is a recursive algorithm i.e the current best estimate is updated whenever a new observation is obtained.

The filter prediction and update algorithm requires a few basic calculations of which only the conditional means and variances of the filtering and prediction density need to be stored in each step of the iteration.
To describe the filtering procedure, let

$$m_t = E(\theta_t|D_t) \tag{3}$$

be the optimal estimator of θ_t based on D_t and let

$$C_t = E((\theta_t - m_t)(\theta_t - m_t)^T|D_t) \tag{4}$$

be the mean square error matrix of m_t. Let $\theta_{t-1}|y_{1:t-1} \sim N(m_{t-1}, C_{t-1})$, where $y_{1:t-1}$ denote all observations up to time $t-1$. Then the one-step-ahead predictive density $\theta_t|y_{1:t-1}$ is Gaussian with parameters:

$$E(\theta_t|y_{1:t-1}) = m_{t-1} \equiv A_t \tag{5}$$

$$Var(\theta_t|y_{1:t-1}) = C_{t-1} + W_t \equiv R_t \tag{6}$$

The one-step-ahead predictive density of $y_t|y_{1:t-1}$ is Gaussian with parameters:

$$f_t = E(y_t|y_{1:t-1}) = X_t A_t \tag{7}$$

$$Q_t = Var(y_t|y_{1:t-1}) = X_t R_t X_t^{'} + V \tag{8}$$

The filtering density of θ_t given $y_{1:t}$ is Gaussian with parameters:

$$m_t = E(\theta_t|y_{1:t}) = A_t + R_t X_t^{'} Q_t^{-1} e_t \tag{9}$$

$$C_t = Var(\theta_t|y_{1:t}) = R_t - R_t X_t^{'} Q_t^{-1} X_t R_t \tag{10}$$

where $e_t = y_t - f_t$ is the forecast error.

The probability distribution of update is proportional to the product of the time series measurement likelihood and the predicted state

$$p(\theta_t|y_{1:t}) = \frac{p(y_t|\theta_t)p(\theta_t|y_{1:t-1})}{p(y_t|y_{1:t-1})} \tag{11}$$

$$\propto p(y_t|\theta_t)p(\theta_t|y_{1:t-1})$$

The denominator, $p(y_t|y_{1:t-1})$ is constant relative to θ_t and thereby ignored.

This posterior is used to update the prior recursively until convergence is achieved.

3.2 FFBS Algorithm and Gibbs Sampler

The Recursive Forward Filtering Backward Sampling (RFFBS) algorithm [29, 7] adopted is to allow for the implementation of a fast MCMC approach to the specified dynamic linear model. The forward filtering step is the standard Kalman filtering analysis to give $p(\theta_t|D_t)$ at each t, for $t = 1, ..., n$. The backward sampling step uses the Markov's property specified in equation 3.9 above to sample θ_n^* from $p(\theta_n|D_n)$ and then for $t = 1, ..., n-1$, sample θ_t^* from $p(\theta_t|D_t, \theta_{t+1}^*)$ in order to generate samples from the posterior parameter structure.

In particular, denote

$$p(\theta_0, ..., \theta_T|D_T) = \prod_{t=0}^{T} p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T)$$

and note that, by the Markov's property,

$$p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T) = p(\theta_t|\theta_{t+1}, D_T)$$

then, the RFFBS algorithm proceeds as follows:

1. Sample from $p(\theta_T|D_T)$ using the filtering density above. This distribution is $N(a_t, Q_t)$ where:

$$a_t = m_t + C_t G' R_{t+1}^{-1}(\theta_{t+1} - a_{t+1}) \tag{12}$$
$$Q_t = C_t - G_t' R_{t+1}^{-1} G_t C_t \tag{13}$$

2. Sample from $p(\theta_{T-1}|\theta_T, D_T)$.

3. Proceed recursively until we have a complete sample from $p(\theta_0, ..., \theta_T|D_T)$. Since we sampled from t = T to t = 0, recursively, this procedure is referred to as recursive backward sampling.

To sample from $V|\theta$ we impose a gamma prior on V^{-1} and derive the posterior hyperparameters. Let $V^{-1} \sim Gamma(a_0, b_0)$, then

$$V^{-1}|\theta \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - X_t\theta_t)^2)$$

This Gibbs sampler is run for a given set W_t determined from a given value of $\delta \in [0,1]$ as mentioned above. We used M = 12,000 with a burn-in period of 2,000. Convergence was quite quick, happening in a relatively few iterations.

4 Empirical Analyses

4.1 Data Presentation

The data used in this study are Nigerian economic indicators sourced from the Central Bank of Nigeria (CBN). The data includes annual External Reserves (ER), Lending Rate (LR), Gross Domestic Product (GDP), Exchange Rate (EXRT), Capital Expenditure (CE), External Debt (ED), and Treasury-bill Rate (TR) for the period between 1960-2009.

In order to investigate other possible macroeconmic variables that might have contemporaneous prediction effect on GDP alongside external reserve, we include in our model each of the other variables aforementioned in turns.

To avoid spurious regressions, we adjust all the monetary economic variables in our data for inflation before taking logarithms of each.This is because inflation adjustment, or "deflation", is an important tool in the toolkit for analyzing economic data. This is accomplished by dividing all the monetary time series by a price index, such as the Consumer Price Index (CPI). Adjusting for inflation enables us to uncover the real growth in the variables, if any. It also helps us to stabilize the variance of random or seasonal fluctuations and highlight cyclical patterns in the data.

4.2 Parsimonious Model Selection and Discussion of Results

Predictive performance of the variables was assessed using one-step-ahead

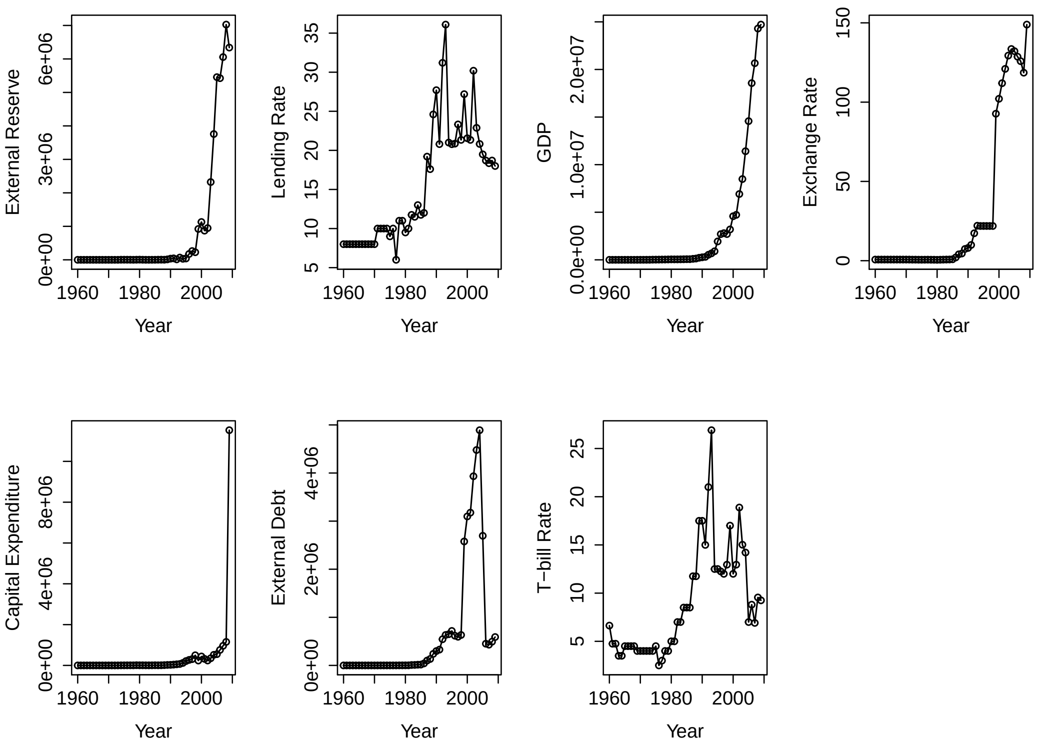

Figure 1: Annual time-series data on External Reserves (ER), Lending Rate (LR), Gross Domestic Product (GDP), Exchange Rate (EXRT), Capital Expenditure (CE), External Debt (ED) and Treasury- bill Rate (TR)

Table 1: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Statistic, Effective Sample Size for Various One-Regressor Models (1960-1969) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	6.116	0.047	-0.103	2702	
2	CE	6.019	0.016	1.131	8841	
3	ED	6.036	0.024	0.833	2314	
4	TR	6.632	0.286	0.835	4486	
5	ERT	5.957	0.054	0.936	1316	
6	LR	blue5.954	0.018	1.011	5944	

Table 2: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Statistic, Effective Sample Size for Various One-Regressor Models(1960-1979) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	blue3.022	0.010	1.250	3674	
2	CE	3.024	0.017	0.964	4584	
3	ED	3.055	0.022	-0.465	1954	
4	TR	3.227	0.047	1.066	5708	
5	ERT	3.017	0.025	-0.207	6659	
6	LR	3.030	0.754	1.011	1112	

Table 3: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-1989) at 12, 000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	blue2.019	6.775	1.030	11536	
2	CE	2.021	0.015	1.053	9426	
3	ED	2.043	0.014	0.464	2369	
4	TR	2.148	0.031	1.142	5353	
5	ERT	2.040	0.020	1.025	3512	
6	LR	2.037	0.594	1.027	2667	

Table 4: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-1999) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	1.521	0.010	0.979	5227	
2	CE	blue1.520	0.036	0.993	8153	
3	ED	1.538	0.011	0.925	2016	
4	TR	1.612	11.000	0.999	8768	
5	ERT	1.535	0.015	1.149	5736	
6	LR	1.537	0.012	-0.881	1901	

Mean Squared Prediction Error (MSPE) from ten dynamic linear models. Our

Table 5: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-2009) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	blue1.222	1.137	1.002	10879	
2	CE	1.264	0.024	1.105	9965	
3	ED	1.236	5.087	1.018	3421	
4	TR	1.309	0.024	-0.401	6018	
5	ERT	1.239	111.536	1.008	10882	
6	LR	1.240	0.010	1.224	1603	

Table 6: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1969) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	6.014	0.019	1.163	4731	
2	ERES + ED	blue 5.957	0.022	0.978	5793	
3	ERES + TR	6.081	0.042	0.941	8054	
4	ERES + ERT	6.103	0.054	0.967	7599	
5	ERES + LR	5.988	0.016	0.999	7375	

Table 7: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1979) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	3.029	0.019	-0.683	5457	
2	ERES + ED	blue3.025	0.040	1.011	9764	
3	ERES + TR	3.045	0.012	-1.155	2659	
4	ERES + ERT	3.070	0.020	0.775	6532	
5	ERES + LR	3.043	0.009	0.907	2410	

Gibbs sampler was run, using the range of values of discount factors $\delta \in \{.01, .02, \ldots, .99\}$ while the δ with the lowest MSPE was chosen in each model in Tables 1 to 10. After the Gibbs sampler was run, we assessed convergence by

Table 8: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1989) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	E SS	
1	ERES + CE	blue2.024	0.015	-0.771	6599	
2	ERES + ED	2.029	0.540	0.995	9013	
3	ERES + TR	2.036	0.008	2.242	2427	
4	ERES + ERT	2.059	0.014	-0.056	4666	
5	ERES + LR	2.051	4.876	1.010	10113	

Table 9: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1999) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	blue1.523	0.015	-1.259	5968	
2	ERES + ED	1.528	0.026	1.005	7810	
3	ERES + TR	1.537	0.009	0.983	3557	
4	ERES + ERT	1.551	11.600	1.007	10363	
5	ERES + LR	1.548	0.008	0.194	2550	

Table 10: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Convergence Diagnostic(CD), Effective Sample Size for Various Two-Regressor Models (1960-2009) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	1.233	2.019	0.987	9049	
2	ERES + ED	blue1.228	0.023	1.020	7455	
3	ERES + TR	1.240	0.010	1.021	3881	
4	ERES + ERT	1.249	0.022	0.990	6175	
5	ERES + LR	1.248	0.010	1.004	4448	

examining trace plots and corroborating the plots with the Geweke test [30, 26]. The tables below contain the estimates of the observation variance(V), the Effective Sample Size(ESS) to ensure we had sufficient replications to estimate V. All of the (absolute) Geweke z statistics (CD) are below the 1.96 threshold, indicating a failure to reject the null hypothesis of stationary means in each time series. Also,the traces of the simulated variances (not shown) do not show any particular sign of non-convergence.

We perform the analysis for varying periods, considering ten years at a time.Table 1 shows the analysis performed for the period 1960-1969.During this period, the model involving lending rate has the lowest MSPE of 5.954 and therefore has the best predictive performance. Table 2 shows the results of analysis perfomed from 1960-1979. Model 1 involving external reserves as predictor has the best performance for this period in terms of MSPE value of 3.022. Table 3 contains results of analysis done for the period 1960-1989. For this period, external reserve still predicts GDP better than other variables.It has the lowest MSPE value of 2.019. In Table 4, thesame analysis was performed for the period 1960-1999. The model containing Capital Expenditure (CE) performs best in predicting GDP for this period with MSPE value of 1.520 but the model containing external reserves has the least observational variance (V) of 0.010. In Table 5, the analysis was done for the the period covering 1960-2009.The model involving external reserve as predictor also performs best in terms of the lowest value of MSPE but surprisingly with a high observational variance.

As we can see from the results in the Tables 1-5, out of all the economic indicators considered, external reserve best predicts economic growth (proxied by GDP) of Nigeria for most of the period under study. Tables 6-10 reveals the result of the models with two regressors involving external reserves and all other variables in order to check for their contemporaneous effects on economic growth . Table 6 shows that the models involving external reserves and external debt has the lowest MSPE for the period 1960-1969.The result is similar in Table 7 for the analysis for period 1960-1979 . In Tables 8 and 9 which covers the periods 1960-1989 and 1960-1999 respectively, the model involving variable combination of external reserves and capital expenditure performs best in terms of lowest MSPE. Finally in Table 10, the result is reverted to reveal the model with external reserve and external debt as predictors having the best performance for the most current period, 1960-2009. Figure 2 hows the estimates of the time varying slopes of external reserves with respect to economic growth over the years considered.

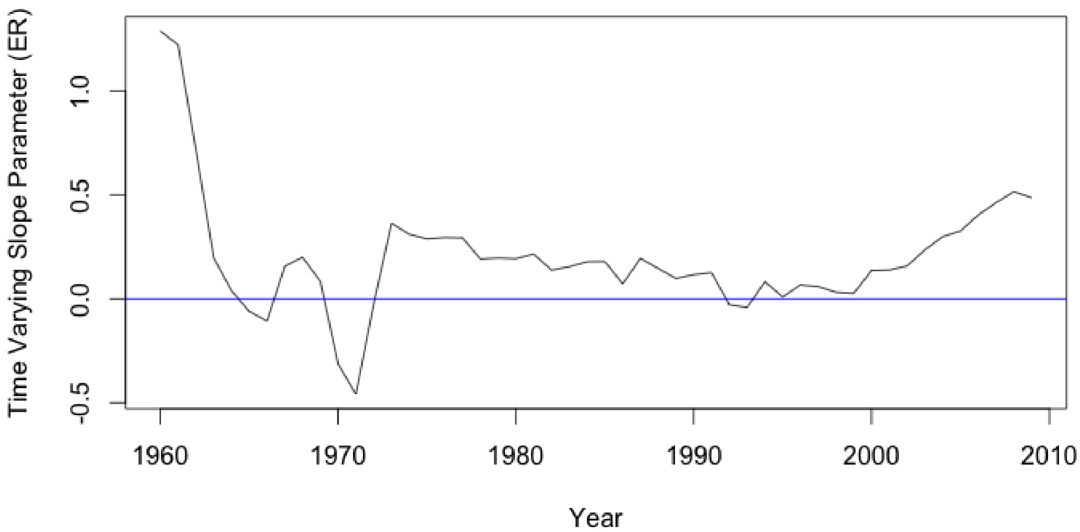

Figure 2: Time Varying Slope Parameters of GDP vs External Reserves (ER).

5 Concluding Remarks

In this paper, we proposed a Bayesian time-varying parameter dynamic linear regression model with application to external reserves-economic growth dynamics. The model was estimated for the Nigerian economy via the Markov chain Monte Carlo (MCMC) method to simulate the predictive posterior estimates of model parameters. In our analysis, we find that external reserves has the highest nexus with economic growth than the other macroeconomic variables considered in terms of predictive performance over the years. This result underscores the importance of external reserves to economic growth. Further more, the economic indicator that best predicts economic growth (GDP) when combined with external reserve is external debt for most of the period under consideration. This is not unconnected with the fact that foreign exchange reserves are necessary to pay debt and to support certain exchange rate regimes among other factors. It is generally believed that countries with rapidly growing FER/GDP ratios, ceteris paribus, exhibit higher capital productivity and higher rates of economic growth [5]. Moreso, the usable foreign exchange reserve of any nation is an important index in the risk models used by credit rating agencies and international financial institutions. Hence, we recommend that the new regime in Nigeria should continue to formulate appropriate monetary policies, maintain adequate reserves while still expending on capital expenditures that are capable of enhancing good living conditions. Finally, the model and methods in this study can be applied to other dynamic processes.

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