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Solution of the Extended Mathematical Model of Onchocerciasis Disease Using Adomian Decomposition Method

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Abstract

Onchocerciasis disease is a debilitating disease that hampers the well-being and productive capacity of an infected person. This paper looked at the use of Adomian Decomposition Method (ADM) for solving and understanding the dynamic of the disease in a wholly susceptible population. The saturated incidence rate is introduced to the 'existing' mathematical model of the problem, and this result in a usual non-linear ODE. The mathematics of the model shows in the detail of the process of the spread of the disease and the transmission pathways. In the absence of vaccination or any form of treatment, the disease takes over the population in the course of time. The computation is carried out and graphical results showing healthy class and infected class are presented and discussed.

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1 Introduction

Onchocerciasis disease also known as river blindness is caused by filial nematode called Onchocerca Volvulus. Heymann [1] say Onchocerciasis is transmitted by black flies, which acts as an intermediate host, living near fast moving bodies of water.

Kindhauser [2] described the disease as an adult worms which form fibrous nodules in subcutaneous tissues or near the bones and joints of their hosts and may live for up to 14years causing chronic and non fatal disease. Many of the clinical system of river blindness are a result of the inflammatory response of the immune system to the migration of thousands and thousands of larvae called microfilariae, discharged by adult worms. The migration and deaths of microfilariae damage surrounding tissue or organs causing intense itching and disfigurement, ocular degeneration and blindness occur when microfilariae migrate to the eyes.

Thylefors et al [3] estimated that Onchocerciasis is the second leading infectious cause of blindness in the world only proceeded by blinding trachoma, which is endemic in 36 countries in Africa, Arabian peninsula and the Aericas, but its distribution is highly concentrated on the poorest regions of the world, 30 out of 36 countries are in sub-Sahara African Countries where approximately 99% of all those infected live in pain and the disease has had a major impact on the economic and social fabric of endemic communities.

Akoh et al [4] and Mbah [5] put it that ,if the host has been in the endemic area of Onchocerciasis disease for a long time he must have quite good concentration of the microfilariae in his body. Thus the age of the host will be considered in terms of length of time of domicile and exposure in the area and this is a very important factor. The number of the microfilariae picked at each meal will highly depends on the location of the body since different parts of the body of the host have different concentration of the microfilariae.

World health organisation [6] report provided country-based and global estimates of the number of person infected and the number blind due to Onchocerciasis disease, but no information on the burden of Onchocercal skin disease. The estimates were based on published and reported data of highly varying quality, ranging from extensive, reliable data from the OCP area and national prevalence surveys in Liberia and Nigeria to crude estimates based on very little information for some of the endemic countries. However during the 1990s, there was substantial progress in Onchocerciasis control due to large scale chemotheraphy with lrermectin coupled with vector control. Nearly all endemic Countries are covered and monitored by three major control programmes, such as:

(a) Onchocerciasis Control Programme (OCP)
(b) African Programme for the Onchocerciasis Control (APOC).
(c) Onchocerciasis Control Programme of the Americas (OCPA).

With the success of OCP IN West Africa, the largest numbers of infected persons are found in some APOC countries such as Nigeria, Cameroon, Ethiopia and Uganda. For this reason, it was re-estimated that the prevalence of blinding trachoma for the year 2000 based on the existing data sources by taking into account the effect of control programmes.

Remme et al [7] estimate the prevalence of blindness and low vision by age for 338 OCP villages. On average, the prevalence of low vision is 1.78times the prevalence of blindness as measured by visual acuity alone and there is no statistically significant trend with age in the ratio between the prevalence of low vision and of blindness.

Abou-Gareeb et al [8], Onchocerciasis is more common among males than among females and is estimated to lead to more disabling sequelae among the people. Also Onchocercal blindness appears many years after the infection and thus age distribution of cases are skewed among older age groups.

2 The Model

$$\frac{dS}{dt} = b - \frac{\beta SI}{1+\alpha I} - \mu S\,, \quad S(0) = S_0 \tag{1.0}$$

$$\frac{dI}{dt} = \frac{\beta SI}{1+\alpha I} - \mu I\,, \quad I(0) = I_0 \tag{1.1}$$

where S is susceptible class, I is the infected class, b is the birth rate, β is the infection rate of the susceptible, μ is the natural death rate unrelated to the disease, $\frac{1}{1+\alpha I}$ is the extended incident rate and α is the death rate related to the disease.

Solution:

Considering two cases where the population is kept constant and when the population is not constant.

Cases 1: When $b = 0$, then

$$\frac{dS}{dt} = -\frac{\beta SI}{1+\alpha I} - \mu S \tag{1.2}$$

$$\frac{dI}{dt} = \frac{\beta SI}{1+\alpha I} - \mu I \tag{1.3}$$

But

$$(1+\alpha I)^{-1} = 1 - \alpha I + \alpha^2 I^2 + \ldots \tag{1.4}$$

Considering the first three terms in (1.4), we have

$$dS = \left(-\beta SI\left(1-\alpha I+\alpha^2 I^2\right) - \mu S\right)dt \tag{1.5}$$

$$dI = \left(\beta SI\left(1-\alpha I+\alpha^2 I^2\right) - \mu I\right)dt \tag{1.6}$$

And taking,

$$S = \sum_{n=0}^{\infty} S_n, \quad I = \sum_{n=0}^{\infty} I_n \tag{1.7}$$

If, $S(0) = x,\ \ I(0) = y$

Then,

$$S(t) = x + \int_0^t (-\beta\sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I_n + \beta\alpha\sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I^2{}_n)dt - \int_0^t (\beta\alpha^2 \sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I_n{}^3 + \mu\sum_{n=0}^{\infty} S_n)dt \quad (1.8)$$

$$I(t) = y + \int_0^t (\beta\sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I_n - \beta\alpha\sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I^2{}_n)dt + \int_0^t (\beta\alpha^2 \sum_{n=0}^{\infty} S_n \sum_{n=0}^{\infty} I_n{}^3 - \mu\sum_{n=0}^{\infty} I_n)dt \quad (1.9)$$

Hence,

$$S_0 = x, I_0 = y \quad (1.10)$$

Taking

$$A_n = \sum_{n=0}^{\infty} S_n I_n, B_n = \sum_{n=0}^{\infty} S_n I_n{}^2, C_n = \sum_{n=0}^{\infty} S_n I_n{}^3 \quad (1.11)$$

Put (1.11) into (1.8) and (1.9), we have

$$S_{n+1} = -\int_0^t \beta A_n dt + \int_0^t \beta\alpha B_n dt - \int_0^t \beta\alpha^2 C_n dt - \int_0^t \mu S_n dt \quad (1.12)$$

$$I_{n+1} = \int_0^t \beta A_n dt - \int_0^t \beta\alpha B_n dt + \int_0^t \beta\alpha^2 C_n dt - \int_0^t \mu I_n dt \quad (1.13)$$

But

$$\begin{aligned} A_0 &= S_0 I_0 \\ A_1 &= S_0 I_1 + S_1 I_0 \\ A_2 &= S_0 I_2 + S_1 I_1 + S_2 I_0 \end{aligned} \quad (1.14)$$

$$\begin{aligned} B_0 &= S_0 I_0{}^2 \\ B_1 &= 2S_0 I_0 I_1 + S_1 I_0{}^2 \\ B_2 &= S_2 I_0{}^2 + 2S_1 I_1 I_0 + S_0 I_1{}^2 + 2I_0 I_2 S_0 \end{aligned} \quad (1.15)$$

$$\begin{aligned} C_0 &= S_0 I_0{}^3 \\ C_1 &= 3S_0 I_0{}^2 I_1 + S_1 I_0{}^3 \end{aligned} \quad (1.16)$$

When n=0 in (1.12) and (1.13), we have

$$S_1 = -\beta xyt + \beta\alpha xy^2 t - \beta\alpha^2 xy^3 t - \mu xt$$
$$I_1 = \beta xyt - \beta\alpha xy^2 t + \beta\alpha^2 xy^3 t - \mu yt \tag{1.17}$$

When n=1 in (1.12) and (1.13), we have

$$\begin{aligned} S_2 = &-\frac{\beta^2 x^2 yt^2}{2} + \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} - \frac{7\beta^2\alpha^2 x^2 y^3 t^2}{2} + \frac{3\beta xy\mu t^2}{2} \\ &+ \frac{\beta^2 xy^2 t^2}{2} - \frac{\beta^2 \alpha xy^3 t^2}{2} + \frac{3\beta^2\alpha^2 xy^4 t^2}{2} \\ &- 2\beta\alpha\mu xy^2 t^2 - \frac{\beta^2\alpha xy^3 t^2}{2} - \beta^2\alpha^3 xy^5 t^2 + \frac{5\beta^2\alpha^3 x^2 y^4 t^2}{2} \\ &- \frac{3\beta^2\alpha^4 x^2 y^5 t^2}{2} + \frac{5\beta\alpha^2\mu xy^3 t^2}{2} + \frac{\beta^2\alpha^4 xy^6 t^2}{2} + \frac{\mu^2 xt^2}{2} \end{aligned} \tag{1.18}$$

$$\begin{aligned} I_2 = &\frac{\beta^2 x^2 yt^2}{2} - \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} + \frac{7\beta^2\alpha^2 x^2 y^3 t^2}{2} - \frac{3\beta xy\mu t^2}{2} \\ &- \frac{\beta^2 xy^2 t^2}{2} + \frac{\beta^2 \alpha xy^3 t^2}{2} - \frac{3\beta^2\alpha^2 xy^4 t^2}{2} + 2\beta\alpha\mu xy^2 t^2 \\ &+ \frac{\beta^2\alpha xy^3 t^2}{2} + \beta^2\alpha^3 xy^5 t^2 - \frac{5\beta^2\alpha^3 x^2 y^4 t^2}{2} + \frac{3\beta^2\alpha^4 x^2 y^5 t^2}{2} \\ &- \frac{5\beta\alpha^2\mu xy^3 t^2}{2} - \frac{\beta^2\alpha^4 xy^6 t^2}{2} - \frac{\mu^2 yt^2}{2} \end{aligned} \tag{1.19}$$

Since,

$$S(t) = S_0 + S_1 + S_2 + \dots\dots\dots\dots$$
$$I(t) = I_0 + I_1 + I_2 + \dots\dots\dots\dots \tag{1.20}$$

Putting (1.10), (1.17), (1.18) and (1.19) into (1.20), we have

$$\begin{aligned} S(t) = &x - \beta xyt + \beta\alpha xy^2 t - \beta\alpha^2 xy^3 t - \mu xt - \frac{\beta^2 x^2 yt^2}{2} \\ &+ \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} - \frac{7\beta^2\alpha^2 x^2 y^3 t^2}{2} + \frac{3\beta xy\mu t^2}{2} + \frac{\beta^2 xy^2 t^2}{2} \\ &- \frac{\beta^2 \alpha xy^3 t^2}{2} + \frac{3\beta^2\alpha^2 xy^4 t^2}{2} - 2\beta\alpha\mu xy^2 t^2 - \frac{\beta^2\alpha xy^3 t^2}{2} \\ &- \beta^2\alpha^3 xy^5 t^2 + \frac{5\beta^2\alpha^3 x^2 y^4 t^2}{2} - \frac{3\beta^2\alpha^4 x^2 y^5 t^2}{2} \\ &+ \frac{5\beta\alpha^2\mu xy^3 t^2}{2} + \frac{\beta^2\alpha^4 xy^6 t^2}{2} + \frac{\mu^2 xt^2}{2} \end{aligned} \tag{1.21}$$

$$I(t) = y + \beta xyt - \beta\alpha xy^2t + \beta\alpha^2xy^3t - \mu yt + \frac{\beta^2x^2yt^2}{2}$$
$$-\frac{3\beta^2\alpha x^2y^2t^2}{2} + \frac{7\beta^2\alpha^2x^2y^3t^2}{2} - \frac{3\beta xy\mu t^2}{2} - \frac{\beta^2xy^2t^2}{2}$$
$$+\frac{\beta^2\alpha xy^3t^2}{2} - \frac{3\beta^2\alpha^2xy^4t^2}{2} + 2\beta\alpha\mu xy^2t^2 + \frac{\beta^2\alpha xy^3t^2}{2} \tag{1.22}$$
$$+\beta^2\alpha^3xy^5t^2 - \frac{5\beta^2\alpha^3x^2y^4t^2}{2} + \frac{3\beta^2\alpha^4x^2y^5t^2}{2}$$
$$-\frac{5\beta\alpha^2\mu xy^3t^2}{2} - \frac{\beta^2\alpha^4xy^6t^2}{2} - \frac{\mu^2yt^2}{2}$$

Cases 2: When $b \neq 0$

$$\frac{dS}{dt} = b - \beta SI(1+\alpha I)^{-1} - \mu S \tag{2.1}$$

$$\frac{dI}{dt} = \beta SI(1+\alpha I)^{-1} - \mu I \tag{2.2}$$

Considering the first three terms of the expansion, we have

$$dS = \left(b - \beta SI + \beta\alpha SI^2 - \beta\alpha^2SI^3 - \mu S\right)dt$$
$$dI = \left(\beta SI - \beta\alpha SI^2 + \beta\alpha^2SI^3 - \mu I\right)dt \tag{2.3}$$

If, $S(0) = x, I(0) = y$

Then,

$$S(t) = x + \int_0^t (b - \beta\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I_n + \beta\alpha\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I^2{}_n)dt$$
$$-\int_0^t (\beta\alpha^2\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I_n{}^3 + \mu\sum_{n=0}^{\infty}S_n)dt \tag{2.4}$$

$$I(t) = y + \int_0^t (\beta\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I_n - \beta\alpha\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I^2{}_n)dt$$
$$+\int_0^t (\beta\alpha^2\sum_{n=0}^{\infty}S_n\sum_{n=0}^{\infty}I_n{}^3 - \mu\sum_{n=0}^{\infty}I_n)dt \tag{2.5}$$

Taking

$$A_n = \sum_{n=0}^{\infty} S_n I_n, B_n = \sum_{n=0}^{\infty} S_n I_n{}^2, C_n = \sum_{n=0}^{\infty} S_n I_n{}^3 \tag{2.6}$$

So,

$$\begin{aligned} S_0 &= x + \int_0^t b\,dt \Rightarrow x + bt \\ I_0 &= y \end{aligned} \tag{2.7}$$

It follows from (2.4) and (2.5) that,

$$\begin{aligned} S_{n+1} &= -\int_0^t \beta A_n dt + \int_0^t \beta\alpha B_n dt - \int_0^t \beta\alpha^2 C_n dt - \int_0^t \mu S_n dt \\ I_{n+1} &= \int_0^t \beta A_n dt - \int_0^t \beta\alpha B_n dt + \int_0^t \beta\alpha^2 C_n dt - \int_0^t \mu I_n dt \end{aligned} \tag{2.8}$$

When n=0 in (2.8), we have

$$S_1 = -\beta xyt - \frac{\beta byt^2}{2} + \beta\alpha xy^2 t + \frac{\beta\alpha by^2 t^2}{2} - \beta\alpha^2 xy^3 t - \frac{\beta\alpha^2 by^3 t^2}{2} - \mu xt - \frac{\mu bt^2}{2} \tag{2.9a}$$

$$I_1 = \beta xyt + \frac{\beta byt^2}{2} - \beta\alpha xy^2 t - \frac{\beta\alpha by^2 t^2}{2} + \beta\alpha^2 xy^3 t + \frac{\beta\alpha^2 by^3 t^2}{2} - \mu yt \tag{2.9b}$$

From (2.8) put n=1, we have

$$\begin{aligned} S_2 = &-\frac{\beta^2 b^2 yt^4}{8} - \frac{\beta^2 b^2 xt^3}{2} - \frac{\beta^2 x^2 yt^2}{2} + \frac{\beta^2 by^2 t^3}{6} + \frac{\beta^2 xy^2 t^2}{2} + \frac{3\beta^2 \alpha b^2 y^2 t^4}{8} \\ &+ \frac{3\beta\alpha bxy^2 t^3}{2} + \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} - \frac{\beta^2 \alpha by^3 t^3}{3} - \beta^2 \alpha xy^3 t^2 - \frac{3\beta^2 \alpha^2 b^2 y^3 t^4}{4} \\ &- 3\beta^2 \alpha^2 bxy^3 t^3 - 3\beta^2 \alpha^2 x^2 y^3 t^2 + \frac{\beta^2 \alpha^2 by^4 t^3}{2} + \frac{3\beta^2 \alpha^2 xy^4 t^2}{2} \\ &+ \frac{5\beta^2 \alpha^3 y^4 t^4}{8} + \frac{5\beta^2 \alpha^3 bxy^4 t^3}{2} - \frac{\beta^2 \alpha^3 by^5 t^3}{3} - \beta^2 \alpha^3 xy^5 t^2 - \frac{3\beta^2 \alpha^4 b^2 y^5 t^4}{8} \\ &- \frac{3\beta^2 \alpha^4 bxy^5 t^3}{2} - \frac{3\beta^2 \alpha^4 x^2 y^5 t^2}{2} + \frac{\beta^2 \alpha^4 by^6 t^3}{6} + \frac{\beta^2 \alpha^4 xy^6 t^2}{2} \\ &+ \frac{5\beta^2 \alpha^3 x^2 y^3 t^2}{2} + \frac{2\beta b\mu yt^3}{3} + \frac{3\beta\mu xyt^2}{2} - \beta\alpha\mu y^2 t^3 - 2\beta\alpha\mu xy^2 t^2 \\ &+ \frac{4\beta b\mu y^3 t^3}{3} + \frac{5\beta\alpha^2 \mu xy^3 t^2}{2} - \frac{\mu^2 bt^3}{6} + \frac{\mu^2 xt^2}{2} \end{aligned} \tag{2.10}$$

$$I_2 = \frac{\beta^2 b^2 yt^4}{8} + \frac{\beta^2 b^2 xt^3}{2} + \frac{\beta^2 x^2 yt^2}{2} - \frac{\beta^2 by^2 t^3}{6} - \frac{\beta^2 xy^2 t^2}{2} - \frac{3\beta^2 \alpha b^2 y^2 t^4}{8}$$
$$- \frac{3\beta\alpha bxy^2 t^3}{2} - \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} + \frac{\beta^2 \alpha by^3 t^3}{3} + \beta^2 \alpha xy^3 t^2$$
$$+ \frac{3\beta^2 \alpha^2 b^2 y^3 t^4}{4} + 3\beta^2 \alpha^2 bxy^3 t^3 + 3\beta^2 \alpha^2 x^2 y^3 t^2 - \frac{\beta^2 \alpha^2 by^4 t^3}{2}$$
$$- \frac{3\beta^2 \alpha^2 xy^4 t^2}{2} - \frac{5\beta^2 \alpha^3 y^4 t^4}{8} - \frac{5\beta^2 \alpha^3 bxy^4 t^3}{2} + \frac{\beta^2 \alpha^3 by^5 t^3}{3}$$
$$+ \beta^2 \alpha^3 xy^5 t^2 + \frac{3\beta^2 \alpha^4 b^2 y^5 t^4}{8} + \frac{3\beta^2 \alpha^4 bxy^5 t^3}{2} + \frac{3\beta^2 \alpha^4 x^2 y^5 t^2}{2}$$
$$- \frac{\beta^2 \alpha^4 by^6 t^3}{6} - \frac{\beta^2 \alpha^4 xy^6 t^2}{2} - \frac{5\beta^2 \alpha^3 x^2 y^3 t^2}{2} - \frac{2\beta b\mu yt^3}{3} - \frac{3\beta\mu xyt^2}{2}$$
$$+ \beta\alpha\mu y^2 t^3 + 2\beta\alpha\mu xy^2 t^2 - \frac{4\beta b\mu y^3 t^3}{3} - \frac{5\beta\alpha^2 \mu xy^3 t^2}{2} + \frac{\mu^2 xt^2}{2} \quad (2.11)$$

Then,

$$S(t) = x + bt - \beta xyt - \frac{\beta byt^2}{2} + \beta\alpha xy^2 t + \frac{\beta\alpha by^2 t^2}{2} - \beta\alpha^2 xy^3 t - \frac{\beta\alpha^2 by^3 t^2}{2}$$
$$- \mu xt - \frac{\mu bt^2}{2} - \frac{\beta^2 b^2 yt^4}{8} - \frac{\beta^2 b^2 xt^3}{2} - \frac{\beta^2 x^2 yt^2}{2} + \frac{\beta^2 by^2 t^3}{6}$$
$$+ \frac{\beta^2 xy^2 t^2}{2} + \frac{3\beta^2 \alpha b^2 y^2 t^4}{8} + \frac{3\beta\alpha bxy^2 t^3}{2} + \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} - \frac{\beta^2 \alpha by^3 t^3}{3}$$
$$- \beta^2 \alpha xy^3 t^2 - \frac{3\beta^2 \alpha^2 b^2 y^3 t^4}{4} - 3\beta^2 \alpha^2 bxy^3 t^3 - 3\beta^2 \alpha^2 x^2 y^3 t^2 + \frac{\beta^2 \alpha^2 by^4 t^3}{2}$$
$$+ \frac{3\beta^2 \alpha^2 xy^4 t^2}{2} + \frac{5\beta^2 \alpha^3 y^4 t^4}{8} + \frac{5\beta^2 \alpha^3 bxy^4 t^3}{2} - \frac{\beta^2 \alpha^3 by^5 t^3}{3} - \beta^2 \alpha^3 xy^5 t^2$$
$$- \frac{3\beta^2 \alpha^4 b^2 y^5 t^4}{8} - \frac{3\beta^2 \alpha^4 bxy^5 t^3}{2} - \frac{3\beta^2 \alpha^4 x^2 y^5 t^2}{2} + \frac{\beta^2 \alpha^4 by^6 t^3}{6}$$
$$+ \frac{\beta^2 \alpha^4 xy^6 t^2}{2} + \frac{5\beta^2 \alpha^3 x^2 y^3 t^2}{2} + \frac{2\beta b\mu yt^3}{3} + \frac{3\beta\mu xyt^2}{2} - \beta\alpha\mu y^2 t^3$$
$$- 2\beta\alpha\mu xy^2 t^2 + \frac{4\beta b\mu y^3 t^3}{3} + \frac{5\beta\alpha^2 \mu xy^3 t^2}{2} - \frac{\mu^2 bt^3}{6} + \frac{\mu^2 xt^2}{2} \quad (2.12)$$

$$I(t) = y + \beta xyt + \frac{\beta byt^2}{2} - \beta\alpha xy^2 t - \frac{\beta\alpha by^2t^2}{2} + \beta\alpha^2 xy^3 t + \frac{\beta\alpha^2 by^3 t^2}{2}$$
$$- \mu yt + \frac{\beta^2 b^2 yt^4}{8} + \frac{\beta^2 b^2 xt^3}{2} + \frac{\beta^2 x^2 yt^2}{2} - \frac{\beta^2 by^2 t^3}{6} - \frac{\beta^2 xy^2 t^2}{2}$$
$$- \frac{3\beta^2 \alpha b^2 y^2 t^4}{8} - \frac{3\beta\alpha bxy^2 t^3}{2} - \frac{3\beta^2 \alpha x^2 y^2 t^2}{2} + \frac{\beta^2 \alpha by^3 t^3}{3} + \beta^2 \alpha xy^3 t^2$$
$$+ \frac{3\beta^2 \alpha^2 b^2 y^3 t^4}{4} + 3\beta^2 \alpha^2 bxy^3 t^3 + 3\beta^2 \alpha^2 x^2 y^3 t^2 - \frac{\beta^2 \alpha^2 by^4 t^3}{2} - \frac{3\beta^2 \alpha^2 xy^4 t^2}{2}$$
$$- \frac{5\beta^2 \alpha^3 y^4 t^4}{8} - \frac{5\beta^2 \alpha^3 bxy^4 t^3}{2} + \frac{\beta^2 \alpha^3 by^5 t^3}{3} + \beta^2 \alpha^3 xy^5 t^2 + \frac{3\beta^2 \alpha^4 b^2 y^5 t^4}{8}$$
$$+ \frac{3\beta^2 \alpha^4 bxy^5 t^3}{2} + \frac{3\beta^2 \alpha^4 x^2 y^5 t^2}{2} - \frac{\beta^2 \alpha^4 by^6 t^3}{6} - \frac{\beta^2 \alpha^4 xy^6 t^2}{2} - \frac{5\beta^2 \alpha^3 x^2 y^3 t^2}{2}$$
$$- \frac{2\beta b\mu yt^3}{3} - \frac{3\beta\mu xyt^2}{2} + \beta\alpha\mu y^2 t^3 + 2\beta\alpha\mu xy^2 t^2 - \frac{4\beta b\mu y^3 t^3}{3}$$
$$- \frac{5\beta\alpha^2 \mu xy^3 t^2}{2} + \frac{\mu^2 xt^2}{2} \tag{2.13}$$

3 Discussion of Results

Figure 1 shows that populations of susceptible decreases gradually while the population of the infected increase with time, when $b = 0$.

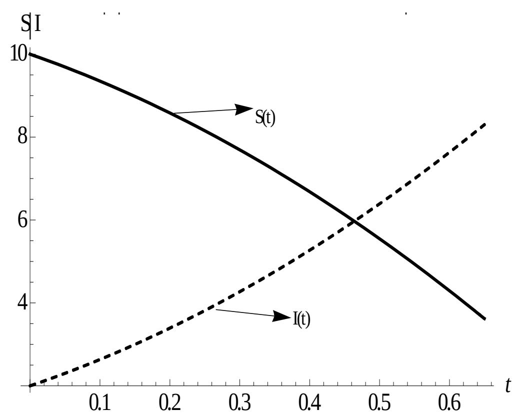

Figure 1 : $\beta = 0.3$, $x = 10$, $y = 2.0$, $\alpha = 0.02$, $\mu = 0.01$

Figure 2 shows that populations of susceptible ’shoot up’ initially, decreases gradually while the population of the infected start to increase with time, when $b \neq 0$.

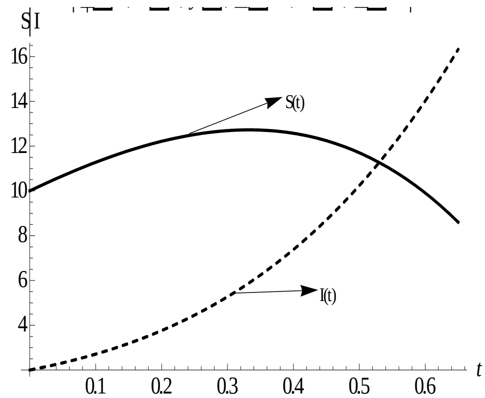

Figure 2: $\beta = 0.3$, $x = 10$, $y = 2$, $\alpha = 0.02$, $b = 2.0$ $\mu = 0.01$

5 Conclusion

Using the Adomian decomposition method of solution, this paper improves our understanding of the dynamic of Onchocerciasis disease in a wholly susceptible population with varying important parameters that affect the propagation of the disease.

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