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An EOQ model for linear deterioration rate of consumption with permissible delay in payments with special discounts

R. Babu Krishnaraj[1] and K. Ramasamy[2]

Abstract

An EOQ model of inventory for items with linear rate of deterioration is considered, taking the demand rate to be dependent on the stocjk level at any instant of time. The Payments are permitted to be delayed up to certain period of time. Shortages are not allowed. After the certain period, a special discount is permitted to reduce the stock in inventory because if the stock is holded, an interest is paid at a pre-decided rate depending on the items on hand. To avoid the interest payment, we introduce the discount to clear the items in stock.

Mathematics Subject Classification: 90B05

[1] Research Scholar, Kongunadu Arts & Science College, Coimbatore – 641 029, Tamil Nadu, India, e-mail:bkr200883@rediff.com
[2] Department Mathematics, Kongunadu Arts & Science College, Coimbatore – 641 029, Tamil Nadu, India

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1 Introduction

Generally, in deterministic inventory models, the demand is either uniform or time dependent and has nothing to do with the status of inventory at hand Silver and Peterson [22] have noted that the sales volume tends to be proportional to the level of inventory. Many researchers in the field of inventories have attempted to analyse the situation of dependence of demand rate on stock level.

Gupta and Vrat [11] have assumed the demand rate to be a function of order quantity. Mandal and Phaujdar [16] have investigated a model with the assumption of demand as linear function of the inventory level.

Baker and Urban [2] have taken this type of demand represented by a non-linear function of stock level. There are other authors who have taken into consideration also the deterioration of items with shortages allowed. Datta and Pal [5] have taken the effect of deterioration with demand as a linear function of stock level. They have assumed constant deterioration rate. Jacob and Jacob [13] have studied an order level inventory model with linearly time dependent rate of deterioration, demand taken as a function of on-hand inventory. They have used the criterion of profit maximization.

Gupta and Agarwal [9] have studied an inventory model where demand is assumed to be constant during the procurement of inventory and after the production stops, demand depends upon stock level. Models with stock dependent demand have also been studied by Padmanabhan and Vrat [17], Ray and Chaudhuri [18], Sarker et al. [19], Giri and chaudhuri [7] and Mandal and Maiti [15].

Goyal [8] was first to study an EOQ model under the condition of permissible delay in payments. Gupta and Jauhari [10] have included this criterion of delayed payments in their study of a model with power pattern of demand. Discounted cash flow approach for optimal inventory policy with trade credit was studied by Chung [4]. Shinn et al. [21] extended Goyal's [8] model and considered the quantity discount. Aggarwal and Jaggi [1] and Hwang and Shinn

[12] dealt with deterministic models with uniform rate of deterioration. Shah and shah [20] studied the probabilistic model of inventory with permissible delay in payments. They have treated time and deterioration of units as continuous variable and demand as a random variable. Jamal et al. [14] have generalized the model of Aggarwal and Jaggi [1] by allowing shortages. Chang and Dye [3] have developed a model under deterioration with backlogging rate assumed to be inversely proportional to the waiting time for the next replenishment. Recently, Dye [6] has considered a model with stock dependent demand and partial backlogging under the condition of delayed payment. We have considered a model here that, allowing credit facility with stock dependent demand. But the objective is, after the time period to settle the amout, a special discount is permitted to clear the remaining stock, to avoid the interest payment. Hence the average cost is reduced.

2 Notations and Assumptions

The model is developed under the following notations and assumptions:

1. Replenishment is instantaneous. The lot size is of Z units per replenishment.
2. Demand rate R(I) is linear function of I such that,

$$R(I) = \begin{cases} \alpha + I\beta, & I \geq 0 \\ \alpha, & otherwise. \end{cases}$$

$\alpha > 0,\ 0 < \beta > 1$ and I denotes the on hand inventory level at time t.

3. t_p is the time period allowed to the receiver for settling of the account. During time period, t_p interest is earned on the accumulate sales revenue at the rate of i_e per unit revenue per time unit. After period t_p interest is to be paid at the rate of $i_p (i_p \rangle i_e)$ per unit investment per time unit on the capital tied in the remaining stock.
4. p is the purchasing price per unit.

5. c is the selling price per unit.
6. h is the holding cost per unit time.
7. A is the ordering cost per order.
8. shortages are not allowed.
9. Deterioration starts when the items arrive in stock and it is taken to be slowly varying with time linearly, i.e., $\theta(t)=\gamma t,(0\langle\gamma\ll 1)$. Second and higher degree terms of β and γ are neglected.
10. s special discount price per unit.

3 Model Development

The differential equation governing the stock status for the period (0, T) can be written as:

$$\frac{dI}{dt}+\gamma tI=-(\alpha+\beta I),0\leq t\leq T \tag{1}$$

The solution of the differential equation (1) is,

$$\frac{dI}{dt}+(\gamma t+\beta)=-\alpha$$

$$I(t)=-\alpha e^{-(\gamma\frac{t^2}{2}+\beta t)}\int_t^T e^{(\gamma\frac{t^2}{2}+\beta t)}dt \tag{2}$$

We get after simplification,

$$I(t)=\alpha\left[T+\frac{\gamma T^2}{6}+\frac{\beta T^2}{2}-t+\frac{\gamma t^3}{6}+\frac{\beta t^2}{2}-\beta tT-\frac{\gamma Tt^2}{2}\right] \tag{3}$$

At t-0, I(0)=Z, therefore,

$$\text{Z} = \alpha\left[T+\frac{\gamma T^3}{6}+\frac{\beta T^2}{2}\right] \tag{4}$$

Hence, total amount of deteriorated units is,

$$D = Z - \int_0^T (\alpha + \beta I)dt$$

$$= Z - \alpha\int_0^T [1+\beta(T+\tfrac{\gamma T^3}{6}+\frac{\beta T^2}{2}-t-\frac{\gamma t^3}{3}+\frac{\beta t^2}{2}-\beta tT-\frac{\gamma Tt^2}{2})]dt$$

$$= Z - \alpha\left[t+\beta tT+\frac{\beta\gamma T^3 t}{6}+\frac{\beta^2 T^2 t}{2}-\frac{\beta t^2}{2}-\frac{\beta\gamma t^4}{12}+\frac{\beta^2 t^3}{6}-\frac{\beta^2 t^2 T}{2}-\frac{\gamma\beta T t^3}{6}\right]_0^T$$

$$= \alpha(T+\frac{\gamma T^3}{6}+\frac{\beta T^2}{2}) - \alpha(T+\beta T^2-\frac{\beta T^2}{2})$$

The simplification yields the relation,

$$D = \frac{\alpha\gamma T^3}{6} \tag{5}$$

The average inventory per time unit of the system during the period T is,

$$A(I) = \frac{1}{T}\int_0^T I(t)dt$$

$$= \frac{\alpha}{T}\int_0^T \left(T+\frac{\gamma T^3}{6}+\frac{\beta T^2}{2}-t+\frac{\gamma t^3}{3}+\frac{\beta t^2}{2}-\beta tT-\frac{\gamma Tt^2}{2}\right)dt$$

$$= \frac{\alpha}{T}\left(Tt+\frac{\gamma T^3 t}{6}+\frac{\beta T^2 t}{2}-\frac{t^2}{2}+\frac{\gamma t^4}{12}+\frac{\beta t^3}{6}-\frac{\beta t^2 T}{2}-\frac{\gamma T t^3}{6}\right)_0^T$$

$$= \frac{\alpha}{T}\left(T^2+\frac{\gamma T^4}{6}+\frac{\beta T^3}{2}-\frac{T^2}{2}+\frac{\gamma T^4}{12}+\frac{\beta T^3}{6}-\frac{\beta T^3}{2}-\frac{\gamma T^4}{6}\right)$$

$$= \frac{\alpha}{T}\left(\frac{T^2}{2}+\frac{\gamma T^4}{12}+\frac{\beta T^3}{6}\right)$$

$$= \alpha\left(\frac{T}{2}+\frac{\gamma T^3}{12}+\frac{\beta T^2}{6}\right) \tag{6}$$

The author discussed the various possibilities separately regarding the period of permissible delay in payments in the following three parameters,

(i) $t_p < T$, (ii) $t_p > T$, (iii) $t_p = T$.

But in the case (ii) and case (iii), the time period allowed to the receiver for settling of an account t_p is greater than or equal to the maximum permissible duration T. Hence in these cases, there is no need for remaining stock, Since, after period t_p, interest is to be paid at the rate of $i_p(i_p > i_e)$ per unit investment per time unit on the capital tied in the remaining stock. Hence the maintenance cost may not be increased in both of these cases. But in case (i) the is a chance of remaining stock because of the condition
t_p < T.

Our aim in this problem is , as per the case (i), if the remaining stock occurs, to avoid the payment of interest on the capital tied in the remaining stock, if we allow special discount for the remaining stock, there is no need to pay the interest for the remaining items, it will reduce the average cost of maintaining inventory.

We consider the possibility:

Let I_e denote the average accumulated sales per unit time of the system during T, interest on which is earned during t_p. Then I_e is given by,

$$I_e = \frac{1}{T}\int_0^{t_p}\int_0^{t}(\alpha + \beta I)dxdt$$

$$= \frac{1}{T}\int_0^{t_p}\int_0^{t}\left(1 + \beta(T + \frac{\gamma T^3}{6} + \frac{\beta T^2}{2} - x - \frac{\gamma x^3}{3} + \frac{\beta x^2}{2} - \beta xT - \frac{\gamma T x^2}{2})\right)dxdt$$

$$= \frac{\alpha}{T}\int_0^{t_p}\left(x + \beta Tx - \frac{x^2}{2}\right)_0^t dt$$

$$= \frac{\alpha}{T}\int_0^{t_p}\left(t + \beta Tt - \frac{t^2}{2}\right)dt$$

$$= \frac{\alpha}{T}\left(\frac{t^2}{2} + \frac{\beta T t^2}{2} - \frac{\beta t_p^3}{6}\right)_0^{t_p}$$

$$I_e = \frac{\alpha}{T}\left(\frac{t_p^2}{2} + \frac{\beta T t_p^2}{2} - \frac{\beta t_p^3}{6}\right) \quad (7)$$

The average remaining stock per unit time of the system during T is given by,

$$I_p = \frac{1}{T}\int_{t_p}^{T} I(t)dt$$

$$= \frac{\alpha}{T}\int_{t_p}^{T}\left(T + \frac{\gamma T^3}{6} + \frac{\beta T^2}{2} - t + \frac{\gamma t^3}{3} + \frac{\beta t^2}{2} - \beta t T - \frac{\gamma T t^2}{2}\right)dt$$

$$= \frac{\alpha}{T}\left(Tt + \frac{\gamma T^3 t}{6} + \frac{\beta T^2 t}{2} - \frac{t^2}{2} + \frac{\gamma t^4}{12} + \frac{\beta t^3}{6} - \frac{\beta t^2 T}{2} - \frac{\gamma T t^3}{6}\right)_{t_p}^{T}$$

$$= \frac{\alpha}{T}(\frac{T^2}{2} + \frac{\beta T^3}{6} + \frac{\gamma T^4}{12} - Tt_p - \frac{\gamma T^3 t_p}{6} - \frac{\beta T^2 t_p}{2} + \frac{t_p^2}{2} - \frac{\gamma t_p^4}{12} - \frac{\beta t_p^3}{6} + \frac{\beta t_p^2 T}{2} - \frac{\gamma T t_p^3}{6}) \quad (8)$$

The average cost per unit time is given by,

$$C = \frac{pD}{T} + hA(I) + \frac{A}{T} - sI_p - cI_e$$

$$= \frac{p\alpha\gamma T^2}{6} + h\alpha(\frac{T}{2} + \frac{\gamma T^3}{12} + \frac{\beta T^2}{6}) + \frac{A}{T}$$

$$- s\frac{\alpha}{T}(\frac{T^2}{2} + \frac{\beta T^3}{6} + \frac{\gamma T^4}{12} - Tt_p - \frac{\gamma T^3 t_p}{6} - \frac{\beta T^2 t_p}{2} + \frac{t_p^2}{2} - \frac{\gamma t_p^4}{12} - \frac{\beta t_p^3}{6} + \frac{\beta t_p^2 T}{2} - \frac{\gamma T t_p^3}{6})$$

$$- ci_e\frac{\alpha}{T}(\frac{t_p^2}{2} + \frac{\beta T t_p^2}{2} - \frac{\beta t_p^3}{6}) \quad (9)$$

It is verified that for optimality $\frac{dC}{dt} = 0$ and $\frac{d^2C}{dt^2} > 0$.

$$\frac{dC}{dT} = \frac{p\gamma\alpha T}{3} + h\alpha\left(\frac{1}{2} + \frac{\gamma T^2}{4} + \frac{\beta T}{3}\right) - \frac{A}{T^2}$$

$$+ \alpha s(\frac{1}{2} + \frac{\gamma T^2}{4} + \frac{\beta T}{3} - \frac{\gamma T t_p}{3} - \frac{\beta t_p}{2} - \frac{t_p{}^2}{2T^2} + \frac{\gamma t_p^4}{12T^2} + \frac{\beta t_p^3}{6T^2})$$

$$- \frac{\alpha c i_e}{T^2}(\frac{t_p^2}{2} - \frac{\beta t_p^3}{6}) = 0 \quad (10)$$

4 Conclusion

Equation (10) gives the solution of the optimal value of T, when substituted back in equation (9), yield the minimum total average cost of the system. Hence it is observed and verified that if we allow special discounts instead of paying interest for the remaining stock, the total average cost of inventory may be reduced.

References

[1] S.P. Aggarwal and C.K. Jaggi, Ordering policies of deteriorating items under permissible delay in payments, *Journal of the Operational Research Society*, **46**, (1995), 658-662.

[2] R.C. Baker and T.L. Urban, A deterministic inventory system with an inventory level dependent demand rate, *Journal of the Operational Research Society*, **39**(9), (1998), 823-831.

[3] H.J. Chang and C.Y. Dye, An EOQ model for deteriorating items with time varying demand and partial backlogging, *J. Opl. Res. Soc.*, **50**, (1999), 1176-1182.

[4] K.H. Chung, Inventory control and trade credit revisited, Journal of the *Operational Research Society*, **40**, (1989), 495-498.

[5] T.K. Datta, and A.K. Pal, Deterministic inventory system for deteriorating items with inventory level department demand and shortages, *Opsearch*, **27**, (1990), 213-224.

[6] Chung-Yuan Dye, A deteriorating inventory model with stock-dependent demand and partial backlogging under conditions of permissible delay in payments, *Opsearch*, **39**(3- 4), (2002), 189-199.

[7] B.C. Giri, and K.S. Chaudhuri, Deterministic models of perishable inventory with stock-dependent demand rate and non-linear holding cost, *European Journal of Operational Research*, **105**, (1998), 467-477.

[8] S.K. Goyal, Economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society*, **36**, (1985), 335-338.

[9] P.N. Gupta and V.K. Agarwal, An inventory model with stock dependent demand rate, *Ganita Sandesh*, **9**(1), (1995), 21-26.

[10] P.N. Gupta and Rajendra Jauhari, An EOQ model for deteriorating items with power emand pattern subject to permissible delay in payments, *Ganita Sandesh*, **9**(2), (1995), 65-71.

[11] R. Gupta and P. Vrat, Inventory model for stock-dependent consumption rate, *Opsearch*, **23**, (1986), 19-24.

[12] H. Hwang and S.W. Shinn, Retailers pricing and lot sizing policy for exponentially deteriorating product under the condition of permissible delay in payments, *Computers and Operations Research*, **24**, (1997), 539-547.

[13] Jacob Lissy and Jacob Mendus, Ordering policy for items with inventory level dependent demand rate and linear rate of deterioration, *Pure and Applied Mathematika Science*, **XLVII**(1-2), (1988), 826-833.

[14] A.M. Jamal, B.R. Sarkar and S. Wang, An ordering for deteriorating items with allowable shortage and permissible delay in payment, *Journal of the Operational Research Society*, **48**, (1997), 826-833.

[15] M. Mandal and M. Maiti, Inventory of damageable items with variable replenishment rate, Stock-dependent and some units in hand, Applied Mathematical Modelling, 23, (1999), 799-807.

[16] B.N. Mandal and S. Phaujdar, An inventory model for deteriorating items with uniform rate of production and stock dependent demand, *Journal of the Operational Research Society*, **40**, (1989), 483-488.

[17] G. Padmanabhan and P. Vrat, EOQ models for permissible items under stock dependent selling rate, *European Journal of Operational Research*, **86**, (1995), 281-292.

[18] B.R. Sarkar, S. Mukherjee and C.V. Balan, An order-level lot size inventory model with inventory level dependent demand and deterioration, *International Journal of Production Economics*, **48**, (1997), 227-236.

[19] J. Ray and K.S. Chaudhuri, An EOQ model with stock-dependent demand, shortage, inflation and time discounting, *Internatonal Journal of Production Economics*, **53**, (1997), 171-180.

[20] N.H. Shah and Y.K. Shah, A discrete-in-time probability inventory model for deteriorating items under conditions of permissible delay in payments, *International Journal of Systems Science*, **29**, (1998), 121-126.

[21] S.W. Shinn, H.P. Hwang and S. Sung, Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost, *Europan Journal of Operational Research*, **91**, (1996), 528-542.

[22] E.Q. Silver and R. Peterson, *Decisio systems for inventory management and production planning*, 2nd ed., John Wiley, New York, 1985.