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Some Characterization Results On Character Graphs of Groups

T. Gnanaseelan[1] and C. Selvaraj[2]

Abstract

The authors in their work in [4] introduced a graph $\Gamma(G,H)$, where G is a finite group and H is a subgroup of G such that the set of irreducible complex characters of G form the vertex set and two vertices χ and ψ are joined by an edge if their restriction to H, namely χ_H and ψ_H have at least one irreducible character of H as a common constituent. In [8] M. Javarsineh and Ali Iranmanesh have studied the nature of this graph for the groups D_{2n}, U_{6n} and T_{4n}. In this paper we study characterization properties of the graph $\Gamma(G,H)$ and obtain various results relevant to this graph. This paper deals with several ideas and techniques used in representation theory and graph theory.

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[1] Department of Mathematics, Presidency College, Chennai- 600005, India,
e-mail: gnanaseelan58@gmail.com
[2] Department of Mathematics, Presidency College, Chennai- 600005, India,
e-mail: pamc9439@yahoo.co.in

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1 Introduction

For a finite group G, let Irr(G) denote the set of all irreducible complex characters of G. If H is a subgroup of G, the Bratteli diagram $B(G,H)$ defined by O. Bratteli is a bipartite graph having the vertex set $Irr(G) \cup Irr(H)$ and a vertex $\chi \in Irr(G)$ is joined to a vertex $\theta \in Irr(H)$ by m edges if and only if $(\chi_{\mathrm{H}}, \theta) = m$ where χ_{H} is the restriction of χ to H, has been studied by N. Chigira and N. Iori.

Many authors have done research on character degrees of finite groups, like the arithmetic structure and the cardinality with the group theoretic structure of G, the character degree graphs associated with a given group, graphs related to conjugacy classes of a finite group and order of elements in the conjugacy classes.

The authors in [4] defined a graph $\Gamma(G,H)$, known as the relative character graph of G with respect to H. This is a simple graph with vertex set $Irr(G)$ and two vertices χ and ψ are joined by an edge if their restriction to H, namely χ_{H} and ψ_{H} have at least one irreducible character of H as a common constituent.

In [8] M. Javarsineh and Ali Iranmanesh have studied the nature of the graphs $\Gamma(D_{2n},H)$, $\Gamma(U_{6n},H)$ and $\Gamma(T_{4n},H)$ where H is a normal subgroup of the respective groups. In this paper we shall give some characterizations of the graph $\Gamma(G,H)$, and discuss the nature of the character graph when G is a Frobenius group. In fact we shall construct the graphs $\Gamma(G,H)$, and $\Gamma(G,N)$, when G is a Frobenius group with Frobenius complements H and Frobenius kernel N and prove their complementary nature.

2 Preliminary Notes

In this section we give some definitions and lemmas.

Definition 2.1 [4] If G is a finite group and H is a subgroup of G, then the relative character graph denoted by $\Gamma(G,H)$, has the vertex set V = $Irr(G)$, the set of irreducible complex characters of G and two vertices χ and ψ are joined by an edge if their restriction to H, namely χ_H and ψ_H have at least one irreducible character of H as a common constituent.

Lemma 2.2 [4] If N is a normal subgroup of G, then $\Gamma(G,N)$ has k distinct connected components where k is the number of distinct orbits of $Irr(N)$ under the action of G by conjugation.

Lemma 2.3 [4] If H is a subgroup G, then $\Gamma(G,H)$, is connected if and only if $core_G(H)=(1)$, where $core_G(H)$ is the largest normal subgroup of G contained in H.

Definition 2.4 [4] For any $\theta \in Irr(H)$, the collection of all irreducible characters of G which occur as constituents of θ^G is called the induced cover of θ and is denoted by $I(\theta,H)$, where θ^G is the induced character of θ.

Lemma 2.5 (Frobenius reciprocity formula) Let H be a subgroup of a finite group G and suppose that χ is an irreducible character of G and θ is an irreducible character of H. Then $(\theta,\chi_H)_H = (\chi,\theta^G)_G$.

Definition 2.6 Let N be a normal subgroup of G and let $\theta \in Irr(N)$. Then, $I_G(\theta)=\{g \in G / \theta^G = \theta\}$ is called the *inertia group* of θ in G.

Definition 2.7 A finite group G is called a Frobenius group if there is a proper non-trivial subgroup H such that $H \cap H^x = \{1\}$ for all $x \notin H$. The subgroup H is called a Frobenius complement. In this case $N = \left(G - \bigcup_{x\in G} H^x\right) \cup \{1\}$ is a normal subgroup of G, called the *Frobenius kernel*.

S_3, A_4 and the dihedral group D_{2n}, n odd are some examples of Frobenius groups.

If G is a Frobenius group with Kernel N and complement H then we have the following properties.

(i) G = NH is the semidirect product of N by H with $N \cap H = \{1\}$.

(ii) o(H) divides o(N) – 1.

(iii) If θ is a non-principal irreducible character of N, then $\theta^G \in Irr(G)$.

(iv) If $\chi \in Irr(G)$ with Kerχ does not contain N, then $\chi = \theta^G$ for some $\theta \in Irr(N)$.

Definition 2.8 Let $\Gamma_1 = (V_1, E_1)$ and $\Gamma_2 = (V_2, E_2)$ be two graphs. The normal product of Γ_1 and Γ_2, denoted by $\Gamma_1 \circ \Gamma_2$ is defined as follows. The vertex set of $\Gamma_1 \circ \Gamma_2$ is $V_1 \times V_2$. Two vertices $w_1 = (u_1, v_1)$ and $w_1 = (u_2, v_2)$ are adjacent if and only if any one of the following conditions hold.

(i) $u_1 = u_2$ and $v_1 v_2 \in E_2$

(ii) $u_1 u_2 \in E_1$ and $v_1 = v_2$

(iii) $u_1 u_2 \in E_1$ and $v_1 v_2 \in E_2$.

Definition 2.10 If Γ is a finite graph, then the complement $\overline{\Gamma}$ of Γ has the vertex set as that of Γ and two vertices u and v are adjacent in $\overline{\Gamma}$ if and only if they are not adjacent in Γ.

3 Main Results

In what follows we shall prove some theorems reflecting the properties of the graph Γ(G,H). Also we shall discuss some properties of the graphs Γ(G,H) and Γ(G,N) when G is a Frobenius group with Frobenius complement H and Frobenius kernel N.

Theorem 3.1 Two vertices χ and χ′ in Γ(G,H) are adjacent if and only if $(\chi,\chi')_{\mathrm{H}} > 0$.

Proof: Let $\{\theta_j\}$ be the complete set of irreducible characters of H. let $\chi_{\mathrm{H}} = \sum m_i\theta_i$ and $\chi'_{\mathrm{H}} = \sum n_j\theta_j$, where m_i and n_j are positive integers. Then

$(\chi,\chi')_{\mathrm{H}} = (\chi_H, \chi'_H)_{\mathrm{H}} = (\sum m_i\theta_i, \sum n_j\theta_j)_{\mathrm{H}} = \sum m_i n_j (\theta_i,\theta_j)_H = \sum m_i n_j \delta_{ij}$. Now χ and χ' are adjacent in Γ(G,H) if and only if χ_{H} and χ'_{H} have at least one θ_i in common whose multiplicity in χ_{H} is m_i and in χ'_{H} is n_j. That is, if and only if $(\chi,\chi')_{\mathrm{H}} > 0$.

Theorem 3.2 Let H be a subgroup of G and let $x \in G$. If $H^x = xHx^{-1}$ is the conjugate of H, then $\Gamma(G,H) = \Gamma(G,H^x)$.

Proof: Let χ_i and $\chi_j \in Irr(G)$.

Then

$$
\begin{aligned}
(\chi_i,\chi_j)_H &= \frac{1}{o(H)}\sum_{h\in H}\chi_i(h)\chi_j(h^{-1}) = \frac{1}{o(H)}\sum_{h\in H}\chi_i(xhx^{-1})\chi_j(xh^{-1}x^{-1}) \\
&= \frac{1}{o(H)}\sum_{h\in H}\chi_i(xhx^{-1})\chi_j\left((xhx^{-1})^{-1}\right) = \frac{1}{o(H)}\sum_{y\in H^x}\chi_i(y)\chi_j(y^{-1}) \\
&= (\chi_i,\chi_j)_{H^{xx}}
\end{aligned}
$$

where $y = xhx^{-1},\ h \in H$. Therefore χ_i and χ_j are adjacent in Γ(G,H) if and only if χ_i and χ_j are adjacent in $\Gamma(G,H^x)$. Hence $\Gamma(G,H) = \Gamma(G,H^x)$.

Theorem 3.3 Two vertices χ_i and χ_j are adjacent in Γ(G,H) if and only if χ_i and χ_j belong to some induced cover I(θ,H).

Proof: Let χ_i and χ_j be adjacent in Γ(G,H). Then, both $\chi_{i\mathrm{H}}$ and $\chi_{j\mathrm{H}}$ contain some $\theta \in Irr(H)$ as a common constituent. Therefore, by Frobenius reciprocity formula, both χ_i and χ_j are irreducible constituents of θ^G. That is, χ_i and χ_j belong to the induced cover I(θ,H).

Conversely, let χ_i and χ_j belong to some induced cover I(θ,H). Then, χ_i and χ_j are irreducible constituents of θ^G. That is, θ is a common irreducible constituent of $\chi_{i\mathrm{H}}$ and $\chi_{j\mathrm{H}}$. That is, χ_i and χ_j are adjacent in Γ(G,H).

Theorem 3.4 Let H = < x > be a cyclic subgroup of order h of G and let $\rho_1, \rho_2, ..., \rho_v$ be the complete set of irreducible representations of G. Then Γ(G,H) is complete if and only if each $\rho_i(x)$ has 1 as an eigen value.

Proof: If Γ(G,H) is complete then χ_1 is adjacent to all χ_i, $i > 1$. But $\chi_{1\mathrm{H}} = \theta_1$. Therefore, if χ_1 is adjacent to χ_i and χ_j, $i, j > 1$ then θ_1 is common to both $\chi_{1\mathrm{H}}$ and $\chi_{i\mathrm{H}}$ and common to both $\chi_{1\mathrm{H}}$ and $\chi_{j\mathrm{H}}$. This implies θ_1 is common to both $\chi_{i\mathrm{H}}$ and $\chi_{j\mathrm{H}}$. Therefore the adjacency of χ_1 to all $\chi_i, i > 1$ is also a sufficient condition for the graph Γ(G,H) to be complete. Therefore Γ(G,H) is complete if an only if $(\chi_i, \chi_1)_{\mathrm{H}} = \frac{1}{h}\sum_{s \in H} \chi_i(s) > 0$ for each i>1.

Let ω be a primitive h^{th} root of unity, and let $\omega^{j_1}, \omega^{j_2}, \ldots, \omega^{j_{n_i}}$ (repetition allowed) be the eigenvalues of $\rho_i(x)$, where $\deg \rho_i = n_i$. Then for each element x^k in H, $\chi_i(x^k) = \omega^{k j_1} + \omega^{k j_2} + \cdots + \omega^{k j_{n_i}}$. Therefore,

$$\sum_{s \in H} \chi_i(s) = \sum_{k=0}^{h-1} \chi_i(x^k) = \sum_{k=0}^{h-1} (\omega^{k j_1}) + \sum_{k=0}^{h-1} (\omega^{k j_2}) + \cdots + \sum_{k=0}^{h-1} (\omega^{k j_{n_i}}) = mh$$

for some $m > 0$ if and only if 1 is an eigenvalue of $\rho_i(x)$ for each $i > 1$.

For example, if we take $G = S_4$, the symmetric group on four letters and H = { 1, (12)(34)}, then Γ(G,H) is a complete graph with 5 vertices.

Theorem 3.5 If H_1 is a subgroup of of G_1 and H_2 is a subgroup of G_1 then, $\Gamma(G_1 \times G_2, H_1 \times H_2) = \Gamma(G_1, H_1) \circ \Gamma(G_2, H_2)$

Proof: Let $\sigma_1, \sigma_2, ..., \sigma_h$ be the vertices of $\Gamma(G_1, H_1)$ and $\psi_1, \psi_2, ..., \psi_k$ be the vertices of $\Gamma(G_2, H_2)$. Then the collection $\{\sigma_i \psi_j\}$ will be the vertex set for $\Gamma(G_1 \times G_2, H_1 \times H_2)$. We map the vertex $\sigma_i \psi_j$ onto the vertex (σ_i, ψ_j) of $\Gamma(G_1, H_1) \circ \Gamma(G_2, H_2)$. We shall now prove that $\sigma_i \psi_j$ and $\sigma_p \psi_q$ are adjacent if and only if (σ_i, ψ_j) and (σ_p, ψ_q) are adjacent.

Let $\sigma_i \psi_j$ and $\sigma_p \psi_q$ are adjacent. This means that $(\sigma_i \psi_j, \sigma_p \psi_q)_{H_1 \times H_2} \neq 0$ (using Theorem 3.1). But $(\sigma_i \psi_j, \sigma_p \psi_q)_{H_1 \times H_2} = (\sigma_i \sigma_p)_{H_1} (\psi_j \psi_q)_{H_2}$. Therefore we have $(\sigma_i, \sigma_p)_{H_1} \neq 0$ and $(\psi_j, \psi_q)_{H_2} \neq 0$. This shows that σ_i and σ_p are adjacent in $\Gamma(G_1, H_1)$ and ψ_j and ψ_q are adjacent in $\Gamma(G_2, H_2)$. Therefore the vertices (σ_i, ψ_j) and (σ_p, ψ_q) are adjacent in $\Gamma(G_1, H_1) \circ \Gamma(G_2, H_2)$.

Conversely, assume that the vertices (σ_i, ψ_j) and (σ_p, ψ_q) are adjacent in $\Gamma(G_1, H_1) \circ \Gamma(G_2, H_2)$. If $\sigma_i = \sigma_p$ and ψ_j and ψ_q are adjacent in $\Gamma(G_2, H_2)$ then, $(\sigma_i, \sigma_p)_{H_1} \neq 0$ and $(\psi_j, \psi_q)_{H_2} \neq 0$. Therefore $(\sigma_i \sigma_p)_{H_1} (\psi_j \psi_q)_{H_2} \neq 0$. That is, $(\sigma_i \psi_j, \sigma_p \psi_q)_{H_1 \times H_2} \neq 0$. That is, $\sigma_i \psi_j$ and $\sigma_p \psi_q$ are adjacent in $\Gamma(G_1 \times G_2, H_1 \times H_2)$. If σ_i and σ_p are adjacent in $\Gamma(G_1, H_1)$ and ψ_j and ψ_q, then by a similar argument we can prove that $\sigma_i \psi_j$ and $\sigma_p \psi_q$ are adjacent in $\Gamma(G_1 \times G_2, H_1 \times H_2)$. Finally, if σ_i and σ_p are adjacent in $\Gamma(G_1, H_1)$ and ψ_j and ψ_q are adjacent in $\Gamma(G_2, H_2)$, then $(\sigma_i, \sigma_p)_{H_1} \neq 0$ and $(\psi_j, \psi_q)_{H_2} \neq 0$. As in the previous cases this will also lead to the adjacency of $\sigma_i\psi_j$ and $\sigma_p\psi_q$ in $\Gamma(G_1 \times G_2, H_1 \times H_2)$. Hence the theorem is true.

Lemma 3.6 If G is a Frobenius group with Kernel N and complement H then

[1] Γ(G,H) is connected.

[2] Γ(G,N) is disconnected.

Proof: (i) Let $x \notin H$. Then $H \cap H^x = (1)$ and hence $core_G(H) = \bigcap_{x\in G} H^x = (1)$.
Therefore Γ(G,H) is connected.

(ii) Since N is normal Γ(G,N) is disconnected.

Lemma 3.7 Let $A = \{\chi \in Irr(G) / \ K\text{er}\,\chi \supset N\}$ and $B = \{\chi \in Irr(G) / K\text{er}\,\chi \not\supset N\}$. Then $|A| = |Irr(H)|$ and $|B| = t/h$, where $t+1 = |Irr(N)|$, h = o(H) and t/h is the number of non-principal orbits of $Irr(N)$ under the action of H.

Theorem 3.8 Let $n = |Irr(H)|$ and $m = t/h$. Then Γ(G,N) consists of the complete subgraph K_n and m isolated vertices.

Proof: If N is a normal subgroup of G, then Γ(G,N) has k distinct connected components where k is the number of distinct orbits of $Irr(N)$ under the action of H by conjugation. By Clifford's theorem, the component of Γ(G,N) corresponding to the principal orbit 1_N consists precisely of the irreducible characters of G whose kernel contains N. That is, this component is a complete subgraph K_n of Γ(G,N), whose vertices are exactly the elements of A.

By lemma 3.7, $Irr(N)$ has exactly m non-principal orbits. For each such orbit any representative will induce an irreducible character of G as per property (iii), each character belongs to B. Since there are m non-principal orbits, there are m isolated vertices in Γ(G,N). From lemma 3.7, |B|= m and hence Γ(G,N) has exactly m isolated vertices.

Theorem 3.9 The connected graph Γ(G,H) consists of a complete subgraph K_m together with 'n' vertices each of which is adjacent to every one of the vertices in K_m. The number of edges in Γ(G,H) is $nm+\binom{m}{2}$.

Proof: Each χ in *A* contains *N* in its kernel. Therefore such characters arise as extensions of irreducible characters of the group G/N, which is isomorphic to *H*. This gives rise to a one-to-one correspondence between the irreducible characters of *H* and the elements of *A*. Hence any two distinct elements of *A* are not adjacent, since their restriction to *H* is irreducible.

Next, let χ' be any element of *B*. Then from (iv), it follows that $\chi'=\theta^G$ for some irreducible character θ of *N*, $\neq 1_N$. Hence from the properties of characters induced from normal subgroups, we have $\chi'(x)=0$ for all $x \notin N$. In particular, $\chi'(x)=0$ for all $x\in H$, $x\neq 1$, since $N\cap H$ = (1). Hence χ' is an integral multiple of the regular character RegH of *H*. Therefore each irreducible character of *H* occurs in χ'_H and hence the irreducible characters in *B* form a complete subgraph K_m. Also each element of *A* is *H*-irreducible and hence any irreducible character in *A* is adjacent to χ'. Therefore any element of *A* is adjacent to every element of *B*.

Since any element of *A* is adjacent to every element of *B* and no two elements of *A* are not adjacent we get nm edges. Also any two elements of *B* are adjacent. Therefore there are $\binom{m}{2}$ edges. Hence there are $nm+\binom{m}{2}$ edges in Γ(G,H).

Theorem 3.10 Let G = NH be a Frobenius group. Then, $\Gamma(G,N)=\overline{\Gamma}(G,H)$. Conversely, if G is a semi direct product NH where N is a normal subgroup of G and H is not normal such that $\Gamma(G,N)=\overline{\Gamma}(G,H)$, then G is a Frobenius group with kernel N and complement H.

Proof: (We shall follow the notations used in Theorem 4.5). In Γ(G,H) any vertex in *A* is adjacent to all vertices in *B* and no two vertices in A are adjacent. Also, $B \cup \{1_G\}$ = I(1,H). In fact, each χ' in *B* is such that χ'_H is an integral multiple of the regular character Reg H and hence contains 1_H and therefore $\chi' \in$ I(1, H). Also, any $\chi' \neq 1_G$ in I(1, H) must belong to *B*, since otherwise $\chi'_{\mathrm{H}} \in Irr(\mathrm{H})$ and non-principal by the one-to-one correspondence, which cannot happen since χ'_H contains 1_H by our choice of χ' in I(1, H). From property (iv) any χ in *B* is induced from some $\theta \in Irr(N)$ and hence no two vertices in *B* are adjacent in Γ(G,N). Also for any χ in *A* the only irreducible constituent of χ_N is 1_N and 1_N does not occur as a constituent of χ'_{N}, χ' in *B*. Therefore no χ in *A* is adjacent to any χ' in *B* in Γ(G,N). Now *A* is precisely the induced cover $\mathrm{I}(1, N)$. Therefore any two irreducible in *A* are adjacent in Γ(G,N). Thus Γ(G,N) is precisely the complement of the graph Γ(G,H).

Conversely, since $B \cup \{1_G\}$ = I(1,H), any two χ_j and χ_k, $j \neq k$ in *B* are adjacent in Γ(G,H) and hence they are not adjacent in Γ(G,N), as Γ(G,H) and Γ(G,N) are complements of each other. Therefore, for $\chi \in B$, $\chi_{\mathrm{N}} = e\sum \theta_i$, for some integer $e \geq 1$, $\theta_i \in Irr(N)$ and $\theta_{\mathrm{i}} \neq 1$ are distinct conjugates. Hence, for any *i*, $\theta_i^G = e\chi$ because, if $\eta \in B$ occurs in the right hand side, then η and χ must be adjacent in Γ(G,N), which is not true, and also no $\eta \in A$ can occur in the right hand side. Therefore, $\theta_i^G = e\chi$ vanishes on G – N and hence on H – {1}, which shows that χ_{H} is an integral multiple of RegH. But, $Ind_H^G 1 = 1 + \alpha_1\chi_1 + \alpha_2\chi_2 + \cdots + \alpha_m\chi_m$, $\chi_i \in B$. Therefore, deg($\sum \alpha_j\chi_j$) = [G: H] – 1 = o(N) – 1. Since each χ_j is an integral multiple of RegH, o(H) divides deg χ_j for each *j*. Hence

o(H) divides o(N) -1. (1)

Now assume that G is not a Frobenius group. Then there exists $x \in H, x \neq 1$ and $u \in N$, $u \neq 1$ such that $xux^{-1} = u$. Therefore by Brauer's theorem, there exists $\theta \in Irr(N)$, $\theta \neq 1$ such that $\theta^x = \theta$. Let degθ= k and let T be the inertia group of θ. Then T is a subgroup of G properly containing N. By Mackey's irreducibility criterion, θ^G is not irreducible and hence $\theta^G = e\chi$, $\chi \in \mathrm{B}$, $e > 1$ and $\chi_N = e\sum_{i=1}^{t}\theta_i$, $t = [G:T]$ and hence degχ = ekt. Then, $\theta^T = e\psi$; $\psi^G = \chi$ and $\psi_N = e\theta$, $\psi \in Irr(T)$. Since [T:N] k = deg θ^{T} = e degψ= e^2k, we have [T:N] = e^2 and hence o(H) = [G: N] = [G: T] [T: N] = te^2. Since χ_{H} = a RegH, a is an integer, we have e t k = a t e^2. This implies e divides k. Since deg θ = k divides o(N), e divides o(N), so that o(N) = b e, b is an integer. From (1) o(H) divides o(N) – 1. That is, te^2 divides o(N) – 1 and hence o(N) – 1 = d te^2, d is an integer. That is, be – 1 = d te^2. This shows that e divides 1, which is not possible, since e > 1. Hence G must be a Frobenius group with kernel N and complement H.

The graphs Γ(G,H) and Γ(G,N) are given in Figure 1 and Figure 2 respectively where G = D_{18}, the dihedral group with 18 elements.

$D_{18} = \{1, x, x^2, ..., x^8, y, yx, ..., yx^8\}$; $x^9 = 1$, $y^2 = 1$, $yxy = x^{-1}$ implies $yx^k y = x^{-k}$, $(yx^k)^2 = 1$. D_{18} has 6 irreducible characters of which 2 are linear (degree 1) and 4 are of degree 2. The linear characters are given Table 1.

4 Labels of figures and tables

Table 1: Linear characters of D_{18}

	x^k	yx^k
χ_1	1	1
χ_2	1	-1

The irreducible characters of degree 2 are given by

$$\chi_h(x^k) = 2\cos\frac{2\pi hk}{9}, \quad \chi_h(yx^k) = 0 \ .$$

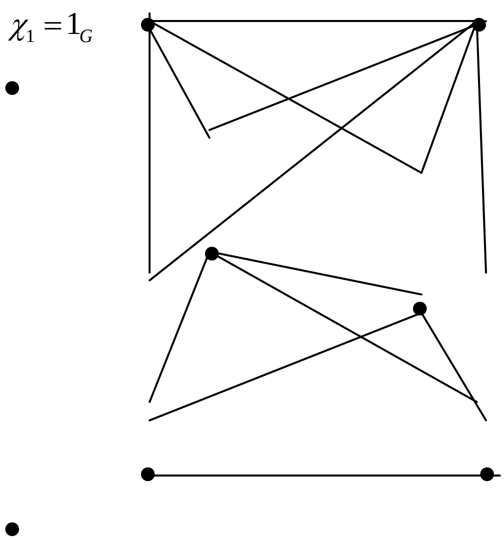

Figure 1

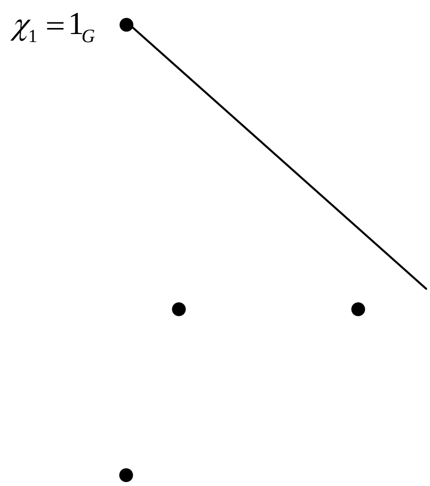

Figure 2

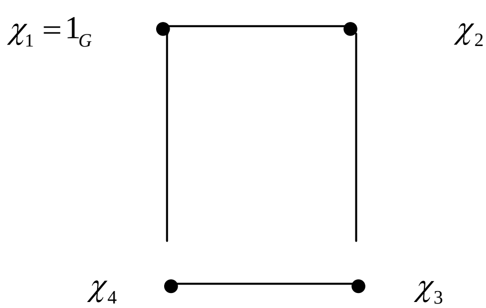

Figure 3

5 Conclusion

We conclude this paper with the following basic question. Given a graph Γ, can we find a group G such that Γ is isomorphic to Γ(G,H) for some subgroup H of G ?

Consider the graph with four vertices given in Figure 3. In this graph χ_1 and χ_2 are adjacent. Therefore, there exists some $\theta \in Irr(H)$ such that θ is an irreducible constituent of both χ_{1H} and χ_{2H}. But $\chi_{1H} = 1_H$. Therefore 1_H is a constituent of both and χ_{1H} and χ_{2H}. By a similar argument we have that 1_H is a constituent of both χ_{1H} and χ_{4H}. Therefore χ_2 and χ_4 must be adjacent in Γ(G,H). Hence we cannot find a group G and a subgroup H of G such that Γ(G,H) is isomorphic to the above graph.

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