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New Oscillation Criteria for Certain Second-Order Nonlinear Dynamic Equations on Time Scales

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Abstract

In this paper, we are concerned with the oscillation of a class of second-order nonlinear dynamic equations on time scales and obtain several sufficient conditions for the oscillation of the equations by developing a generalized Riccati transformation technique. Our results improve and extend some recent results in the literature.

Mathematics Subject Classification: 39A10

Keywords: Oscillation, Dynamic equation, Time scale

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1 Introduction

The study of dynamic equations on time scales has recently received a lot of attention. The theory of dynamic equations not only can unify the theories of differential equations and difference equations, but it is also able to extend these classical cases to cases "in between," e.g., to the so-called q-difference equations. The general idea is to prove a result for a dynamic equation where the domain of the unknown function is a time scale $\mathbb{T}$. In this way results not only related to the set of real numbers $\mathbb{R}$ or the set of integers $\mathbb{Z}$ but those pertaining to more general time scales are obtained. There are many applications of dynamic equations on time scales to biology, quantum mechanics, electrical engineering, neural networks, heat transfer, combinatorics, social sciences and so on. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. A book on the subject of time scales, by Bohner and Peterson [1], summarizes and organizes much of time scale calculus, see also the book by Bohner and Peterson [2] for advances in dynamic equations on time scales.

The purpose of this paper is to investigate the oscillation of the following second-order nonlinear dynamic equation

$$\left(r(t)\,|\,x^{\Delta}(t)\,|^{\alpha-1}\,x^{\Delta}(t)\right)^{\Delta}+q(t)\,|\,x(t)\,|^{\beta-1}\,x(t)=0 \tag{1.1}$$

on an arbitrary time scale $\mathbb{T}$, where the following conditions are assumed to hold: (S_1) α, $\beta>0$ are constants and $\sup\mathbb{T}=\infty$; (S_2) r and q are positive rd-continuous functions defined on the time scale interval $[t_0,\infty)$.

By a solution of (1.1) we mean a nontrivial real-valued function $x\in C^1_{rd}[t_x,\infty)$ for a certain $t_x\ge t_0$, which has the property that $r\,|\,x^{\Delta}\,|^{\alpha-1}\,x^{\Delta}\in C^1_{rd}[t_x,\infty)$ and satisfies (1.1) on $[t_x,\infty)$. Our attention is restricted to those solutions of (1.1) which exist on the half-line $[t_x,\infty)$ and satisfy $\sup\{|\,x(t)\,|: t>t_*\}>0$ for any $t_*\ge t_x$. A solution x of (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative; otherwise it is nonoscillatory. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

In the past years, the oscillation theory of dynamic equations has been developed very rapidly. For some papers on the subject, we refer to [3-8, 10-12] and the references cited therein. In 2005, Saker [4] presented some oscillation criteria for (1.1) when $\alpha = \beta > 1$ is an odd positive integer and (S_2) holds. In 2008, Hassan [5] obtained some sufficient conditions for the oscillation of (1.1) when $\alpha = \beta$ is a quotient of odd positive integers and (S_2) holds. Hassan [5] improved and extended the results of Saker [4]. Recently, Grace et al. [6] established several new oscillation criteria for (1.1) when α, β are quotients of odd positive integers and (S_2) holds.

However, the cases considered by [4-6] are some special cases of (1.1), and all the results of [4-6] can not be applied to (1.1) when α, β are not equal to quotients of odd positive integers. Thus, it is of great interest to investigate the oscillation of (1.1) when α, $\beta > 0$ are constants. In this paper, we will establish some new oscillation criteria for (1.1) when α, $\beta > 0$ are constants. Our results improve and extend the results of [4-6].

The following lemmas are useful in the proof of our main results.

Lemma 1.1 (Bohner and Peterson [1], p. 32, Theorem 1.87) *Let* $f : \mathbb{R} \to \mathbb{R}$ *be continuously differentiable and suppose* $g : \mathbb{T} \to \mathbb{R}$ *is delta differentiable. Then* $f \circ g : \mathbb{T} \to \mathbb{R}$ *is delta differentiable and satisfies*

$$(f \circ g)^{\Delta}(t) = \left\{ \int_0^1 f'\left(g(t) + h\mu(t) g^{\Delta}(t) \right) dh \right\} g^{\Delta}(t),$$

where $\mu(t) := \sigma(t) - t$ *is the graininess function on* $\mathbb{T}$, *here* $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$ *is the forward jump operator on* $\mathbb{T}$.

Lemma 1.2 (Hardy et al. [9]) *If* X *and* Y *are nonnegative, then*

$$\lambda X Y^{\lambda - 1} - X^{\lambda} \le (\lambda - 1) Y^{\lambda} \quad \text{when } \lambda > 1,$$

where the equality holds if and only if $X = Y$.

2 Main Results

Theorem 2.1 *Assume that (S1), (S2) and the following condition hold:*

$$\int_{t_0}^{\infty} r^{-1/\alpha}(t)\Delta t = \infty \tag{2.1}$$

Furthermore, assume that there exists a positive nondecreasing delta differentiable function φ *such that for all* $T > t_1 \geq t_0$,

$$\limsup_{t\to\infty} \int_T^t \left[q(s)\varphi(s) - \frac{(\alpha/\beta)^\alpha r(s)(\varphi^\Delta(s))^{\alpha+1}}{(\alpha+1)^{\alpha+1}(\varphi(s)v(s))^\alpha} \right] \Delta s = \infty, \tag{2.2}$$

where $v(t) := \begin{cases} c_1, & \text{if } \alpha < \beta, \\ 1, & \text{if } \alpha = \beta, \\ c_2(u^\sigma(t))^{(\alpha-\beta)/\alpha}, & \text{if } \alpha > \beta, \end{cases}$ *here* c_1 *and* c_2 *are any positive constants,* σ *is the forward jump operator on* $\mathbb{T}$, $u(t) := \left(\int_{t_1}^{t} r^{-1/\alpha}(s)\Delta s \right)^{-1}$ *and* $u^\sigma := u \circ \sigma$. *Then (1.1) is oscillatory.*

Proof Let x be a nonoscillatory solution of (1.1). Without loss of generality, we may assume that x is an eventually positive solution of (1.1). Then there exists $t_1 \geq t_0$ such that $x(t) > 0$ for $t \in [t_1, \infty)$. Therefore, from (1.1) we have

$$(r(t)|x^\Delta(t)|^{\alpha-1} x^\Delta(t))^\Delta = -q(t)x^\beta(t) < 0 \quad \text{for} \quad t \geq t_1.$$

Thus, we see that $r(t)|x^\Delta(t)|^{\alpha-1} x^\Delta(t)$ is strictly decreasing on $[t_1, \infty)$ and is eventually of one sign. We claim $x^\Delta(t) > 0$ for $t \in [t_1, \infty)$. Assume on the contrary, then there exists $t_2 \geq t_1$ such that $x^\Delta(t_2) \leq 0$. Take $t_3 > t_2$. Then we obtain $r(t)|x^\Delta(t)|^{\alpha-1} x^\Delta(t) \leq r(t_3)|x^\Delta(t_3)|^{\alpha-1} x^\Delta(t_3) < r(t_2)|x^\Delta(t_2)|^{\alpha-1} x^\Delta(t_2)$ for $t \in [t_3, \infty)$. Let $M := r(t_3)|x^\Delta(t_3)|^{\alpha-1} x^\Delta(t_3) < 0$. Then we get $x^\Delta(t) \leq -(-M)^{1/\alpha} r^{-1/\alpha}(t)$ for $t \in [t_3, \infty)$. Integrating both sides of the last inequality from t_3 to t, we have $x(t) - x(t_3) \leq -(-M)^{1/\alpha} \int_{t_3}^{t} r^{-1/\alpha}(s)\Delta s$ for $t \in [t_3, \infty)$. Letting $t \to \infty$ and using (2.1), we conclude $\lim_{t\to\infty} x(t) = -\infty$. This contradicts the fact that $x(t)>0$ for $t\in[t_1,\infty)$. Thus, we have $x^{\Delta}(t)>0$ for $t\in[t_1,\infty)$. Let $w(t):=r(t)(x^{\Delta}(t))^{\alpha}\dfrac{\varphi(t)}{x^{\beta}(t)}$ for $t\in[t_1,\infty)$. Then by the formulas

$$(FG)^{\Delta}=F^{\Delta}G+F^{\sigma}G^{\Delta} \text{ and } \left(F/G\right)^{\Delta}=F^{\Delta}/G^{\sigma}-FG^{\Delta}/(GG^{\sigma})$$

for the delta derivatives of the product FG and the quotient F/G of differentiable functions F and G, where σ is the forward jump operator on $\mathbb{T}$, $F^{\sigma}:=F\circ\sigma$ and $G^{\sigma}:=G\circ\sigma$, we get

$$\begin{aligned} w^{\Delta}&=\left(r(x^{\Delta})^{\alpha}\right)^{\Delta}\frac{\varphi}{x^{\beta}}+\left(r(x^{\Delta})^{\alpha}\right)^{\sigma}\left(\frac{\varphi}{x^{\beta}}\right)^{\Delta}\\ &=\left(r(x^{\Delta})^{\alpha}\right)^{\Delta}\frac{\varphi}{x^{\beta}}+\left(r(x^{\Delta})^{\alpha}\right)^{\sigma}\left(\frac{\varphi^{\Delta}}{(x^{\beta})^{\sigma}}-\frac{\varphi(x^{\beta})^{\Delta}}{x^{\beta}(x^{\beta})^{\sigma}}\right) \text{ on } [t_1,\infty). \end{aligned}$$

For $t\geq t_1$, since $(r(t)(x^{\Delta}(t))^{\alpha})^{\Delta}=-q(t)x^{\beta}(t)$, we have

$$w^{\Delta}=-q\varphi+\left(r(x^{\Delta})^{\alpha}\right)^{\sigma}\left(\frac{\varphi^{\Delta}}{(x^{\beta})^{\sigma}}-\frac{\varphi(x^{\beta})^{\Delta}}{x^{\beta}(x^{\beta})^{\sigma}}\right)=-q\varphi+\frac{w^{\sigma}}{\varphi^{\sigma}}\varphi^{\Delta}-\varphi\frac{w^{\sigma}}{\varphi^{\sigma}}\frac{(x^{\beta})^{\Delta}}{x^{\beta}}. \quad (2.3)$$

For $t\in[t_1,\infty)$, since $0<x(t)\leq x^{\sigma}(t)$, by Lemma 1.1 and by the formula $x^{\sigma}(t)=x(t)+\mu(t)x^{\Delta}(t)$ we obtain

$$\begin{aligned} (x^{\beta}(t))^{\Delta}&=\beta x^{\Delta}(t)\int_0^1[x(t)+h\mu(t)x^{\Delta}(t)]^{\beta-1}dh=\beta x^{\Delta}(t)\int_0^1[(1-h)x(t)+hx^{\sigma}(t)]^{\beta-1}dh\\ &\geq\begin{cases}\beta x^{\Delta}(t)\int_0^1(x^{\sigma}(t))^{\beta-1}dh, & \text{if } 0<\beta\leq 1,\\ \beta x^{\Delta}(t)\int_0^1 x^{\beta-1}(t)dh, & \text{if } \beta>1\end{cases}=\begin{cases}\beta(x^{\sigma}(t))^{\beta-1}x^{\Delta}(t), & \text{if } 0<\beta\leq 1,\\ \beta x^{\beta-1}(t)x^{\Delta}(t), & \text{if } \beta>1.\end{cases} \end{aligned}$$

Therefore, we conclude $\dfrac{(x^{\beta}(t))^{\Delta}}{x^{\beta}(t)}\geq\begin{cases}\beta(x^{\sigma}(t))^{\beta-1}x^{\Delta}(t)/x^{\beta}(t), & \text{if } 0<\beta\leq 1,\\ \beta x^{\Delta}(t)/x(t), & \text{if } \beta>1\end{cases}$ for $t\in[t_1,\infty)$. Since $0<x(t)\leq x^{\sigma}(t)$ for $t\in[t_1,\infty)$, we obtain $\dfrac{(x^{\beta}(t))^{\Delta}}{x^{\beta}(t)}\geq\dfrac{\beta x^{\Delta}(t)}{x^{\sigma}(t)}$ for all $\beta>0$ and for $t\in[t_1,\infty)$. Hence, from (2.3) we find

$$w^{\Delta}\leq -q\varphi+\frac{w^{\sigma}}{\varphi^{\sigma}}\varphi^{\Delta}-\beta\varphi\frac{w^{\sigma}}{\varphi^{\sigma}}\frac{x^{\Delta}}{x^{\sigma}} \text{ on } [t_1,\infty). \quad (2.4)$$

From the definition of w we get $r^{1/\alpha}x^{\Delta}=(wx^{\beta}/\varphi)^{1/\alpha}$. Since $r^{1/\alpha}(t)x^{\Delta}(t)$ is strictly decreasing and $t\le\sigma(t)$ on $[t_1,\infty)$, we have $r^{1/\alpha}x^{\Delta}\ge(r^{1/\alpha}x^{\Delta})^{\sigma}$ $=[w^{\sigma}(x^{\sigma})^{\beta}/\varphi^{\sigma}]^{1/\alpha}$ on $[t_1,\infty)$. Thus, from (2.4) we obtain

$$w^{\Delta}\le -q\varphi+\frac{w^{\sigma}}{\varphi^{\sigma}}\varphi^{\Delta}-\frac{\beta\varphi(w^{\sigma})^{(1+\alpha)/\alpha}}{r^{1/\alpha}(\varphi^{\sigma})^{(1+\alpha)/\alpha}}(x^{\sigma})^{(\beta-\alpha)/\alpha}\quad\text{for}\quad t\in[t_1,\infty). \tag{2.5}$$

Next, we consider the following three cases:

Case (i). Let $\alpha<\beta$. For $t\in[t_1,\infty)$, since $x^{\sigma}(t)\ge x(t)\ge x(t_1)>0$, we have

$$(x^{\sigma})^{(\beta-\alpha)/\alpha}(t)\ge(x(t_1))^{(\beta-\alpha)/\alpha}:=c_1. \tag{2.6}$$

Case (ii). Let $\alpha=\beta$. Then, for $t\in[t_1,\infty)$ we get

$$(x^{\sigma})^{(\beta-\alpha)/\alpha}(t)=1. \tag{2.7}$$

Case (iii). Let $\alpha>\beta$. Since $r(t)(x^{\Delta}(t))^{\alpha}$ is strictly decreasing on $[t_1,\infty)$, we have

$$r(t)(x^{\Delta}(t))^{\alpha}\le r(t_1)(x^{\Delta}(t_1))^{\alpha}:=b\quad\text{for}\quad t\in[t_1,\infty).$$

Hence, we obtain $x^{\Delta}(t)\le b^{1/\alpha}r^{-1/\alpha}(t)$ for $t\in[t_1,\infty)$. Integrating both sides of the last inequality from t_1 to t, we have $x(t)\le x(t_1)+b^{1/\alpha}\int_{t_1}^{t}r^{-1/\alpha}(s)\Delta s$ for $t\in[t_1,\infty)$. Therefore, there exist a constant $b_1>0$ and $t_4>t_1$ such that $x(t)\le b_1\int_{t_1}^{t}r^{-1/\alpha}(s)\Delta s:=b_1u^{-1}(t)$ for $t\in[t_4,\infty)$. Hence, for $t\in[t_4,\infty)$ we get

$$(x^{\sigma}(t))^{(\beta-\alpha)/\alpha}\ge c_2(u^{\sigma}(t))^{(\alpha-\beta)/\alpha}, \tag{2.8}$$

where $c_2:=(b_1)^{(\beta-\alpha)/\alpha}$. Thus, for all $\alpha,\beta>0$ and for $t\in[t_4,\infty)$, from (2.5)-(2.8) it follows that

$$w^{\Delta}\le -q\varphi+\frac{w^{\sigma}}{\varphi^{\sigma}}\varphi^{\Delta}-\frac{\beta\varphi(w^{\sigma})^{(1+\alpha)/\alpha}}{r^{1/\alpha}(\varphi^{\sigma})^{(1+\alpha)/\alpha}}v. \tag{2.9}$$

Taking $\lambda=(\alpha+1)/\alpha$, $X=\dfrac{(\beta\varphi v)^{1/\lambda}}{r^{1/(\alpha+1)}\varphi^{\sigma}}w^{\sigma}$ and $Y=\dfrac{(\varphi^{\Delta})^{\alpha}r^{1/\lambda}}{\lambda^{\alpha}(\beta\varphi v)^{\alpha/\lambda}}$, by (2.9) and Lemma 1.2 we obtain

$$w^{\Delta}\le -q\varphi+\frac{(\alpha/\beta)^{\alpha}r(\varphi^{\Delta})^{\alpha+1}}{(\alpha+1)^{\alpha+1}(\varphi v)^{\alpha}}\quad \text{on } [t_4,\infty).$$

Integrating both sides of the last inequality from t_4 to t, we obtain

$$\int_{t_4}^{t}\left[q(s)\varphi(s)-\frac{(\alpha/\beta)^{\alpha}r(s)(\varphi^{\Delta}(s))^{\alpha+1}}{(\alpha+1)^{\alpha+1}(\varphi(s)v(s))^{\alpha}}\right]\Delta s\le w(t_4)-w(t)<w(t_4)\quad \text{for } t\in[t_4,\infty).$$

Therefore, we get $\limsup\limits_{t\to\infty}\int_{t_4}^{t}\left[q(s)\varphi(s)-\dfrac{(\alpha/\beta)^{\alpha}r(s)(\varphi^{\Delta}(s))^{\alpha+1}}{(\alpha+1)^{\alpha+1}(\varphi(s)v(s))^{\alpha}}\right]\Delta s\le w(t_4)<\infty$, which contradicts (2.2). The proof is complete. □

Next, we consider the case when

$$\int_{t_0}^{\infty}r^{-1/\alpha}(t)\Delta t<\infty \tag{2.10}$$

holds, which implies that (2.1) doesn't hold.

Theorem 2.2 *Assume that (S_1), (S_2) and (2.10) hold. Furthermore, assume that there exists a positive nondecreasing delta differentiable function* φ *such that for all* $T>t_1\ge t_0$*, (2.2) and the following condition hold:*

$$\int_{T}^{\infty}\left(r^{-1}(z)\int_{T}^{z}k^{\beta}(s)q(s)\Delta s\right)^{1/\alpha}\Delta z=\infty \tag{2.11}$$

where $k(t):=\int_{t}^{\infty}r^{-1/\alpha}(s)\Delta s$*. Then (1.1) is oscillatory.*

Proof Assume that x is a nonoscillatory solution of (1.1). Without loss of generality, we may assume that x is an eventually positive solution of (1.1). Then there exists $t_1\ge t_0$ such that $x(t)>0$ for $t\in[t_1,\infty)$. Proceeding as in the proof of Theorem 2.1, we obtain that $r(t)|x^{\Delta}(t)|^{\alpha-1}x^{\Delta}(t)$ is strictly decreasing on $[t_1,\infty)$ and is eventually of one sign. Therefore, there are two cases for the sign of $x^{\Delta}(t)$. The proof when $x^{\Delta}(t)$ is eventually positive is similar to that of Theorem 2.1 and hence is omitted.

Next, assume that $x^{\Delta}(t)$ is eventually negative. Then there exists $t_2 \geq t_1$ such that $x^{\Delta}(t) < 0$ for $t \in [t_2, \infty)$. Thus, from (1.1) we have $\left(r(t)(-x^{\Delta}(t))^{\alpha}\right)^{\Delta} = q(t)x^{\beta}(t) > 0$ for $t \in [t_2, \infty)$, which implies that $r(t)(-x^{\Delta}(t))^{\alpha}$ is strictly increasing on $[t_2, \infty)$. Hence, we have $r(s)(-x^{\Delta}(s))^{\alpha} \geq r(t)(-x^{\Delta}(t))^{\alpha}$ for $s \geq t \geq t_2$. Then for $s \geq t \geq t_2$ we conclude $-x^{\Delta}(s) \geq r^{-1/\alpha}(s)r^{1/\alpha}(t)(-x^{\Delta}(t))$. Integrating both sides of the last inequality from $t \geq t_2$ to $z \geq t$ and letting $z \to \infty$, we get

$$x(t) \geq \left(\int_t^{\infty} r^{-1/\alpha}(s)\Delta s\right) r^{1/\alpha}(t)(-x^{\Delta}(t)) := k(t)r^{1/\alpha}(t)(-x^{\Delta}(t))$$
$$\geq k(t)r^{1/\alpha}(t_2)(-x^{\Delta}(t_2)) := ck(t)$$

for $t \in [t_2, \infty)$, where $c := -r^{1/\alpha}(t_2)x^{\Delta}(t_2) > 0$. Thus, from (1.1) we obtain $\left(r(t)(-x^{\Delta}(t))^{\alpha}\right)^{\Delta} = q(t)x^{\beta}(t) \geq c^{\beta}k^{\beta}(t)q(t)$ for $t \in [t_2, \infty)$. Integrating both sides of the last inequality from t_2 to t, we have

$$r(t)(-x^{\Delta}(t))^{\alpha} \geq r(t_2)(-x^{\Delta}(t_2))^{\alpha} + c^{\beta}\int_{t_2}^{t} k^{\beta}(s)q(s)\Delta s > c^{\beta}\int_{t_2}^{t} k^{\beta}(s)q(s)\Delta s$$

for $t \in [t_2, \infty)$. Hence, we obtain $-x^{\Delta}(t) > \left(r^{-1}(t)c^{\beta}\int_{t_2}^{t} k^{\beta}(s)q(s)\Delta s\right)^{1/\alpha}$ for $t \in [t_2, \infty)$. Integrating both sides of the last inequality from t_2 to t, we find $x(t) \leq x(t_2) - c^{\beta/\alpha}\int_{t_2}^{t}\left(r^{-1}(z)\int_{t_2}^{z} k^{\beta}(s)q(s)\Delta s\right)^{1/\alpha}\Delta z$ for $t \in [t_2, \infty)$. Letting $t \to \infty$ and using (2.11), we see $\lim_{t\to\infty} x(t) = -\infty$. This contradicts the fact that $x(t) > 0$ for $t \in [t_1, \infty)$. The proof is complete. □

Remark 2.1 From Theorems 2.1 and 2.2, we can obtain many different sufficient conditions for the oscillation of (1.1) with different choices of the function φ.

For instance, let $\varphi(t)=1$, then $\varphi^{\Delta}(t)=0$ and Theorems 2.1 and 2.2 imply the following results, respectively.

Corollary 2.1 *Suppose that (S_1), (S_2) and (2.1) hold. Furthermore, assume that* $\int_{t_0}^{\infty} q(t)\Delta t=\infty$. *Then (1.1) is oscillatory.*

Corollary 2.2 *Suppose that (S_1), (S_2), (2.10) and (2.11) hold. Furthermore, assume that* $\int_{t_0}^{\infty} q(t)\Delta t=\infty$. *Then (1.1) is oscillatory.*

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