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Bi- Parabolic thermal effect on vibration of visco-elastic square plate

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Abstract

The analysis presented here is to study the effect of thermal gradient on the vibration of visco-elastic square plate (having clamped boundary condition on all the four edges) of variable thickness whose thickness varies linearly in both directions. Two dimensional thermal effects on frequency of free vibrations of a visco-elastic square plate is considered. It is also considered that the temperature varies parabolically in two directions and thickness of square plate varies linearly in two directions. An approximate but quite convenient frequency equation is derived for a square plate (clamped at all the edges) by using Rayleigh-Ritz technique with a two-term deflection function. Both the modes of the frequency are calculated by the latest computational technique, MATLAB, for the various

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values of taper parameters and temperature gradient. All the results are presented in the graphs.

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1 Introduction

The study of vibration of plates has acquired great importance in the field of research, engineering and space technology. The visco-elastic behaviors of some materials invigorated scientists for modern designs and analysis techniques and their application to many practical problems. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications.

A study of the literature on vibration problems shows that the visco-elastic plates with thickness variation in two directions has received rather less attention than that of in one direction. Recently, Gupta and Lalit Kumar [3] studied Thermal effects on the vibration of non-homogeneous visco-elastic rectangular plate of linearly varying thickness. Leissa [5] gave different models on the vibration of plates. Gupta and Anupam Khanna [1] discussed thermal effect on vibrations of parallelogram plate of linearly varying thickness. The effect of thermal gradient on the frequencies of an orthotropic plate of linearly varying thickness has been discussed by Tomar and Gupta [7]. Vibration of rectangular plates by the Ritz method was given by Young [8]. Tomar and Gupta [6] discussed the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions. An analysis is presented on the vibration of clamped visco-elastic rectangular plate with parabolic thickness variations by Gupta and Anupam Khanna [2]. A. Khanna, A. Kumar and M. Bhatia [4] recently presented an analysis on two dimensional thermal effect with two dimensional varying thickness of visco- elastic square plate .

The aim of present investigation is to study the parabolically two dimensional thermal effect on the vibration of visco-elastic square plate whose thickness varies linearly in both directions. It is assumed that the plate is clamped on all the four edges and its temperature varies linearly in both the directions. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity (E).

For various numerical values of thermal gradient and taper constants; frequency for the first two modes of vibration are calculated with the help of MATLAB. All the numerical calculations will be carried out using the material constants of alloy 'Duralium'. All results are shown in Graphs.

2 Equation of Motion and Analysis

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is given by equation (2.1) respectively:

$$\begin{aligned} &D_1\left(W,_{xxxx}+2W,_{xxyy}+W,_{yyyy}\right)+2D_1,_x\left(W,_{xxx}+W,_{xyy}\right) \\ &+2D_{1,y}\left(W,_{yyy}+W,_{yxx}\right)+D_{1,xx}(W,_{xx}+\nu W,_{yy}) \\ &+D_{1,yy}(W,_{yy}+\nu W,_{xx})+2(1-\nu)D_{1,xy}W,_{xy}-\rho hp^2W = 0 \end{aligned} \tag{2.1}$$

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness.

Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3/12(1-\nu^2) \tag{2.2}$$

and corresponding two-term deflection function is taken as

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \quad (2.3)$$

Assuming that the square plate of engineering material has a steady two dimensional parabolic temperature distribution i.e.

$$\tau=\tau_0(1-x^2/a^2)(1-y^2/a^2) \quad (2.4)$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "a" is the length of a side of square plate.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as

$$E=E_0\left(1-\gamma\tau\right) \quad (2.5)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variation (2.5) become

$$E=E_0[1-a(1-x/a)(1-y/a)] \quad (2.6)$$

where, $a=\gamma\tau_0$, $(0\le a<1)$ thermal gradient.

It is assumed that thickness also varies linearly in two directions as shown below:

$$h=h_0(1+\beta_1 x/a)(1+\beta_2 y/a) \quad (2.7)$$

where, β_1 and β_2 are taper parameters in x and y directions respectively and $h=h_0$ at $x=y=0$. Now, put the value of E and h from equation (2.6) and (2.7) in the equation (2.2), one obtain

$$D_1=[E_0[1-a(1-x^2/a^2)(1-y^2/a^2)]h_0^{\,3}(1+\beta_1 x/a)^3(1+\beta_2 y/a)^3]/12(1-v^2) \quad (2.8)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^*-T^*)=0 \quad (2.9)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{array}{l} W = W_{,x} = 0, \;\; x = 0, \; a \\ W = W_{,y} = 0, \;\; y = 0, \; a \end{array}\right\} \tag{2.10}$$

Now assuming the non-dimensional variables as

$$X = x/a, \quad Y = y/a, \quad \overline{W} = W/a, \quad \overline{h} = h/a \tag{2.11}$$

The kinetic energy T^* and strain energy V^* are [5]

$$T^* = (1/2)\rho p^2 \overline{h_0} a^5 \int_0^1 \int_0^1 [(1+\beta_1 X)(1+\beta_2 Y)\overline{W^2}] dY dX \tag{2.12}$$

and

$$V^* = Q\int_0^1 \int_0^1 [1 - a(1-X^2)(1-Y^2)](1+\beta_1 X)^3 (1+\beta_2 Y)^3 \{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1-v)(\overline{W},_{XY})^2\} dY dX \tag{2.13}$$

where, $Q = E_0 h_0{}^3 a^3 / 24(1-v^2)$.

Using equations (2.12) and (2.13) in equation (2.9), one get

$$(V^{**} - \lambda^2 T^{**}) = 0 \tag{2.14}$$

where,

$$V^{**} = \int_0^1 \int_0^1 [1 - a(1-X^2)(1-Y^2)](1+\beta_1 X)^3 (1+\beta_2 Y)^3 \{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1-v)(\overline{W},_{XY})^2\} dY dX \tag{2.15}$$

and

$$T^{**} = \int_0^1 \int_0^1 [(1+\beta_1 X)(1+\beta_2 Y)\overline{W^2}] dY dX \tag{2.16}$$

Here, $\lambda^2 = 12\rho(1-v^2)a^2 / E_0 h_0{}^2$ is a frequency parameter.

Equation (2.16) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of *W*. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n, \quad n = 1, 2 \tag{2.17}$$

On simplifying (2.17), one gets

$$bn_1 A_1 + bn_2 A_2 = 0, \quad n = 1, 2 \tag{2.18}$$

where, bn_1 , bn_2 ($n = 1, 2$) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (2.18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \tag{2.19}$$

with the help of equation (2.19), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1(Mode1) and λ_2(Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3 Results and Discussion

All calculations are carried out with the help of latest Matrix Laboratory computer software i.e. *MATLAB*. Computation has been done for frequency of visco-elastic square plate for different values of taper constants β_1 and β_2, thermal gradients a, at different points for first two modes of vibrations have been calculated numerically.

In Figure 1: It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = 0.0$ both modes of vibrations.

In Figure 2: Also it is obvious to understand the decrement in frequency for $\beta_1 = \beta_2 = 0.6$. But it is also noticed that value of frequency is increased with the increment in β_1 and β_2.

In Figure 3: It is evident that frequency decreases continuously as thermal gradient increases, $\beta_1 = 0.4$, $\beta_2 = 0.2$ respectively with the two modes of vibration.

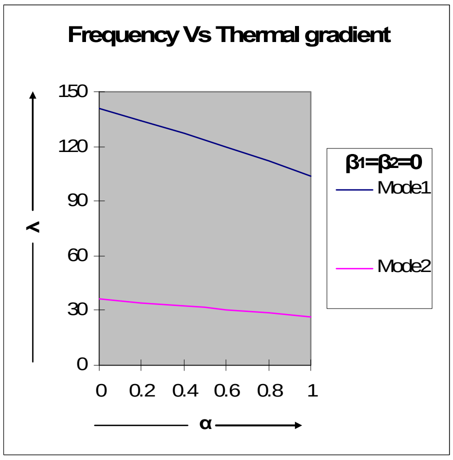

Figure 1: Frequency vs. thermal gradient at $\beta_1 = \beta_2 = 0.0$

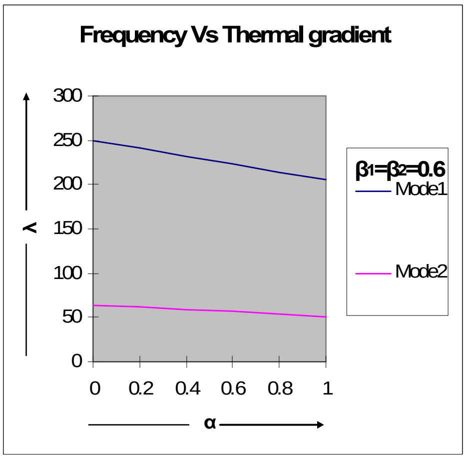

Figure 2: Frequency vs. thermal gradient at $\beta_1 = \beta_2 = 0.6$

In Figure 4: Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_2 from 0.0 to 1.0.

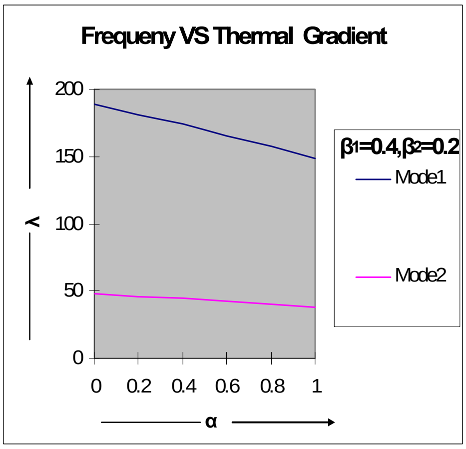

Figure 3: Frequency vs. thermal gradient at $\beta_1 = 0.4$, $\beta_2 = 0.2$

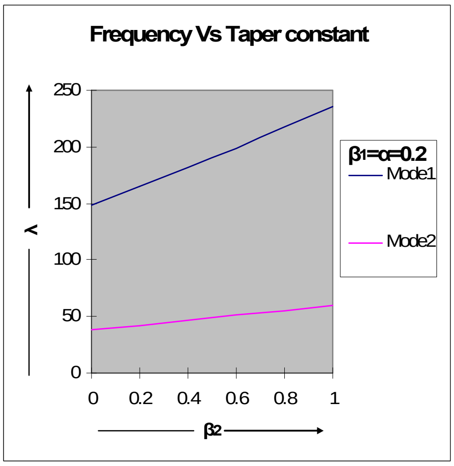

Figure 4: Frequency vs. Taper constant at $\beta_1 = 0.2$, $a = 0.2$

4 Conclusion

Results of present paper are compared with paper [8]. It is interesting to note that value of frequency has greater value in this paper as compared to [8]. So, main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

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