Journal of Applied Mathematics & Bioinformatics, vol.1, no.1, 2011, 65-78
ISSN: 1792-6602 (print), 1792-6939 (online)

A regularized Dynamical System Method for nonlinear Ill-posed Hammerstein type operator equations

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Abstract

A combination of Dynamical System Method(DSM) and a regularization method has been considered for obtaining a stable approximate solution for ill-posed Hammerstein type operator equations. By choosing the regularization parameter according to an adaptive scheme considered by Pereverzev and Schock (2005) an order optimal error estimate has been obtained. Moreover the method that we consider converges exponentially compared to the linear convergence obtained by George and Nair (2008).

Mathematics Subject Classification : 47J06, 47J25, 65J20, 65R10
Keywords: Ill-posed Hammerstein type operator, Dynamical System Method, Adaptive scheme, Regularization, Source conditions.

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Article Info: *Revised* : April 5, 2011. *Published online* : May 31, 2011

1 Introduction

In this paper we consider the problem of obtaining an approximate solution for the nonlinear ill-posed Hammerstein type operator ([4], [5], [6],[9]) equation

$$KF(x) = y. \tag{1}$$

where $F : D(F) \subset X \mapsto Z$ is nonlinear and $K : Z \mapsto Y$ is a bounded linear operator where we take X, Y, Z to be Hilbert spaces.

In [9], George and Nair, studied a modified form of Newton Lavrentiev Regularization method for obtaining approximations for a solution $\hat{x} \in D(F)$ of (1), which satisfies

$$\|F(\hat{x}) - F(x_0)\| = \min\{\|F(x) - F(x_0)\| : KF(x) = y, x \in D(F)\}$$

where x_0 is an initial guess.

In this paper we consider the special case where $X = Z$. We assume throughout that the solution $\hat{x}$ of (1) satisfies

$$\|\hat{x} - x_0\| = \min\{\|x - x_0\| : KF(x) = y, x \in D(F)\}$$

and that $y^\delta \in Y$ are the available noisy data with

$$\|y - y^\delta\| \leq \delta.$$

The method considered in [9] gives linear convergence and the method considered in [6] gives quadratic convergence.

Recall that a sequence (x_n) in X with $\lim x_n = x^*$ is said to be convergent of order $p > 1$, if there exist positive reals β, γ, such that for all $n \in \mathbf{N}$

$$\|x_n - x^*\| \leq \beta e^{-\gamma p^n}.$$

If the sequence (x_n) has the property, that

$$\|x_n - x^*\| \leq \beta q^n, \qquad 0 < q < 1$$

then (x_n) is said to be linearly convergent. For an extensive discussion of convergence rate see Kelley [12].

In this paper we consider a combination of a modified form of DSM and a regularization method for obtaining a stable approximate solution for (1).

Moreover the method that we considered in this paper converges exponentially compared to the linear convergence obtained in [9].

Organization of this paper is as follows. In section 2, we introduce the DSM method. In section 3 we provide an error estimate and in section 4 we derive error bounds under general source conditions by choosing the regularization parameter by an a priori manner as well as by an adaptive scheme proposed by Pereverzev and Schock in [14]. In section 5 we provide an application and finally the paper ends with conclusion in section 6.

2 Dynamical System Method

Observe that the solution x of (1) can be obtained by first solving

$$Kz = y \tag{2}$$

for z and then solving the nonlinear equation

$$F(x) = z. \tag{3}$$

For solving (2), we consider the regularized solution of (2) with y^δ in place of y as

$$z_\alpha^\delta = (K^*K + \alpha I)^{-1}K^*y^\delta, \qquad \alpha > 0, \delta > 0. \tag{4}$$

Note that (4) is the Tikhonov regularization of (2).

For solving (3), in [6], George and Kunhanandan considered $x_{n,\alpha}^\delta$, defined iteratively by

$$x_{n+1,\alpha}^\delta = x_{n,\alpha}^\delta - F'(x_{n,\alpha}^\delta)^{-1}(F(x_{n,\alpha}^\delta) - z_\alpha^\delta), n \in \mathbb{N} \tag{5}$$

with $x_{0,\alpha}^\delta = x_0$.

Note that the iteration (5) is the Newton's method for the nonlinear problem

$$F(x) - z_\alpha^\delta = 0. \tag{6}$$

The difficult and expensive part of the solution is inverting $F'(.)$ at each iterate $x_{n,\alpha}^\delta$. In [15] (cf. section 2.4.6, page 59), Ramm considered a method called

Dynamical System Method (DSM), which avoids, inverting of the operator $F'(.)$. In this paper we consider a method which is a combination of a modified form of DSM and the Tikhonov regularization. The DSM consists of finding (cf. [15],[13]) a nonlinear locally Lipschitz operator $\Phi(u,t)$, such that the Cauchy problem:

$$u'(t) = \Phi(u,t), \qquad u(0) = u_0 \tag{7}$$

has the following three properties:

$$\exists u(t) \forall t \geq 0, \quad \exists u(\infty), \quad F(u(\infty)) = 0,$$

i.e., (7) is globally uniquely solvable, its unique solution has a limit at infinity $u(\infty)$, and this limit solves (6). We assume that(6) is well posed, so (6) has a solution say x_α^δ, such that $x_\alpha^\delta \in B_R(x_0)$.

Throughout this paper we will be using the following assumptions.

Assumption 2.1. $F'(x_0)^{-1}$ exist and is a bounded operator with $\|F'(x_0)^{-1}\| =: \beta$.

Assumption 2.2. There exists a constant $k_0 > 0$ such that for every $x, u \in B_R(x_0)$ and $v \in X$, there exists an element $\Phi(x,u,v) \in X$ such that

$$[F'(x) - F'(u)]v = F'(u)\Phi(x,u,v), \|\Phi(x,u,v)\| \leq k_0\|v\|\|x-u\|.$$

The following Lemma based on the above assumption is used in proving our main result.

Lemma 2.3. Let $R > 0$ and $x, u \in \overline{B_R(x_0)}$. Then

$$F'(x_0)(u-x) - (F(u) - F(x)) = F'(x_0)\int_0^1 \Phi(x + t(u-x), x_0, x-u)dt.$$

Proof. We know by fundamental theorem of integration, that $F(u) - F(x) = \int_0^1 F'(x + t(u-x))(u-x)dt$. Hence by Assumption 2.2

$$\begin{aligned} F'(x_0)(u-x) - (F(u) - F(x)) &= \int_0^1 [F'(x+t(u-x)) - F'(x_0)](x-u)dt \\ &= F'(x_0)\int_0^1 \Phi(x+t(u-x), x_0, x-u)dt. \end{aligned}$$

The next assumption on source condition is based on a source function φ and a property of the source function φ. We will be using this assumption to obtain an error estimate for $\|F(\hat{x}) - z_\alpha^\delta\|$.

Assumption 2.4. There exists a continuous, strictly monotonically increasing function $\varphi : (0, a] \to (0, \infty)$ with $a \geq \|K^*K\|$ satisfying;

- $\lim_{\lambda \to 0} \varphi(\lambda) = 0$
- $$\sup_{\lambda \geq 0} \frac{\alpha\varphi(\lambda)}{\lambda + \alpha} \leq \varphi(\alpha), \qquad \forall \lambda \in (0, a].$$
- there exists $v \in X$ such that
$$F(\hat{x}) = \varphi(K^*K)v$$

We assume that $F \in C^2_{loc}$ i.e., $\forall x \in B_R(x_0)$,

$$\|F^{(j)}(x)\| \leq M_j, \qquad j = 1, 2. \tag{8}$$

Hereafter we assume that $\beta < \frac{1}{2}$ and

$$R < \frac{2(1 - 2\beta)}{\beta M_2 + 2k_0}. \tag{9}$$

In this paper we consider the following Cauchy's problem

$$x'(t) = -(F'(x_0) + \varepsilon(t)I)^{-1}(F(x) - z^\delta_\alpha), \qquad x(0) = x_0 \tag{10}$$

where x_0 is an initial approximation for x^δ_α and

$$\varepsilon : [0, \infty) \to [0, K] \tag{11}$$

is monotonic increasing function with $\varepsilon(0) = 0$ and

$$0 < K \leq \min\{\frac{1 - k_0 R}{2\beta}, 1\}. \tag{12}$$

Remark 2.5. Note that (9) implies $R < \frac{1}{k_0}$ and (12) implies $\beta\varepsilon(t) < 1$.

In order to have a local solution for the Cauchy problem (10), we make use of the following theorem.

Theorem 2.6. ([13], Theorem 2.1) Let X be a real Banach space, U be an open subset of X, and $x_0 \in U$. Let $\Phi : U \times \mathbb{R}^+ \to X$ be of class C^1 that is bounded on bounded sets. Then the following hold.

- There exists a maximal interval J containing 0 such that the initial value problem
$$x'(t) = \Phi(x(t), t), \quad x(0) = x_0,$$
has a unique solution $x(t) \in U$ for all $t \in J$.

- If J has the right end point, say τ, and $x_\tau := \lim_{t\to\tau} x(t)$ exists, then x_τ is on the boundary of U.

The following Proposition establishes the existence and uniqueness of the solution of the Cauchy problem (10).

Proposition 2.7. Let $\varepsilon(t)$ be as in (11), F maps bounded sets onto bounded sets. Then there exists a maximal interval $J \subseteq [0, \infty)$ such that (10) has a unique solution $x(t)$ for all $t \in J$.

Proof. Let

$$\Phi = -(F'(x_0) + \varepsilon(t)I)^{-1}(F(x) - z_\alpha^\delta), \qquad x \in B_R(x_0), \quad t \in \mathbb{R}^+.$$

Then $\Phi : B_R(x_0) \times \mathbb{R}^+ \to X$ is of class C^1. Because F is bounded on bounded sets and since $\beta\varepsilon(t) < 1$, we have

$$\begin{aligned} \|(F'(x_0) + \varepsilon(t)I)^{-1}\| &\leq \|F'(x_0)^{-1}\|\|(I + \varepsilon(t)F'(x_0)^{-1})^{-1}\| \\ &\leq \frac{\beta}{1 - \beta\varepsilon(t)}. \end{aligned} \tag{13}$$

That is $(F'(x_0) + \varepsilon(t)I)$ has a bounded inverse for every $t \in \mathbb{R}^+$. So Φ is bounded on bounded sets. Hence the conclusion follows by applying Theorem 2.6.

Let $x(t) - x_\alpha^\delta := w$ and $\|w\| := g$. Then by Taylor Theorem (cf.[1], Theorem 1.1.20)

$$F(x(t)) - z_\alpha^\delta = F(x(t)) - F(x_\alpha^\delta) = F'(x_\alpha^\delta)(x(t) - x_\alpha^\delta) + T(x(t), x_\alpha^\delta)$$

where $T(x(t), x_\alpha^\delta) = \int_0^1 F''(\lambda x(t) + (1-\lambda)x_\alpha^\delta)(x(t) - x_\alpha^\delta)^2(1-\lambda)d\lambda$.

Observe that

$$w'(t) = x'(t) = -(F'(x_0) + \varepsilon(t)I)^{-1}[F'(x_\alpha^\delta)(x(t) - x_\alpha^\delta) + T(x(t), x_\alpha^\delta)]$$

and hence

$$\begin{aligned}
gg' &= \frac{1}{2}\frac{dg^2}{dt} \\
&= \frac{1}{2}\frac{d}{dt}\langle w, w\rangle \\
&= \langle w, w'\rangle \\
&= \langle w, -(F'(x_0)+\varepsilon(t)I)^{-1}[F'(x_\alpha^\delta)(x(t)-x_\alpha^\delta)+T(x(t),x_\alpha^\delta)]\rangle \\
&= \langle w, -w\rangle + \langle w, \Lambda w\rangle + \langle w, -(F'(x_0)+\varepsilon(t)I)^{-1}T(x(t),x_\alpha^\delta)\rangle \\
&\leq -\|w\|^2 + \|\Lambda\|\|w\|^2 + \|(F'(x_0)+\varepsilon(t)I)^{-1}T(x(t),x_\alpha^\delta)\|\|w\| \\
&\leq -g^2 + \|\Lambda\| g^2 + \|(F'(x_0)+\varepsilon(t)I)^{-1}T(x(t),x_\alpha^\delta)\| g \qquad (14)
\end{aligned}$$

where $\Lambda = I - (F'(x_0)+\varepsilon(t)I)^{-1}F'(x_\alpha^\delta)$. Note that

$$\begin{aligned}
\|\Lambda\| &\leq \sup_{\|v\|\leq 1} \|(F'(x_0)+\varepsilon(t)I)^{-1}[(F'(x_0)-F'(x_\alpha^\delta))+\varepsilon(t)I]v\| \\
&\leq \|(F'(x_0)+\varepsilon(t)I)^{-1}(F'(x_0)-F'(x_\alpha^\delta))\| \\
&\quad + \|(F'(x_0)+\varepsilon(t)I)^{-1}\varepsilon(t)I)v\| \\
&\leq \|(F'(x_0)+\varepsilon(t)I)^{-1}(F'(x_0)-F'(x_\alpha^\delta))\| \\
&\quad + \|(F'(x_0)+\varepsilon(t)I)^{-1}\varepsilon(t)Iv\| \\
&\leq \|(F'(x_0)+\varepsilon(t)I)^{-1}F'(x_0)\Phi(x_\alpha^\delta, x_0, v)\| \\
&\quad + \|(F'(x_0)+\varepsilon(t)I)^{-1}\varepsilon(t)v\| \\
&\leq \frac{k_0R+\beta\varepsilon(t)}{1-\beta\varepsilon(t)}, \qquad (15)
\end{aligned}$$

the last step follows from Assumption 2.2, (13) and the inequality $\|(I+\varepsilon(t)F'(x_0)^{-1})^{-1}\| \leq \frac{1}{1-\beta\varepsilon(t)}$. Again by (13) and (8)

$$\begin{aligned}
\|(F'(x_0)+\varepsilon(t)I)^{-1}T(x(t),x_\alpha^\delta)\| &\leq \frac{\beta}{1-\beta\varepsilon(t)}\|T(x(t),x_\alpha^\delta)\| \\
&\leq \frac{\beta}{1-\beta\varepsilon(t)}\frac{M_2\|x(t)-x_\alpha^\delta\|^2}{2} \\
&\leq \frac{\beta}{1-\beta\varepsilon(t)}\frac{M_2 g^2}{2}. \qquad (16)
\end{aligned}$$

Therefore by (14), (15) and (16) we have

$$gg' \leq -g^2 + (\frac{k_0R+\beta\varepsilon(t)}{1-\beta\varepsilon(t)})g^2 + \frac{\beta}{1-\beta\varepsilon(t)}\frac{M_2}{2}g^3$$

and hence

$$g' \leq -\gamma g + c_0 g^2 \tag{17}$$

where $\gamma := 1 - (\frac{k_0 R + \beta\varepsilon(t)}{1-\beta\varepsilon(t)}) > 0$ and $c_0 := \frac{\beta}{1-\beta\varepsilon(t)}\frac{M_2}{2}$. So by (17)

$$g(t) \leq re^{-\gamma t} \tag{18}$$

where $r = \frac{g(0)}{1-\frac{c_0 g(0)}{\gamma}}$. Note that $g(0) = \|x_0 - x_\alpha^\delta\| \leq R$ and hence condition (9) implies $\frac{c_0 g(0)}{\gamma} < 1$.

The above discussion leads to the following Theorem.

Theorem 2.8. If (8) and the Assumptions of Proposition 2.7 hold. Then (10) has a unique global solution $x(t)$ and $x(t)$ converges to x_α^δ. Further

$$\|x(t) - x_\alpha^\delta\| \leq re^{-\gamma t}$$

where r is as in (18).

Theorem 2.9. Suppose (8) and (9) hold. If, in addition, $\|\hat{x} - x_0\| \leq R$ then

$$\|\hat{x} - x_\alpha^\delta\| \leq \frac{\beta}{1-\kappa_0 R}\|F(\hat{x}) - z_\alpha^\delta\|.$$

Proof. Observe that

$$\begin{aligned}
\hat{x} - x_\alpha^\delta &= \hat{x} - x_\alpha^\delta - F'(x_0)^{-1}(z_\alpha^\delta - F(x_\alpha^\delta)) \\
&= F'(x_0)^{-1}[F'(x_0)(\hat{x} - x_\alpha^\delta) - (z_\alpha^\delta - F(x_\alpha^\delta))] \\
&= F'(x_0)^{-1}[F'(x_0)(\hat{x} - x_\alpha^\delta) - (F(\hat{x}) - F(x_\alpha^\delta) + z_\alpha^\delta - F(\hat{x}))] \\
&\leq \|F'(x_0)^{-1}[F'(x_0)(\hat{x} - x_\alpha^\delta) - (F(\hat{x}) - F(x_\alpha^\delta))]\| \\
&\quad + \|F'(x_0)^{-1}(z_\alpha^\delta - F(\hat{x}))\|.
\end{aligned}$$

So by Lemma 2.3 and Assumption 2.2 we have

$$\|\hat{x} - x_\alpha^\delta\| \leq k_0 R\|\hat{x} - x_\alpha^\delta\| + \beta\|F(\hat{x}) - z_\alpha^\delta)\|$$

and hence the result follows.

3 Error Analysis

The following Theorem is a consequence of Theorem 2.8 and Theorem 2.9.

Theorem 3.1. Suppose (8), (9) and the Assumptions in Theorem 2.8 hold. If, in addition, $\|\hat{x} - x_0\| \leq R$ then

$$\|\hat{x} - x(t)\| \leq \frac{\beta}{1 - \kappa_0 R}\|F(\hat{x}) - z_\alpha^\delta\| + re^{-\gamma t}.$$

In view of the estimate in the Theorem 3.1, the next task is to find an estimate $\|F(\hat{x}) - z_\alpha^\delta\|$. For this, let us introduce the notation;

$$z_\alpha := (K^*K + \alpha I)^{-1}K^*y.$$

We observe that

$$\begin{aligned} \|F(\hat{x}) - z_\alpha^\delta\| &\leq \|F(\hat{x}) - z_\alpha\| + \|z_\alpha - z_\alpha^\delta\| \\ &\leq \|F(\hat{x}) - z_\alpha\| + \frac{\delta}{\sqrt{\alpha}}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} F(\hat{x}) - z_\alpha &= F(\hat{x}) - (K^*K + \alpha I)^{-1}K^*KF(\hat{x}) \\ &= [I - (K^*K + \alpha I)^{-1}K^*K]F(\hat{x}) \\ &= \alpha(K^*K + \alpha I)^{-1}F(\hat{x}). \end{aligned}$$

So by Assumption 2.4.

$$\begin{aligned} \|F(\hat{x}) - z_\alpha\| &\leq \|\alpha(K^*K + \alpha I)^{-1}\varphi(K^*K)v\| \\ &\leq \sup_{0 < \lambda \leq \|K\|^2} \frac{\alpha\varphi(\lambda)}{\lambda + \alpha}\|v\| \leq \varphi(\alpha)\|v\|. \end{aligned}$$

Therefore by (19) we have

$$\|F(\hat{x}) - z_\alpha^\delta\| \leq \|v\|\varphi(\alpha) + \frac{\delta}{\sqrt{\alpha}}. \tag{20}$$

So, we have the following theorem.

Theorem 3.2. Under the assumptions of Theorem 3.1 and (20),

$$\|\hat{x} - x(t)\| \leq \frac{C\beta}{1 - k_0 R}(\varphi(\alpha) + \frac{\delta}{\sqrt{\alpha}}) + re^{-\gamma t},$$

where $C = \max\{\|v\|, 1\}$.

4 Error Bounds Under Source Conditions

Note that the estimate $\varphi(\alpha) + \frac{\delta}{\sqrt{\alpha}}$ in Theorem 3.2 attains minimum for the choice $\alpha := \alpha_\delta$ which satisfies $\varphi(\alpha_\delta) = \frac{\delta}{\sqrt{\alpha_\delta}}$. Let $\psi(\lambda) := \lambda\sqrt{\varphi^{-1}(\lambda)}, 0 < \lambda \leq \|K\|^2$. Then we have $\delta = \sqrt{\alpha_\delta}\varphi(\alpha_\delta) = \psi(\varphi(\alpha_\delta))$, and

$$\alpha_\delta = \varphi^{-1}(\psi^{-1}(\delta)). \tag{21}$$

So the relation (20) leads to

$$\|F(\hat{x}) - z_\alpha^\delta\| \leq 2\psi^{-1}(\delta).$$

Theorem 3.2 and the above observation leads to the following.

Theorem 4.1. Let $\psi(\lambda) := \lambda\sqrt{\varphi^{-1}(\lambda)}, 0 < \lambda \leq \|K\|^2$. and the assumptions of Theorem 3.1 are satisfied. For $\delta > 0$, let $\alpha_\delta = \varphi^{-1}(\psi^{-1}(\delta))$. If

$$T := min\{t : e^{-\gamma t} < \frac{\delta}{\sqrt{\alpha_\delta}}\},$$

then

$$\|\hat{x} - x(T)\| = O(\psi^{-1}(\delta)).$$

4.1 An adaptive choice of the parameter

The error estimate in the above Theorem has optimal order with respect to δ. Unfortunately, an a priori parameter choice (21) cannot be used in practice since the smoothness properties of the unknown solution $\hat{x}$ reflected in the function φ are generally unknown. There exist many parameter choice strategies in the literature, for example see [2], [10], [11], [7], [8], [16] and [17].

In [14], Pereverzev and Schock considered an adaptive selection of the parameter which does not involve even the regularization method in an explicit manner. In this method the regularization parameter α_i are selected from some finite set $\{\alpha_i : 0 < \alpha_0 < \alpha_1 < \cdots < \alpha_N\}$ and the corresponding regularized solution, say $z_{\alpha_i}^\delta$ are studied on-line. Later George and Nair [9] and George and Kunhanandan [6], considered the adaptive method for selecting the regularization parameter for approximately solving Hammerstein-type operator equations. In this paper also we consider the adaptive method for selecting the regularization parameter α.

Let $i \in \{0, 1, 2, \cdots, N\}$ and $\alpha_i = \mu^{2i}\alpha_0$ where $\mu > 1$. Let

$$l := max\{i : \varphi(\alpha_i) \leq \frac{\delta}{\sqrt{\alpha_i}}\} \tag{22}$$

and

$$k := max\{i : \|z^\delta_{\alpha_i} - z^\delta_{\alpha_j}\| \leq \frac{4\delta}{\sqrt{\alpha_j}}, j = 0, 1, 2, \cdots, i\}. \tag{23}$$

We need the following Theorem from [6].

Theorem 4.2. ([6], Theorem 4.3)Let l be as in (22), k be as in (23) and $z^\delta_{\alpha_k}$ be as in (4) with $\alpha = \alpha_k$. Then $l \leq k$ and

$$\|F(\hat{x}) - z^\delta_{\alpha_k}\| \leq (2 + \frac{4\mu}{\mu - 1})\mu\psi^{-1}(\delta).$$

Theorem 4.3. Let $\psi(\lambda) := \lambda\sqrt{\varphi^{-1}(\lambda)}, 0 < \lambda \leq \|K\|^2$. and the assumptions of Theorem 3.1 and (23) are satisfied. Let

$$T := min\{t : e^{-\gamma t} < \frac{\delta}{\sqrt{\alpha_k}}\},$$

and $x(T)$ be the solution of the Cauchy's problem (10) with $z^\delta_{\alpha_k}$ in place of z^δ_α. Then

$$\|\hat{x} - x(T)\| = O(\psi^{-1}(\delta)).$$

5 Applications

In this section we consider a specific example of Hammerstein type operator equation, which satisfies the assumptions of this paper. Consider the nonlinear operator equation

$$\int_\Omega k(s, t)f_\lambda(t, x(t))dt = y(s) \tag{24}$$

with $\Omega \in \mathbb{R}$ is a bounded domain, $k : \Omega \times \Omega \to \mathbb{R}$ is a measurable kernel and $f_\lambda : \Omega \times \mathbb{R} \to \mathbb{R}$ is defined as $f_\lambda(s, x) = b(s)g(x(s)) + \lambda c(s)$, where $0 \neq c \in L^p, 0 < b \in L^{\frac{p}{p-q}}$ for some $q \in (2, p)$ (cf. [3]) and g is a differentiable function such that $g'(x_0(t)) > \kappa > 0, \forall t \in \Omega$.

Note that (24) is of the form $KF(x) = y$ where $K : L^2(\Omega) \to L^2(\Omega)$ is given by

$$K(x(s)) = \int_\Omega k(s,t)x(t)dt$$

and $F : H^1(\Omega) \to L^2(\Omega)$ is given by

$$F(x(s)) = f_\lambda(s, x(s)).$$

Observe that

$$F'(x)h(s) = f_{\lambda x}(s, x(s))h(s) = b(s)g'(x(s))h(s),$$

so for $x, y \in B_R(x_0)$ and $h \in H^1(\Omega)$,

$$\begin{aligned}
[F'(x) - F'(z)]h(s) &= b(s)[g'(x(s)) - g'(z(s))]h(s) \\
&= b(s)g'(z(s))[\frac{g'(x(s))}{g'(z(s))} - 1]h(s) \\
&= F'(z)\Phi(x, z, h)
\end{aligned}$$

where $\Phi(x, z, h) = [\frac{g'(x(s))}{g'(z(s))} - 1]h(s)$. Thus F satisfies the Assumption 2.2. Further note that, since $g'(x_0(t)) > \kappa > 0, \forall t \in \Omega$, $F'(x_0)^{-1}$ exists and is a bounded operator.

6 Conclusion

We presented a method, which is a combination of Dynamical System Method studied extensively by Ramm and his collaborators and Tikhonov regularization method for obtaining a stable approximate solution for nonlinear ill-posed Hammerstein type operator equations. This method avoids the inversion of $F'(.)$ at every iterate in the Newton's type method.

ACKNOWLEDGEMENTS. One of the authors, Ms.Shobha, thanks National Institute of Technology Karnataka, India, for the financial support.

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