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GARCH Modelling of Conditional Correlations and Volatility of Exchange rates in BRICS Countries[1]

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Abstract

We examine the nature of BRICS currency returns using a *t*-DCC model and investigate whether multivariate volatility models can characterize and quantify market risk. We initially consider a multivariate *normal*-DCC model and show that it cannot adequately capture the fat tails prevalent in financial time series data such as exchange rates. We then consider a multivariate *t*-version of the Gaussian dynamic conditional correlation (DCC) proposed by [1] and successfully implemented by [2] and [3]. We find that the *t*-DCC model (dynamic conditional correlation based on the *t*-distribution) out performs the *normal*-DCC model. The former passes most diagnostic tests although it barely passes the Kolmogorov-Smirnov goodness-of-fit test.

JEL classification numbers: C51, G10, G11
Keywords: Correlations and Volatilities; MGARCH (Multivariate General Autoregressive Conditional Heteroscedasticity), Multivariate t (*t*-DCC), Kolmogorov-Smirnov test (KS_N), Value at Risk (V**a**R) diagnostics, ML – Maximum Likelihood

1 Introduction

Although any grouping of countries (such BRICS) involves some degree of arbitrary selection; the country and population size coupled with economic growth potential often acts as a common framework. There are at least two identifiable strengths to BRICS economies that are worth examining. First, BRICS countries produce 25% of global Gross Domestic Product (GDP), an increase of 15% from 1990. It is estimated that by 2020, they will account for about 37%-38% of global GDP with the current population of 3 billion with income per capita ranging from $7,710

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[1] The acronym BRIC is shorthand for emerging economies of Brazil, Russia, India and China. Almost a decade later in December 2010, South Africa joined the group resulting in the acronym BRICS. A few people have suggested that South Africa was added just to represent the African continent.

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to $13,689. Second, these economies have reasons for creating a club or grouping of their own to act as a counterweight in multilateral diplomacy, particularly in dealing with the U.S. and the EU.

Since BRICS currency markets are now globally integrated, they are likely to be affected by developments in each other's market.[3] For investors, less international correlation between currency market returns mean that investors may reduce currency portfolio risk more by diversifying internationally instead of wholly investing in the domestic currency market. Since the level of gains from international diversification to reduce risk depends on the international *correlation structure*, the paper provides empirical estimates. The correlation structure between currency returns is widely used in finance and financial management, to establish efficient frontiers of portfolio currency holdings. The paper provides time-varying (*dynamic*) conditional correlation estimates of BRICS currency market returns. The fact that currency markets are related, there is likely to volatility across such markets.[4] To account for such effects, the multivariate model estimates a measure of conditional volatility. Thus, we employ a multivariate *t*-DCC model for conditional correlations in returns and conditional volatility.

Table 1 presents summary statistics of standardized daily returns (%) and devolatized daily returns (%). For the non-devolatized returns, the results show excessive kurtosis with the real, the renminbi, and rand values closer to 3, the value for the Gaussian (normal) distribution. However, devolatized returns do not show excess kurtosis of similar magnitude to standardized returns. We also note that the means and standard deviations (SD) of devolatized returns lie between 0 and 1. It is clear that devolatized returns are successful in achieving near Gaussianity. This means that the estimation of correlation and volatilities conditional on devolatized returns are likely to be more meaningful when we employ a multivariate *t*-distribution rather than the standard multivariate normal distribution. In the paper, the models used are written as the *t*-DCC and *normal*-DCC models respectively. For the *t*-DCC model, we estimate an unrestricted DCC (1, 1) model with asset-specific volatility parameters $\lambda_1 = (\lambda_{11}, \ldots\ldots, \lambda_{15}$ and $\lambda_2 = (\lambda_{21}, \ldots\ldots, \lambda_{25}$ and conditional correlation parameters (ϕ_1 and ϕ_2) plus the term, v for the degrees of freedom, conditioned by the *t*-distribution.

Table 1: Summary Statistics for the Standard Returns (%) and Devolatized Returns (%) from 01-Jan-2008 to 27- 13

Currencies	Standardized Daily Returns				Devolatized Daily Returns			
	Mean	SD	Skew	Ex-Kurt	Mean	SD	Skew	Ex-Kurt
Real (RBRA)	0.032	0.521	-0.020	2.942	0.035	0.998	-0.062	0.392
Ruble (RRRU)	0.018	0.644	0.018	1.947	0.006	0.992	-0.039	0.279
Rupee (RRUP)	0.025	0.501	0.012	1.898	-0.017	1.00	0.160	0.224
Renminbi(RREM)	-0.015	0.239	0.010	2.829	-0.003	0.979	0.260	-0.219
Rand (RZAR)	-0.028	0.391	0.030	7.422	-0.029	1.00	0.310	0.611

[3] In the paper, terms such as assets, currencies, and exchange rates are used interchangeably.
[4] Since volatility is a non-observable variable, it is usually proxied for in two ways: (a) using the square of daily equity returns (r^2_{it}) or (b) the standard error of intra-daily returns (realized volatilities) ($\sigma_{it}^{realized}$) as in (7) below.

Table 2 reports the correlation matrix of currency returns. All BRICS currency returns are positively related. Returns from the South African rand are highly correlated with Russian ruble returns (0. 67030); the Brazilian real with the South African rand (0. 63669) and Brazilian real with the Russia ruble (0. 56937). The Chinese renminbi is least correlated with the currencies of the other four countries that make up BRICS. The explanation may lie in the fact that during this period the Chinese currency was tightly regulated by the government. It remains to be seen whether these relationships can be captured by conditional correlations from the *t-DCC* model.

Table 2: Estimated Correlation Matrix of Exchange Rate Variables
1041 observations used for estimation from 02-Oct-09 to 27-Sep-13

	RBRA	RRUP	RREM	RZAR	RRRU
RBRA	1.0000				
RRUP	.42299	1.0000			
RREM	.11391	.27649	1.0000		
RZAR	.63669	.47100	.15725	1.0000	
RRRU	.56937	.46645	.16396	.67030	1.0000

Table 3 reports descriptive statistics for exchange rates for BRICS currencies. During this time, the renminbi exhibited the lowest volatility (as measured by the standard deviation) while the Brazilian real, the Russian ruble, the South African rand, and the Indian rupee all exhibit high volatility.

Table 3: Sample period: 1023 observations from 02-Oct-09 to 27-Sep-13

Variable(s)	Brazilian Real	Indian Rupee	Chinese Renminbi	South African Rand	Russian Ruble
Maximum	4.0257	2.5434	.41498	4.4367	.4933
Minimum	-3.6077	-2.0291	-.58581	-3.9848	-2.5998
Mean	.013508	.0098319	-.0082088	.011137	.0030058
Std. Deviation	.69426	.44236	.086973	.87826	.58467
Skewness	.17638	.066769	-.77731	.15934	.17429
Kurtosis - 3	4.7101	3.8685	6.9512	3.0627	3.0851
Coef of Variation	51.39	44.99	10.59	78.86	194.51

RBRA = rate of returns for the Brazilian Real; rrup = rate of returns for the Indian Rupee; rrem = rate of returns for the Chinese renminbi; rzar = rate of returns for the South African Rand, and RRRU = rate of returns for the Russian Ruble.

A few empirical results are noteworthy. First, our results indicate that the *t*-DCC model is preferred over the normal-DCC model in estimating conditional volatilities and correlations of exchange rates. Second, both $\hat{\pi}_N$ and $\hat{z}_\pi$ (tests of the validation of the *t*-DCC model) provide support for the *t*-DCC model despite the 2008 financial crisis. However, the model barely passes the non-parametric Kolmogorov-Smirnov (KS_N) test which tests whether probability transform estimates, $\hat{U}_t$ are uniformly distributed over the range (0, 1). Third, from Figures 1 and 2 it is clear that all currency returns correlations are positively related. Fourth, the model shows that around April 2010 and March 2012 there were sharp spikes in volatility.

In Figure 3 and 4, the rand experienced the highest spike. During this period, renminbi had the lowest volatility. Fifth, conditional correlations of ruble (in currency returns) fell during the financial crisis but picked up from July 2010. Finally, the rand-renminbi conditional correlation are positive but very low compared with the rand correlations with other currencies. It suggests that South African investors would have been better diversifying in the ruble, the rupee, and the real and that renminbi investors would not be investing in South Africa. However, these results should be treated with caution since exchange rates are affected by many other variables.

Table 4: Maximized log-likelihood Values of the *t-DCC Model* Estimated with Daily Returns over 01-Jan-2008 to 30-Dec-2011

Currencies	Standardized Daily Returns			Devolatized Daily Returns		
	Normal	t-distr.	df	Normal	t-distr.	df
All five currencies	-1200.5	-979.6	4.231 (0.2716)	-1073.3	-579.6819	2.7570 (0.2768)
The df is the estimated degrees of freedom and standard errors are given in round brackets.						

Table 4 presents the statistical significance of the multivariate t-distribution in the analysis of return volatilities. For BRICS currencies for standardized returns (z_{it} in (6)), the maximized log-likelihood is -1200.5 (normal distribution) and -979.5 (*t*-distribution). The maximized log-likelihood for devolatized returns ($\tilde{r}_{it}$ in (7)) are -1073.3 and -579.6819 for the *normal* and *t*-distributions respectively. Similarly, the estimated degrees of freedom are 4.2319 and 2.7570 respectively. These values are way below the value of 30 that would be expected for the multivariate normal distribution. The value of -979.5 is lower than -579.6819 for the *t*-distribution. Thus, the use of devolatized daily returns under the multivariate *t*-distribution is preferred and used in the paper.

The process of modeling conditional correlations across currency returns and conditional volatilities is a major function of currency portfolio managers and those tasked with reducing risks under the Value at Risk (VaR) strategies. If there is more than one currency in a portfolio, the use of multivariate models is often suggested. The returns to currencies are of five BRICS countries (Brazil, Russia, India, China and South Africa). This paper employs a $t-DCC$ model to estimate conditional volatilities and conditional currency returns.

The estimation of conditional volatilities and currency returns is achieved by the DCC (with time-varying correlation estimates) model by assuming a normal or Gaussian distribution of errors in the variance-covariance matrix Σ_{t-1}. [5]A major shortcoming of this approach is that the Gaussian assumption often fails in financial empirical analysis because of the fat-tailed nature of the distribution of returns. The simple dynamic conditional correlation model ($normal-DCC$) from [1] and [4] is based on a covariance-based method. This bears the risk of modeling bias but the assumed conditional Gaussian marginal distributions are not capable in mimicking the heavy-tails found in financial time series data observed in markets. Despite this shortcoming, [5] found that the conditional Gaussian distribution fits with V**a**R models with reasonable estimates.

The transformation of currency returns to Gaussianity is critical since correlation as a measure of dependence can be misleading in the presence of non-Gaussian currency returns as in (6) below. [3], [2] and Embrechts et al. [6] point out that for correlation to be useful as a measure of dependence, the transformation of currency returns should be made approximately Gaussian. The *t*-DCC model uses devolatized returns that very closely approximate Gaussianity. It is based on de-volatized returns as outlined in [3] and [2].

The literature on multivariate modelling is quite sizable as reviewed in [7] and [8]; the *Riskmetrics* from J.P. Morgan and others, and the multivariate generalized autoregressive conditional heteroscedastic specification (MGARCH) from [9].

The major innovation is the decomposition of the conditional covariance matrix to conditional volatilities and conditional cross-currency returns correlations ($\Sigma_{t-1} = D_{t-1}R_{t-1}D_{t-1}$, see (1) below) where D_{t-1} is a $m\,x\,m$ diagonal matrix of conditional volatilities while R_{t-1} is a symmetric $m\,x\,m$ correlation matrix. The returns to assets is represented by a vector r_t $(= m\,x\,1)$ at time t that have a conditional multivariate $t-$distribution with mean of μ_{t-1}, a non-singular variance-covariance matrix (Σ_{t-1}), and $v_{t-1} > 2$ degrees of freedom. The cross-currency returns are modelled in terms of a fewer number of unknown parameters which resolves *the curse of dimensionality*. The returns are standardized to achieve Gaussianity. [1] shows that with Gaussianity in innovations, the log-likelihood function of the *normal*-DCC model can be maximized in a two-step procedure. In step 1, m univariate GARCH models are estimated separately and step 2 uses the standardized residuals from step 1 to estimate conditional correlations (R_{t-1}).

Under this approach, [1]'s two-step procedure is no longer applicable to a *t–DCC* specification. Following [3] and [2] the obvious approach is to estimate simultaneously all the parameters of the model, including v , the degrees of freedom parameter. This approach solves the *curse of dimensionality* [1] and the absence of Gaussianity (by assuming a t distribution instead).

There is another strand of literature that focuses on volatility spillovers and correlations within a multivariate framework. [11] employed a multivariate stochastic volatility model on high frequency data of four USD exchange rates (the euro, the French franc, pound sterling, and the

[5] [10] introduced a Factor ARCH model to model the structure of the conditional variance matrix. The current paper employs a t-DCC model to examine the dynamic relationship in exchange rate returns and volatilities.

Canadian dollar). They found that the degree of persistence of exchange rates volatility and spillovers tends to change over time. McMillan and Speight (2010) [12] employed the realized variance method (instead of ARCH) to examine the nature and size of interdependence on the pound sterling, the yen and the US dollar. They found that the US dollar dominated the yen and the pound in returns and volatility.

[13] examined volatility spillovers in the deutsche mark (DM) exchange rates of three EMS and three non-EMS exchange rates using a multivariate exponential GARCH model. He found significant volatility spillovers among DM rates except for the yen (non-EMS currency) before German unification. [14] examined volatility spillovers of the DM/$ and ¥/$ exchange rates across regional markets. They found evidence of significant intra- and inter-regional spillovers in these rates. [15] show that macroeconomic and political events do affect the local economy and also exert spillover effects to other markets and thus impact exchange rates. [16] studied Granger causality in-mean and in-variance between the DM and ¥. He found simultaneous causality in-mean interaction and causality in-variance between these two currencies.

[17] used a two-step multivariate GARCH model to examine volatility spillover in various exchange rates relative to the Indian rupee. He found that volatilities in the exchange rate of leading currencies causes volatility in the exchange rate of the rupee. [18] used daily exchange rates of the Canadian dollar, the DM, the French franc, Italian lira, pound sterling and the yen relative to the USD ($) to examine the presence of a long-run volatility trend and volatility spillovers among exchange rates. They found the existence of a long-run trend and volatility spillovers in all European currencies except in the yen. [19] focused on the dynamic nature of returns, volatility, and correlation transmission mechanism among Indian exchange rates relative to the dollar, pound sterling, the euro and the yen. He found time-varying conditional correlations between exchange rate changes overtime with higher volatilities during times of global crises for all USD rates and other exchange rate pairs.

According to [21], the data on financial series (currencies in this case) share some commonalities such as heteroscedasticity; the variation and clustering of volatility over time, and autocorrelation. To the extent that financial volatilities tend to move together over time and across currency markets (clustering) the relevant model is the multivariate modelling framework with estimates that improve decision-making in areas such as portfolio selection, option pricing, hedging, risk management, and currency pricing.

[20] and [22] modified [1]'s DCC model by basing it on the stochastic process of the conditional correlation matrix on *devolatized* residuals rather than on *standardized* residuals. Standardized residuals are obtained by dividing residuals by the conditional standard deviations from the a first-stage GARCH (p, q) model, while devolatized residuals are found by dividing residuals by the square root of the *k*-day moving average of squared residuals.

The paper is organized as follows. Section 2 presents the $t-DCC$ used to provide estimates of conditional volatilities and currency returns using devolatized currency returns. Section 3 offers a brief discussion on recursive relations for real time analysis. Section 4 details the maximum likelihood (ML) estimation of the *normal*-DCC and *t*-DCC model. Section 5 presents VaR diagnostics such as tests of serial correlation and uniform distributions. Section 6 is the empirical application to devolatilized returns. Section 7 presents ML estimates of the *t*-DCC models in subsections: (a) currency-specific estimates; (b) post-estimation evaluation of the *t*-DCC model, and (c) recursive estimates and the V**a**R diagnostics. Section 8 presents the evolution of currency return volatilities and correlations. Section 9 concludes.

2 The t-DCC model

2.1. *t-DCC* Model or Modelling dynamic conditional volatilities and correlations of currency returns

We use currency returns which are standardized by realized volatilities (7) rather than GARCH (1, 1) volatilities (6). Returns in (7) are more likely to be approximately Gaussian than standardized returns [23] and [24]. Since we employ daily data and have no access to intra-daily data, we follow [3] and [2] in getting an estimate of σ_{it} that uses contemporaneous daily returns and their lagged values as in [25]. Earlier [26] presented a generalization of the ARCH methodology from [27]. The main contribution was to allow for past conditional variances in a current conditional variance equation. The $t-DCC$ estimation is applied to five currencies over the period 01-January 2008 to 27-September 2013. The sample is split into an estimation sample (2008 to 2011) and an evaluation sample (2012 to 2013). The results show a strong rejection of the $normal-DCC$ model in favor of the $t-DCC$ model (partly based on the log-likelihood for the normal distribution is -1073.3 (Tables 4) while that of the t-DCC model is -579.6819 (Tables 5)). When subjected to a series of diagnostic tests, it passes a number of VaR tests over the evaluation sample. The data comes from the IMF, International Financial Statistics for all the years.

We now offer a *t*-DCC model as formulated by [3] and [2] and [22] [from the work by [25] and [1].

$$\Sigma_{t-1} = D_{t-1} R_{t-1} D_{t-1} \tag{1}$$

where

$$D_{t-1} = \begin{pmatrix} \sigma_{1,t-1}.......... & & & \\ ....\sigma_{2,t-1}....0 & & & \\ ....0.....\sigma_{3,t-1}.... & & & \\ & & & \sigma_{m,t-1} \end{pmatrix}$$

$$R_{t-1} = \begin{pmatrix} 1 & \rho_{12,t-1} & \rho_{13,t-1} & \cdots & \rho_{1m,t-1} \\ \rho_{21,t-1} & 1 & \rho_{23,t-1} & \cdots & \rho_{2m,t-1} \\ . & & . & & . \\ . & & & . & . \\ . & & & \ddots & . \\ \vdots & \ldots & \ldots & & \rho_{m-1,m,t-1} \\ \rho_{m,1,t-1} & \ldots & & \rho_{m,m-1,t-1} & 1 \end{pmatrix}$$

$R_{t-1} = (\rho_{ij,t-1}) = (\rho_{ji,t-1})$ is the symmetric $m\,x\,m$ correlation matrix and D_{t-1} is $m\,x\,m$ diagonal matrix with $\sigma_{i,t-1}$, $\mathrm{i} = 1,2,\ldots,\mathrm{m}$ representing the conditional volatility of the i-*th* currency return. That is, $\sigma^2_{i,t-1} = V(r_{it} \mid \Omega_{t-1})$ and conditional pair-wise currency return correlations are represented by

$\rho_{ij,t-1} = \dfrac{Cov(r_{it}, r_{jt} \mid \Omega_{t-1})}{\sigma_{i,t-1}\sigma_{j,t-1}}$ where Ω_{t-1} is the information set available at $t-1$. Note that when for $i = j$, $\rho_{ij,t-1} = 1.$

[25] considered a correlation matrix where $R_{t-1} = R$ which defines a constant correlation matrix (CCC) while [1] allows R_{t-1} to be time-varying, suggesting a class of multivariate models known as the dynamic conditional correlation models (DCC). [25]'s multivariate GARCH model assumes that the one-step ahead conditional correlations are constant. [1] relaxed the assumption of constant conditional correlation of the CCC model of [25]. The conditional variances of individual currency returns are estimated as univariate GARCH (p, q) specifications, and the diagonal matrix is formed with their square roots. [28] generalized the DCC model by allowing for the possibility of asymmetric effects on conditional variances and correlations. [6] The decomposition of the variance-covariance matric Σ_{t-1} is critical to the estimation of conditional volatilities and correlation. That is, Σ_{t-1} allows for the separate specification of conditional volatilities and conditional cross-currency returns correlations. One uses the GARCH (1, 1) to model $\sigma^2_{i,t-1}$ as

$$V(r_{it} \mid \Omega_{t-1}) = \sigma_{i,t-1}{}^2 = \bar{\sigma}_i{}^2(1 - \lambda_{1i} - \lambda_{2i}) + \lambda_{1i}\sigma^2{}_{i,t-2} + \lambda_{2i}r^2{}_{i,t-1} \tag{2}$$

where $\bar{\sigma}_i{}^2$ is the **un**conditional variance of the of the i-*th* currency return. In the event that $\lambda_{1i} + \lambda_{2i} = 1,$ the **un**conditional variance ceases to exist in which case we have an integrated

[6] [29] proposed an alternative model which uses a conditionally heteroscedastic model where unobserved common factors are assumed to be heteroskedastic and assumes that the number of common factors are less than the number of currencies.

GARCH (IGARCH) model that is heavily used by finance practitioners and the model is similar to the "exponential smoother" as applied to $r_{it}^{2'} s^2$. That is,

$$\sigma^2_{i,t-1}(\lambda_i) = (1-\lambda_i)\sum_{s=1}^{\infty}\lambda_i^{s-1} r^2_{i,t-s}, \qquad 0<\lambda_i<1, \text{ Or} \tag{3}$$

In recursive form,

$$\sigma^2_{i,t-1}(\lambda_i) = \lambda_i \sigma^2_{i,t-2} + (1-\lambda_i) r^2_{i,t-1} \tag{4}$$

[1] suggested that cross-currency correlations estimates can use the following exponential smoother applied to "standardized returns" to obtain Gaussianity.

$$\hat{\rho}_{ij,t-1}(\phi) = \frac{\sum_{s=1}^{\infty}\phi^{s-1} z_{i,t-s} z_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty}\phi^{s-1} z^2_{i,t-s}}\sqrt{\sum_{s=1}^{\infty}\phi^{s-1} z^2_{\mathrm{j},t-s}}} \tag{5}$$

The standardized returns are represented by

$$z_{it} = \frac{r_{it}}{\sigma_{i,t-1}(\lambda_i)} \tag{6}$$

The unknown parameters that must be estimated are given by $\lambda_1, \lambda_2, \ldots\ldots, \lambda_m$, and ϕ which have been subject to [1]'s two-step procedure. The first stage involves fitting a GARCH (1, 1) model separately to m assets. The second step estimates the coefficient of conditional correlations, ϕ by Maximum Likelihood (ML) methods assuming that currency returns are conditionally Gaussian. However, [3] and [2] point to two major disadvantages of the two-step procedure. First, the normality assumption never holds in daily or weekly returns and it has a tendency to under-estimate portfolio risk.[7] Second, without Gaussianity, the two-step procedure is inefficient.

2.2. Pair-wise correlations based on realized volatilities

[3], [2], and [22] base the specification of cross correlation of volatilities on devolatized returns defined by (7) below. Suppose the realized volatility ($\sigma_{it}^{realized}$) of the i-*th* currency return in day t is defined as standard returns (r_{it}) divided by realized volatilities ($\sigma_{it}^{realized}$) to yield

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{realized}} \tag{7}$$

In (7), devolatilized returns are $\tilde{r}_{it}$ while in (6), standardized returns are represented by z_{it}. Hence, the conditional pair-wise return correlations based on devolatized asset returns is given by

[7] The use of daily data has its cost. For example, there is no accounting for the non-synchronization of daily returns across asset markets in different time zones. The use of weekly or monthly data deals with this issue.

$$\tilde{\rho}_{ij,t-1}(\phi)=\frac{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}_{i,t-s}\tilde{r}_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}^{2}_{i,t-s}}\sqrt{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}^{2}_{j,t-s}}}, \text{ such that } -1<\tilde{\rho}_{ij,t-1}(\phi)<1 \text{ for all values of } |\phi|<1. \quad (8)$$

[3] and [2] offer an alternative formulation of $\rho_{ij,t-1}$ that makes use of realized volatilities as in (8). There is empirical support for this approach that daily returns on foreign exchange assets and currency market returns standardized by realized volatility are approximately Gaussian [23] and [24].

Since we do not have intraday data for the assets examined here, we provide a simple estimate of σ_{it} based on daily returns that take into account all contemporaneous values of $\tilde{r}_{it}$.

$$\tilde{\sigma}^2_{it}(p)=\frac{\sum_{s=0}^{p-1}r^2{}_{i,t-s}}{p} \quad (9a)$$

where p is the lag order which should be chosen very carefully. [2], [3] emphasize that the choice of p is critical since the chosen value must be such that it transforms r_{it} into a Gaussian process. The non-Gaussian behavior found in daily returns is mainly due to jumps in the return process for many markets as reported in [21], [2], and [30]. A choice of p well above 20 does not allow for possible jumps in data to be adequately reflected in $\tilde{\sigma}^2_{it}(p)$, while values of p well below makes r_{it} to behave as an indicator-type looking function (Stavroyiannis et al. (2013)). [21], [2], [3], and [22] note that $\tilde{\sigma}^2_{it}(p)$ is not equivalent to the standard rolling historical estimate of σ_{it} given by

That is, $\tilde{\sigma}^2_{it}(p)$ - $\hat{\sigma}^2_{it}(p)$ = $\dfrac{r_{it}{}^2-r^2{}_{i,t-p}}{p}$ (9b)

when implementing real time analysis, as in recursive formulae augments used in the estimation and evaluation process. It seems that the inclusion of current squared returns $r_{it}{}^2$ (in 9a) in the estimation of $\tilde{\sigma}^2_{it}$ is important in transforming non-Gaussian returns r_{it} into Gaussian $\tilde{r}_{it}$ returns.

3 Recursive relations for real time analysis

The computation of $\rho_{ij,t-1}$ in (5) and (8) as noted by [1] is given by

$$\tilde{\rho}_{ij,t-1}(\phi)=\frac{q_{ij,t-1}}{\sqrt{q_{ii,t-1}q_{jj,t-1}}} \quad (10)$$

where

$$q_{ij,t-1} = \phi q_{ij,t-2} + (1-\phi)\tilde{r}_{i,t-1}\tilde{r}_{j,t-1} \quad (11)$$

It is important to note that $\tilde{\rho}_{ij,t-1}$ is positive definite as the covariance of a typical element of the matrix $q_{ij,t-1}$ is a positive definite. The recursive formula for $\tilde{\rho}_{ij,t-1}(\phi)$ is the same as in (5) except that (10) uses devolatized returns while (5) uses standardized returns (z_{it}). We note that in the above models for pair-wise correlations, $\rho_{ij,t-1}$, these are non-mean reverting. The general specification for pair-wise correlations is given by

$$q_{ij,t-1} = \bar{\rho}_{ij}(1-\phi_1-\phi_2) + \phi_1 q_{ij,t-2} + \phi_2 \tilde{r}_{i,t-1}\tilde{r}_{j,t-1} \quad (12)$$

where $\bar{\rho}_{ij}$ is the **un**conditional correlation of r_{it} and r_{jt} with the restriction that $\phi_1+\phi_2<1$ (mean reversion). There is an expectation that $\phi_1+\phi_2$ will be very close to one. The non-reverting mean case is a special case of $\phi_1+\phi_2=1$. However, it is not possible to be certain that $\phi_1+\phi_2<1$ or not. On the other hand, it is possible to estimate **un**conditional correlations, $\bar{\rho}_{ij}$ by using an expanding window. In the empirical part of the paper, we consider both; the mean reverting and non-mean reverting cases and compare two specifications of conditional correlations using standardized and devolatized returns.

With m daily currency returns in the $m\,x1$ vector, r_t over period $t=1,2,\ldots\ldots T$, $T+1,\ldots\ldots,T+N$, we use the first T_0 observations to calculate (9a) to start the initialization recursive in (12) and obtain estimates of $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}$ in (2) and (12) respectively. Suppose s is the starting point of the recent sample of observations for estimation within the estimation sample (2008 to 2011)[Evaluation sample]. Then it follows that $T>s>T_0>\rho$ where ρ is the size of estimation window so that the estimation window is, $T_e = T-s+1$. Thus, the remaining observations, N (2012 to 2013) can be used for evaluating the *t*-DCC model. Thus, the whole sample equals S_e+S_{ev}. With a rolling window of size w, then $s=T+1-w$ so that the whole estimation can be moved into the future with an update frequency of h.

3.1. Mean-Reverting Conditional Correlations

For the mean-reverting case, we need estimates of the unconditional volatilities and correlation coefficients from (13) and (14) below.

$$\bar{\sigma}_{i,t}^2 = \frac{\sum_{\tau=1}^{r} r^2{}_{i\tau}}{t} \quad (13)$$

$$\bar{\rho}_{ij,t} = \frac{\sum_{\tau=1}^{t} r_{i\tau} r_{j\tau}}{\sqrt{\sum_{\tau=1}^{t} r^2{}_{i\tau}}\sqrt{\sum_{\tau=1}^{t} r^2{}_{j\tau}}}, \quad (14)$$

The index t represents the end of available estimation sample which may be recursively rolling or expanding Pesaran and Pesaran [2], [3], and [21].

4 Maximum Likelihood Estimation of the *normal*-DCC and the *t*-DCC Model

In the non-mean reverting specifications, (2) and (12), the *t*-DCC model has $2m + 3$ unknown parameters made up of $2m$ coefficients $\lambda_1 = (\lambda_{11}, \lambda_{12}, \ldots\ldots, \lambda_{1m})'$ and $\lambda_2 = (\lambda_{21}, \lambda_{22}, \ldots\ldots, \lambda_{2m})'$ that enter the individual currency returns volatilities, and the two coefficients ϕ_1 and ϕ_2 that enter conditional correlations plus the degrees of freedom (v) of the multivariate t distribution.

Following [20], for testing that one of the currency returns has non-mean reverting volatility, let λ_{i1} and λ_{i2} be parameters for the conditional volatility equation of the $i-th$ currency, the relevant test is

$H_0 : \lambda_{i1} + \lambda_{i2} = 1$ against $H_A : \lambda_{i1} + \lambda_{i2} < 1$

Under H_0, the process is non-mean reverting and the unconditional variance for the currency does not exist. In (2) and (12), parameters $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}^2$ are unconditional volatilities and return correlations and could be estimated using the initialization sample (13) and (14). In the non-mean reverting case, the intercepts in (2) and (12) cease to exist.

Suppose we denote the unknown coefficients as follows.

$\theta = (\lambda_1, \lambda_2, \phi_1, \phi_2, v)`$

Given a sample of observations on returns, $r_1, r_2, \ldots\ldots, r_t$ available at time t, the t-log-likelihood function based on decomposing (1) is given by

$$l_t(\theta) = \sum_{\tau=s}^{t} f_\tau(\theta), \tag{15}$$

where $s < t$ is beginning date for the estimation window.[8] With the $t-DCC$ model, $f_\tau(\theta)$ is the density of the multivariate distribution with v degrees of freedom that can be written in terms of $\Sigma_{t-1} = D_{t-1}R_{t-1}D_{t-1}$ as

$$\begin{aligned} f_\tau(\theta) &= \frac{m}{2}\ln(\pi) - \frac{1}{2}\ln|\mathrm{R}_{\tau-1}(\theta)| - \ln|\mathrm{D}_{\tau-1}(\lambda_1,\lambda_2)| + \ln[\Gamma(\frac{m+v}{2})/\Gamma(\frac{v}{2})] \\ &- \frac{m}{2}\ln(v-2) - (\frac{m+v}{2})\ln[1 + \frac{e_\tau'\mathrm{D}^{-1}{}_{\tau-1}(\lambda_1,\lambda_2)R^{-1}{}_{\tau-1}(\theta)\,\mathrm{D}^{-1}{}_{\tau-1}(\lambda_1,\lambda_2)e_\tau}{v-2}] \end{aligned} \tag{16}$$

[8] There is no need to write out the log- likelihood function for a normal distribution since it is only estimated here to show that the results from *t*-DCC are preferred to those from the *normal*-DCC model.

where $e_\tau = r_\tau - \mu_{\tau-1}$ and

$$\ln|D_{\tau-1}(\lambda_1,\lambda_2)| = \sum_{i=1}^{m}\ln[\sigma_{i,\tau-1}(\lambda_{1i,}\lambda_{2i})] \tag{17}$$

As pointed out by [2] and [3] and in surveys by [7] and [8], the multivariate *t* - density is usually written in terms of a scale matrix. However, if we assume that $v > 2,$ then it means that Σ_{t-1} exists to permit the scale matrix to be written in terms of Σ_{t-1}. In [1], R_{t-1} depends on λ_1 and λ_2 in addition to ϕ_1 and ϕ_2 (based on standardized returns) but the specification here is based on devolatilized returns has R_{t-1} depending only on ϕ_1 and ϕ_2 plus the p -the lag order that is used in the devolatization process. The ML estimate of ϕ based on sample observations $r_1, r_2, \ldots\ldots, r_t$ are computable by maximizing $l_t(\theta)$ with respect to ϕ represented by $\hat{\theta}_t$ or simply as

$$\hat{\theta}_t = Arg\max_{\theta}\{l_t(\theta)\},\ \text{for } t = T, T+h, T+2h, \ldots, T+N, \tag{18}$$

where $h,$ the estimation is update frequency and N is the length of the evaluation sample. Note that the standard errors of ML estimates are calculated from the following asymptotic expression.

$$Co\hat{v}(\hat{\theta}_t) = \{\sum_{\tau=s}^{t}[\frac{-\partial^2 f_\tau(\theta)}{\partial\theta\partial\theta'}]_{\theta=\theta_t}\}^{-1}$$

The model is reasonable to estimate in that the number of unknown coefficients of the MGARCH model increases as a *quadratic* function of m while in the standard DCC model, it rises *linearly* with m assets. This fact notwithstanding, the simultaneous estimation of all parameters of the DCC model can and do often gives rise to convergence problems or to a local maxima of the likelihood function $l_t(\theta)$. However, if the standard returns are conditionally Gaussian, it is possible to resort to [1] 's two-stage estimation, albeit with some loss in estimation efficiency. In the multivariate t-distribution adopted here, the degrees of freedom (v) is the same across all currency returns whereas under the two-stage estimation procedure, separate $t-GARCH(1,1)$ can easily lead to different estimates of v [9].

5 Diagnostic Tests of the t-*DCC* Model

Suppose one has a portfolio with m assets with r_t as a *vector* of returns with $m\,x\,1$ *vector* of predetermined weights w_{t-1}. The returns to such a portfolio would be

[9] [3], [2], and [20] note that the marginal distributions found in a multivariate $t-$distribution with v are also $t-$distributed with the same v.

$$\rho_t = w'_{t-1} r_t \tag{19}$$

If the interest is calculating the capital **V**alue **a**t **R**isk (VaR) of a portfolio at *t-1* with probability ($1-\alpha$) represented by $VaR(w_{t-1}, \alpha)$, this requires that

$$\Pr[w'_{t-1} r_t < -VaR(w_{t-1}, \alpha) \mid \Omega_{t-1} \mid] \le \alpha$$

Under these assumptions, the conditional on Ω_{t-1}, then (currency returns) $w'_{t-1} r_t$ or ρ have a Student $t-$ distribution with mean of $w'_{t-1} \mu_{t-1}$ and variance $w'_{t-1} \Sigma_{t-1} w_{t-1}$ with v degrees of freedom. Thus,

$$z_t = \sqrt{\frac{v}{v-2}} \left(\frac{w'_{t-1} r_t - w'_{t-1} \mu_{t-1}}{\sqrt{w'_{t-1} \Sigma_{t-1} w_{t-1}}} \right)$$

which is conditional on Ω_{t-1} and also has a $t-$ distribution with v degrees of freedom with mean $E(z_t \mid \Omega_{t-1}) = 0$ and $Var(z_t \mid \Omega_{t-1}) = v/v-2.$ With the cumulative distribution function (CDF) of a Student t with v degrees of freedom represented by $F_v(z)$, the $VaR(w_{t-1}, \alpha)$ is the solution to

$$F_v = \left(\frac{-VaR(w_{t-1}, \alpha) - w'_{t-1} \mu_{t-1}}{\sqrt{\frac{v-2}{2} (w'_{t-1} \Sigma_{t-1} w_{t-1})}} \right) \le \alpha$$

However, since $F_v(z)$ is a continuous and monotonic function of z , then

$$\left(\frac{-VaR(w_{t-1}, \alpha) - w'_{t-1} \mu_{t-1}}{\sqrt{\frac{v-2}{2} (w'_{t-1} \Sigma_{t-1} w_{t-1})}} \right) = F_v^{-1}(\alpha) = -c_\alpha$$

where c_α is a $\alpha\%$ critical value from the Student t-distribution with v degrees of freedom. The out-of-sample VaR forecast puts $\alpha = 0.99.$ Thus,

$$VaR(w_{t-1}, \alpha) = \tilde{c}_\alpha \sqrt{w'_{t-1}} \Sigma_{t-1} w_{t-1} - w'_{t-1} \mu_{t-1} \tag{20}$$

where $\tilde{c}_\alpha = c_\alpha \sqrt{\frac{v-2}{v}}$

Following [21], [2], [22], [31], and [32], the test of the validity of the $t-DCC$ is calculated recursively by using the V**a**R indicators denoted by (d_t)

$$d_t = I(w'_{t-1} r_t + VaR(w_{t-1}, \alpha)) \tag{21}$$

where $I(B)$ an indicator function that is equal to 1 if $B > 0$ and zero otherwise. The indicator statistics can be computed in-sample or preferably based on recursive out-of-sample one-step ahead forecasts of Σ_{t-1} and μ_{t-1} for pre-determined preferred set of portfolio weights w_{t-1} . In an out-of-sample exercise, the parameters of the mean returns variables (θ) and volatility variables (λ) can be fixed at the start of the evaluation exercise or changed with an update frequency of h periods. Suppose we an evaluation sample, $S_{eval} = \{r_t, t = T+1, T+2, \ldots\ldots, T+N\}$ then the mean hit rate [MHR] is

$$\hat{\pi}_N = \frac{1}{N}\sum_{t=T+1}^{T+N} d_t \tag{22}$$

With a $t-DCC$ model, the estimated mean hit rate, $\hat{\pi}_N$, has a mean of ($1-\alpha$) and variance ($\frac{\alpha(1-\alpha)}{N}$) and the resulting *standardized statistic* is

$$z_\pi = \frac{\sqrt{N[\hat{\pi}-(1-\alpha)]}}{\sqrt{\alpha(1-\alpha)}} \tag{23}$$

This expression has a standard normal distribution if the evaluation sample size N (in our case, 455 observations) is very large. According to [33], [30], [3], and [2], the z_π statistic provides evidence of the performance of Σ_{t-1} and μ_{t-1} in an average **un**conditional setting. On the other hand, [34] has suggested an alternative conditional evaluation procedure based on probability integral transforms.

$$\hat{U}_t = F_v\left(\frac{w'_{t-1} r_t - w'_{t-1}\hat{\mu}_{t-1}}{\sqrt{\frac{v-2}{v} w'_{t-1}\hat{\Sigma}_{t-1} w_{t-1}}}\right), \quad t = T+1, T+2, \ldots\ldots, T+N \tag{24}$$

If the $t-DCC$ model is correctly specified, under the null hypothesis the probability integral transforms estimates (PIT), $\hat{U}_t$ should be not be serially correlated and should have a uniform distribution over the range (0,1) and it is testable. The serial correlation property of $\hat{U}_t$ can be tested by Lagrange multiplier tests by running OLS of $\hat{U}_t$ on an intercept and lagged values of $\hat{U}_{t-1}, \hat{U}_{t-2}, \ldots\ldots, \hat{U}_{t-s}$ [see Table 8]. In this case, the maximum lag length, s can be determined by the AIC information criteria. The uniform distribution of $\hat{U}_t$ over t can be tested using the Kolmogorov-Smirnov (KS_N) statistic defined as $KS_N = \sup_x |F_{\hat{U}}(x) - U(x)|$ where $F_{\hat{U}}(x)$ is the empirical cumulative distribution function (CDF) of $\hat{U}_t$ for $t = T+1,\ T+2, \ldots\ldots,\ T+N$ and $U(x) = x$ is the CDF of the *iid* U (0, 1). If the value of the KS_N statistic is large, it would show that the CDF is not similar to the uniform distribution assumed in the $t-DCC$.[10] However, if the

[10] For more details on the Kolmogorov-Smirnov test and critical values, see [35] and [36].

estimated value of KS_N is below the critical value (say 5%), then it does support the validity of the $t-DCC$.

6 Empirical Application to devolatized currency returns

The rate of return is calculated as follows. If the price of currency is S_t then the returns are defined as $\Delta S_t = \ln(\frac{S_t}{S_{t-1}})\,\text{x}100.$

The calculated rates of returns (standardized and devolatized) are presented in Tables 1. The estimation of the t-DCC model uses devolatized returns.

7 ML estimates of the *t-DCC* models

[22] and [2] point out that weekly or daily return approximately have mean zero serially uncorrelated processes which make it possible to assume that $\mu_{t-1} = 0$. The *t*-DCC model is estimated for 5 five BRICS currency returns over the period 01-Jan-2008 to 27-Sept-2013. The estimation period is 30-Jan-2008 to 30-Dec-2011 (1023 observations) and we use 455 observations (02-Jan-2012 to 27-Sept-2013) for the evaluation of estimated volatilities and correlations model. The V**a**R and distribution diagnostics are used to assess the results from the model. We estimated the unrestricted version of the DCC (1, 1) assuming a normal distribution (*normal*-DCC) with asset-specific volatility parameters $\lambda_1 = (\lambda_{11}, \lambda_{12}, \ldots\ldots, \lambda_{1m})'$ and $\lambda_2 = (\lambda_{21}, \lambda_{22}, \ldots\ldots, \lambda_{2m})'$ with common conditional correlations, ϕ_1 and ϕ_2. In the paper, $m = 5$ and there are no restrictions on decay factors (different volatility for each variable and same for the correlation decay factor). Table 5 presents the maximum likelihood estimates of λ_{i1} ,λ_{i2} for five currency returns and ϕ_1 and ϕ_2. We note that all the currency-specific returns are highly significant with their sum all close to unity. The log-likelihood value is -1073.3. This value is important since we will compare it to the log-likelihood value from the *t*-DCC model.

Table 5: Multivariate GARCH with underlying multivariate Normal distribution (*Normal* DCC-GARCH)

Converged after 42 iterations

Based on 1023 observations from 30-Jan-08 to 30-Dec-11.

The variables (asset returns) in the multivariate GARCH model are:

rbra rrup rrem rzar rrru

Volatility decay factors unrestricted, different for each variable.

Correlation decay factors are unrestricted to allow for mean-reverting conditional correlations.

	ML Estimates	
Currencies	λ_1	λ_2
Real (rbra)	0.92581(0.12987) {71.2851}[0.000]	0.067332(0.0111) {6.0180}[.000]

Ruble (rrru)	0.84929(0.032591) {26.0590}[.000]	0.12884(0.02605) {4.9459}[.000]
Rupee(rrup)	0.88330(0.033083) {26.6996}[.000]	0.098394(0.0251) {3.9114}[.000]
China(rrem)	0.89265(0.012149) {73.4741}[.000]	0.085836(0.0095) {8.9797}[.000]
Rand(rzar)	0.92845(0.018059) {51.4123}[.000]	0.062874(0.0144) {4.3468}[.000]
Correlation parameters $\hat{\phi}_1 = 0.96340(0.002170)$ and $\hat{\phi}_2 = 0.024602(0.001013)$ are such that $1-\hat{\phi}_1-\hat{\phi}_2 = 0.01020(0.001281)$		
The maximized log-likelihood = -1073.3; the standard errors are given in round brackets (.); the t-ratio is given in {.} brackets and probability value is given in square brackets [.].		

Table 6: Multivariate GARCH with underlying multivariate t-distribution (*t-DCC* GARCH Model)

Converged after 39 iterations

Based on 1023 observations from 30-Jan-08 to 30-Dec-11.
The variables (asset returns) in the multivariate GARCH model are:
rbra rrup rrem rzar rrru
Volatility decay factors unrestricted, different for each variable.
Correlation decay factors are unrestricted to allow for mean-reverting conditional correlations.

ML Estimates		
Currencies	λ_1	λ_2
Real (rbra)	0.94828 (0.012427) {76.3066}[.000]	0.048880(0.011003) {4.4422}[.000]
Ruble (rrru)	0.88197(0.028565) {30.8763}[.000]	0.10076(0.022324) {4.5135}[.000]
Rupee(rrup)	0.98423(0.0089048) {110.5276}[.000]	0.01688(0.0072402) {2.3215}[.021]
China(rrem)	0.8027(0.029628) {27.0929}[.000]	0.15359(0.023038) {6.6666}[.000]
Rand(rzar)	0.98365(0.0080896) {121.5935}[.000]	0.015922(0.0064202) {2.4800}[.013]
$\hat{v} = 2.7570(0.4414)$, $\hat{\phi}_1 = 0.97896(0.0014)$, $\hat{\phi}_2 = 0.01104(0.0001)$		
The maximized log-likelihood = -579.6819; the standard errors are given in round brackets (.); the t-ratio is given in {.} brackets and probability value is given in square brackets [.]. Note that ϕ_1 and ϕ_2 are common conditional correlation parameters. Correlation parameters $\hat{\phi}_1 = 0.9789(0.0014)$ an $\hat{\phi}_2 = 0.0110(0.0001)$ are such that $1-\hat{\phi}_1 -\hat{\phi}_2$ = 0.0101(0.0007)		

Table 6 presents all the maximum likelihood estimates of λ_{i1} , λ_{i2} for five currency returns and ϕ_1 and ϕ_2 . We note that all the currency-specific returns are highly significant with their sum all close to unity. The log-likelihood value from the *t*-DCC model is -579.6819 and it is larger than the value from Table 5. The degrees of freedom are 2.7570, well below the value of 30 that is expected for a multivariate normal distribution. As a check on the results in Table 6, similar results were obtained when we estimated a *t*-DCC model on residuals obtained when a regression of currency returns is on returns on their past values (as in Table 8). As the in Table 5, specific-currency returns estimates of the volatility and correlation decay parameters are highly significant and close to 1.

Table 7: Testing for Mean Reversion of Volatility of BRICS currencies

Asset	$1-\hat{\lambda}_1-\hat{\lambda}_2$	Standard Errors	t-ratio [Prob]
Brazilian Real	0.0027112	0.0026958	1.0057[0.315]
Russian Ruble	0.0146990	0.0060821	2.4167[0.016]
Indian Rupee	-0.0014998	0.0012022	-1.2475[0.213]
South African Rand	-0.00001051	0.0010886	-0.0096571[0.992]
China renminbi	0.0000425	0.0000112	2.1847280[0.0115]

Table 7 presents tests for non-mean reversion. The sum of estimates of λ_{1i} and λ_{2i} are almost unit. The hypothesis that $H_0 : \lambda_{i1} + \lambda_{i2} = 1$ (Integrated GARCH) against mean reversion ($H_0 : \lambda_{i1} + \lambda_{i2} < 1$) is rejected for all five currencies. This means that BRICS currencies returns show significant mean-reverting volatility for all assets in these economies.

The evaluation sample from 02-Jan-2012 to 27-Sept-2013 tests is based on the probability integrals transform (PIT), $\hat{U}_t$ as defined by (24). If the *t-DCC* model is correctly specified, then under the null hypothesis, $\hat{U}_t$ has no serial correlation and it is uniformly distributed over (0, 1). $\hat{U}_t$ is obtained by considering an equal-weighted portfolio of all five BRICS currency returns as defined by (19) with a risk tolerance of $\alpha = 0.1$. To test the null hypothesis that $\hat{U}_t$ s are not serial correlated, we use the Lagrange Multiplier test. The value of the CHSQ (12) = 7.9685[.788] and the F Statistic = F (12,442) = 0.65657[.793] are reported in Table 8. Given these values, it is clear the *t-DCC* model specification passes the test.

Table 8: Test of Serial Correlation of Residuals (OLS case)

****

Dependent variable is U-Hat ($\hat{U}_t$)
List of variables in OLS regression:
Intercept
455 observations used for estimation from 02-Jan-12 to 27-Sep-13

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
OLS RES(-1)	.060893	.047528	1.2812[.201]
OLS RES(-2)	.022931	.047618	.48156[.630]
OLS RES(-3)	.0041046	.047593	.08624[.931]
OLS RES(-4)	.7783E-3	.047545	.01637[.987]
OLS RES(-5)	-.023368	.047539	-.4915 [.623]
OLS RES(-6)	-.040307	.047519	-.84822[.397]
OLS RES(-7)	-.038443	.047648	-.80681[.420]
OLS RES(-8)	-.032218	.047670	-.67586[.499]
OLS RES(-9)	.073504	.047695	1.5411[.124]
OLS RES(-10)	-.045262	.047909	-.94474[.345]
OLS RES(-11)	.0023313	.048028	.048542[.961]
OLS RES(-12)	-.040469	.047950	-.84398[.399]

Lagrange Multiplier Statistic CHSQ (12) = 7.9685[.788]
F Statistic F (12,442) = .65657[.793]
U-Hat denotes the probability integral transform.

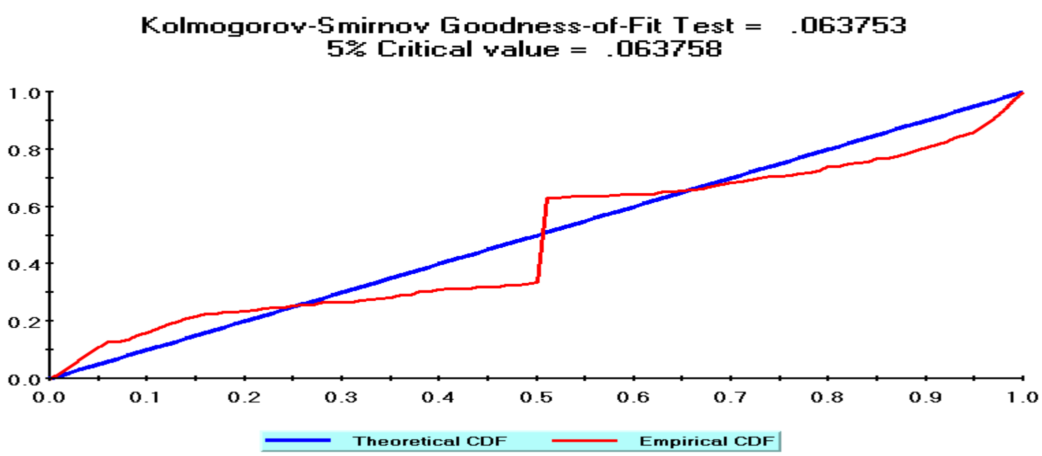

Figure 1: The Kolmogorov-Smirnov goodness-of-fit test for the full t-DCC model [Evaluation Period]

Under the null hypothesis, U-Hat ($\hat{U}_t$) should not display any serial correlation.

In Figure 1, the Kolmogorov-Smirnov test (KS_N) is applied to $\hat{U}_t$ to determine whether the probability integrals transform (PIT) are from a uniform distribution. The value of the KS_N

statistic is 0.0663753 which is barely within the 5% critical value of 0.063758. This means that the null hypothesis that the sample's cumulative density function (CDF) is similar to the uniform distribution cannot be rejected, although barely. Figure 2 shows the histograms of the probability integral transform variable $\hat{U}_t$ with minor violations of a uniform distribution.

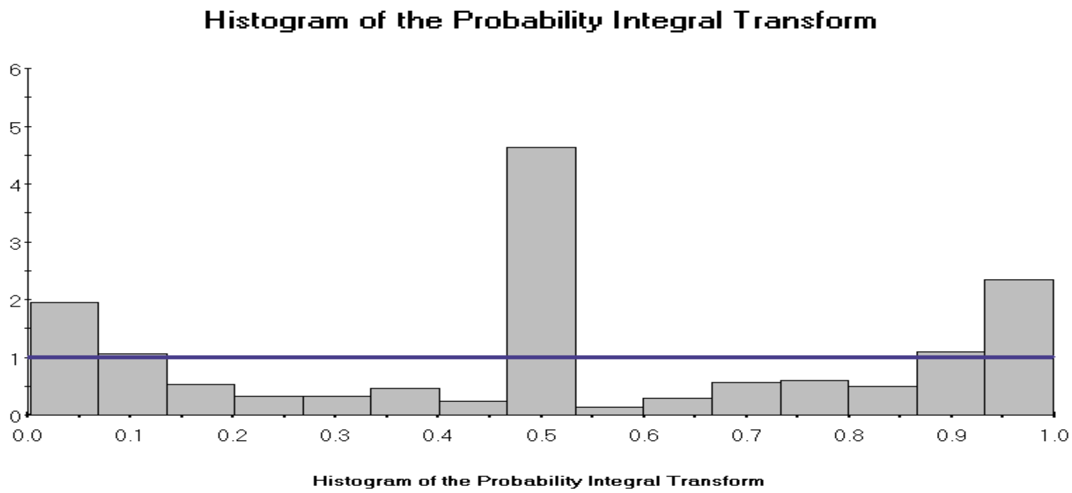

Figure 2: Histogram over the Estimation Period

In Figure 3, we test whether there is any violation of the Value at Risk (V**a**R) constraint as this test focuses on the tail properties of currency returns. With a tolerance probability of $\alpha = 0.01$, Figure 3 shows the risks in these emerging markets shows spikes around June - July 2012, April-May 2013, and September 2013 when the U.S. Federal Reserve Bank indicated that it might begin reducing liquidity (quantitative easing). This announcement sent shock waves in the emerging markets (BRICS) as the U.S. dollar market looked better for investors.

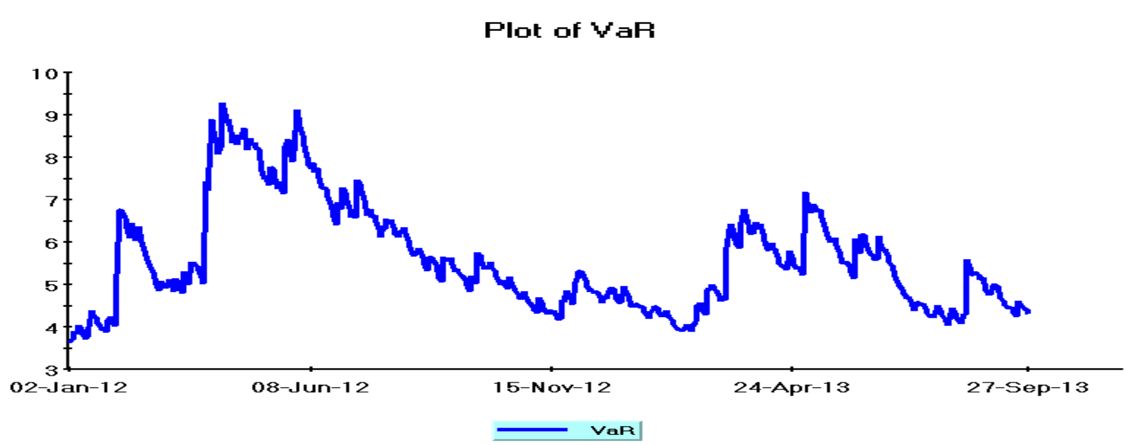

Figure 3: Value at Risk over the Evaluation Period

Table 9: Mean VaR Exceptions and the Associated Diagnostic Test Statistics

Mean Hit Rate ($\hat{\pi}_N$ statistic) =.98462 with expected value of .99000
Standard Normal Test Statistic ($\hat{z}_\pi$) = -1.1544[.248]

Note: Value at Risk for a given probability (0.01) using a 1-step ahead of forecasts of variances and covariance = **7.21** (=asset returns in %s) but 7.47 (using estimated variances and covariance)

In Table 9 there is an additional test of V**a**R violations under a tolerance probability of $\alpha = 0.01$. The $\hat{\pi}$ statistic has a value equal to 0.998 which is very close to its expected value of 0.990. Similarly, the F statistic is 1.1544 with a *p*-value of 0.248. These results provide support for the validity of *t*-DCC model. The $\hat{z}_\pi$ statistic is not significant at *p* = 0.248.

8 Evolution of Currency Return Volatilities and Correlations

The choice was made to present conditional correlations of the South African rand with other currencies to limit the excessive figures that would have been necessary for all currencies. In case of volatilities, the figures presented in the paper show volatilities for all currencies. In order to minimize the impact of initialization on the plots of conditional correlations, initial estimates for 2009 are not shown (Figure 4) and those for 2011 (Figure 5). The same precaution holds for volatilities in Figures 6 and 7. The conditional correlations for the Brazil real, Indian rupee, Russia ruble and South African rand in both the estimation and evaluation periods show high conditional correlation. There is a noticeable dip around 2011 followed by an upward trend. The exception is the rand-renminbi correlation which is lower than all others. It falls below 0.1 in April 2010. Thereafter, it also shows a noticeable upward trend until it coincides with other conditional correlations around September 2011. For the evaluation period, conditional correlations of the rand with the rupee, real, and ruble remain stable between 0.5 and 0.6. Meanwhile, the rand-renminbi correlation remains around 0.1 until October 2012 when it rises to about 0.2. The correlation matrix in Table 2 confirms the low conditional correlation of the rand and renminbi since it is only 0.15725.

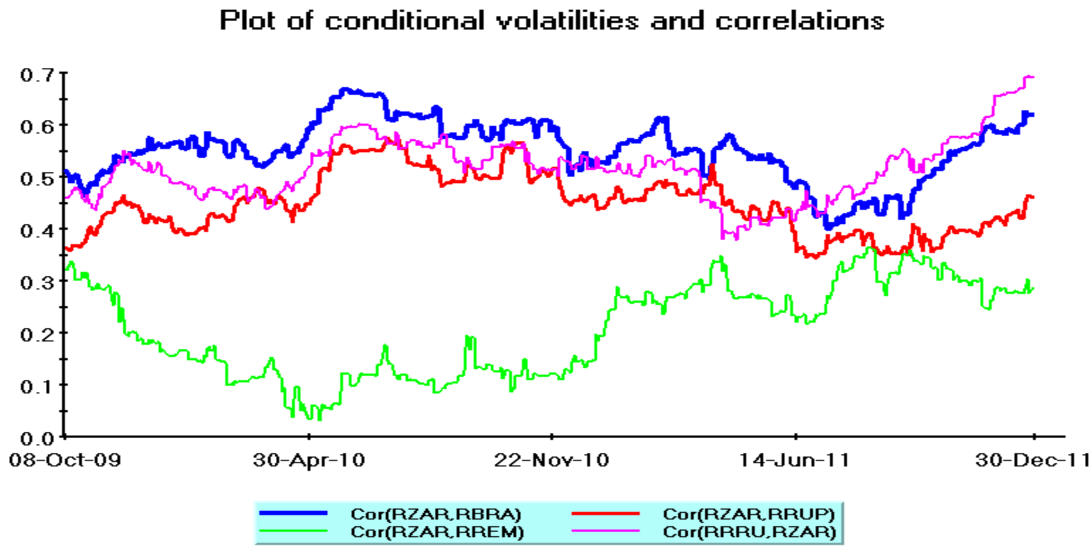

Figure 4: Conditional Correlations of the Rand with other Currencies [Estimation Period]

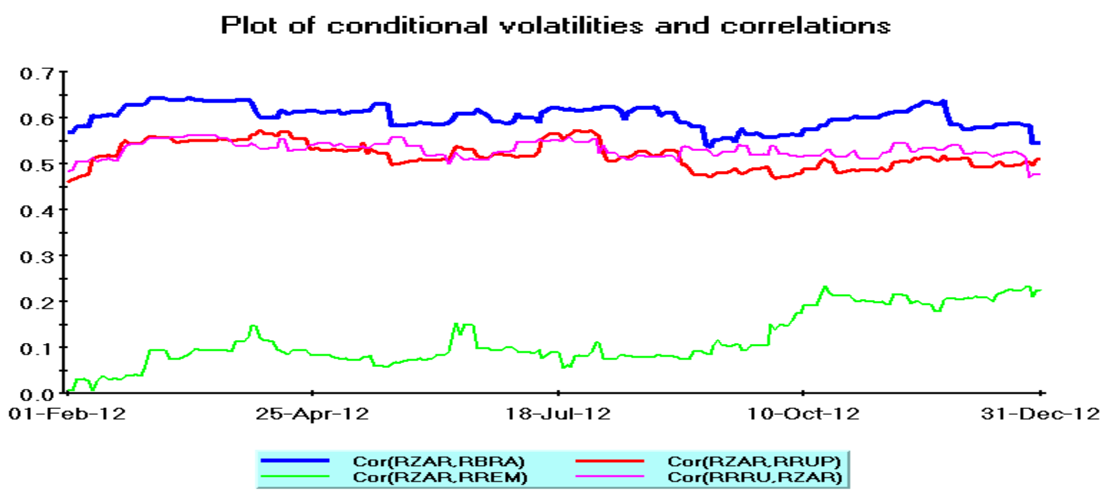

Figure 5: Conditional Correlations of the Rand with other Currencies [Evaluation Period]

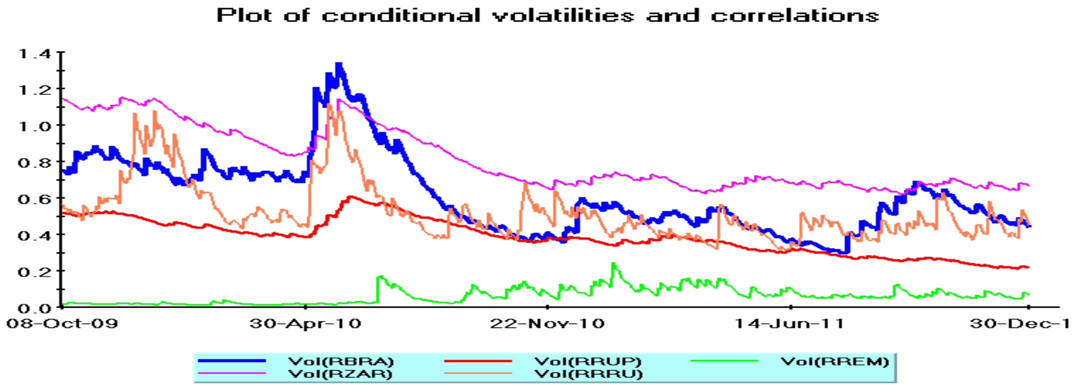

Figure 6: Volatility of the Rand with other Currencies [Estimation Period], 08-Oct-09 to 30-Dec-11

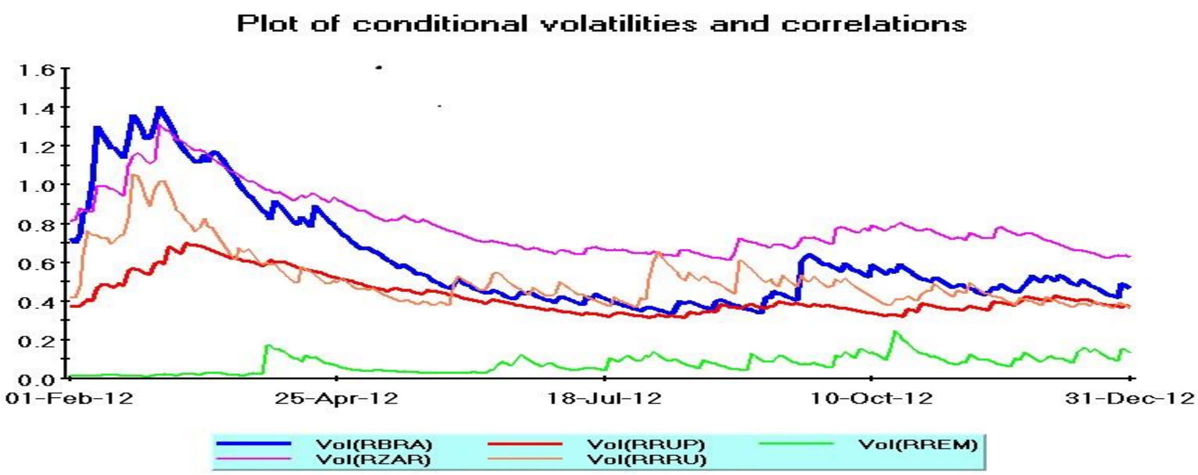

Figure 7: Volatility of the Rand with other Currencies [Evaluation Period], 01-Feb-12 to 31-Dec-11

In case of volatilities over the estimation period, Figure 6 shows the ruble to have a high spike around March 2010 when other currencies exhibit a downward trend. Over the whole estimation period, the rand had the highest volatility (again this is confirmed by Table 2) in contrast with the renminbi that exhibit the least volatility. All currencies show a huge spike around April 2010 at the height of the global financial crisis. After this spike, all currencies exhibit a declining trend (except the renminbi in 2011) until December 2011. For the evaluation period (Figure 7), all currencies exhibit a huge spike around March 2012 followed by a declining trend in volatilities until December 2012. Again, the exception is the renminbi whose volatility is practical zero except for a very small spike in in 2012 as China adjusted its exchange rate albeit by a very small margin.

The declining trends during both the estimation and evaluation periods in Figures 6 and 7 reflect a closer integration of the rand with the real, rupee, and ruble and less so with the renminbi. This result is surprising given public announcements by the South African when it acceded to the BRIC economies at the invitation of China. From the latest export figures, China is now the number one trading partner for South Africa. Fitting a *t*-DCC model on devolatized returns for five BRICS currencies yields estimates of volatilities λs that are very close to 1 in Table 6 except for the renminbi. In contrast, Table 5, the *normal*-DCC model has the renminbi showing higher volatility for the renminbi (λ=0.89265) versus (λ=0.80271) in the *t*-DCC model. In other words, the normal distribution would mask the low volatility of the renminbi. The use of the *t*-DCC model produces a low value for the renminbi that is consistent with all results obtained in this paper from tables, estimates and figures.

9 Concluding Remarks

The paper tested the idea that devolatized returns are a better approach to understanding the volatility of asset markets than standardized returns so widely used in portfolio decision making and risk management (Pesaran and Pesaran, 2010, 2007a, 2007b). Given that the modelling of conditional volatilities and correlations across currency market returns is a critical function of investing and portfolio management in a global economy, Pesaran and Pesaran (2010, 2007a, 2007b) suggest that devolatilized returns within a multivariate *t*-DCC model capture the fat tail properties of currency time series since transforming returns by realized volatility makes the innovations Gaussian. This is a key concept in the application of the *t*-DCC model. The paper applied this approach to the estimation of conditional correlations and volatilities for BRICS currency returns. Our results indicate that the *t*-DCC model is preferred over the normal-DCC model in estimating conditional volatilities and correlations. Second, both $\hat{\pi}_N$ and $\hat{z}_\pi$ [tests for serial correlation and a uniform distribution] provide support for the *t*-DCC model despite the 2008-2009 financial crisis. However, the model barely passes the non-parametric Kolmogorov-Smirnov (KS_N) test that test whether probability transform estimates, $\hat{U}_t$ are uniformly distributed over the range (0, 1). Overall, the results track well correlations and volatilities in BRICS currency returns.

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