On the implementation of asymmetric VaR models for managing and forecasting market risk

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Abstract

This paper investigates the implementation of asymmetric models and skewed distributions when managing market risk using the Value-at-Risk. The comparative analysis of the VaR estimations is executed by consideration of the time dynamics and the sequence of potential violation of the model. The findings of the paper suggest that the consideration of skewed distributions of time series and asymmetric volatility specification result to more accurate estimations of the VaR and hence provide the means for more efficient estimators of the potential losses that an institution is likely to exhibit. The importance of the paper lies on the fact that according to the regulative authorities financial institutions are supposed to adopt internally models for managing more efficiently market risk and this could be achieved by applying asymmetric models on both the volatility of their assets and on the distributions of the examined time series.

1 Introduction

Financial risks are classified into broad categories of market risks, credit risks, liquidity risks, operational risks and legal risks. In recent years, the tremendous growth of trading activity and the well-publicized loss of many financial institutions due to the recent financial crisis have led financial regulators and supervisory committee of banks to favour quantitative techniques which appraise the possible loss that these institutions can incur. One of the most sought-after techniques for managing market risk is the Value at Risk (VaR). Regulative authorities' objectives towards a stable financial system are often exposed to the adverse impact that the overexpansion of trading activities of financial institutions might have on the functioning of financial markets. These worries stem from the increased involvement of banks in the derivatives markets, which are becoming global, more complex, and therefore embed systemic risk; on top of that these instruments do not show up on balance sheets.

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One of the most attractive approaches for modeling VaR is the delta-normal according to which the VaR is calculated as the product of the forecasted volatility and the corresponding percentile of the normal distribution. Although, volatility models exhibit asymmetries on the way that new information is incorporated in the process, most of the extant papers are based on symmetric GARCH models. On top of that, the distributional form which is adopted in most cases is a symmetric one, i.e. the normal or the t-distribution, which apparently does not reflect with accuracy the asymmetric effects that are observed in financial time series.

The aim of this paper is to investigate the effectiveness of VaR models for managing market risk by consideration of the asymmetric effects of financial time series on both the volatility and the distributional form. Based on the family of the autoregressive conditional heteroskedasticity models of Engle (1982) [1], this paper considers many different asymmetric effects on both the volatility process and on the distributional form providing a comparative analysis. According to the empirical findings of the paper the adoption of asymmetric models are more efficient in forecasting market risk, a conclusion that should be considered from both regulators and investors. The rest of the paper contains a literature review in the second chapter, the data and empirical analysis, the empirical findings and finally the conclusion.

2 Literature Review

According to Jorion (1995) [2] market risks arise from changes in the prices of financial assets and liabilities (or volatilities) and are measured by changes in the value of open positions or in earnings. Market risks include basis risk, which occurs when relationships between products used to hedge each other change or break down, and gamma risk, due to nonlinear relationships. (Holders of large positions in derivatives have been hurt by basis and gamma risk, even though they thought they were fully hedged). Market risk can take two forms: absolute risk, measured by the loss potential in dollar terms, and relative risk, relative to a benchmark index. While the former focuses on the volatility of total returns, the latter measures risk in terms of tracking error or deviation from the index. Value-at-Risk has become one of the most sought-after techniques as it provides a simple answer to the following question: with a given probability (say α), what is the predicted financial loss over a given time horizon? The answer is the VaR at level α, which gives an amount in the currency of the traded assets (in dollar terms, for example) and is thus easily understandable. There are several ways for VaR estimation, such as the delta-normal, the historical simulation, the stress testing and the Monte-Carlo approach.

The delta-normal method assumes that all asset returns are normally distributed. A related, problem is the existence of "fat tails" in the distribution of returns on most financial assets. These fat tails are particularly worrisome precisely because VaR attempts to capture the behavior of the portfolio return in the left tail. With fat tails a model based on normal approximation underestimates the proportion of outliers and hence the true VaR.

The historical-simulation method provides a straightforward implementation of full valuation. It consists of going back in time, such as over the last 90 days, and applying current weights to a time series of historical asset returns. This method is relatively simple to implement if historical data have been collected in-house for daily marking to market. The same data can then be stored for later reuse in estimating VaR. As always, the choice of the sample period reflects a trade-off between using longer and shorter sample sizes. Longer intervals increase the accuracy of estimates but could use irrelevant data, thereby missing important changes in the underlying process. For instance, to obtain a monthly VaR, the user would reconstruct historical monthly portfolio returns over, say, the last five years. This method is robust and intuitive and, as such, forms the basis for the Basle 1993 proposals on market risks.

Stress Testing takes a completely opposite approach to the historical simulation method. This method, sometimes called scenario analysis, examines the effect of simulated large movements in key financial variables on the portfolio. It consists of subjectively specifying scenarios of interest to assess possible changes in the value of the portfolio. For instance, one could specify a scenario where the yield curve shifts up by 100 basis points over a month or a doomsday scenario where a currency suddenly devalues by 30 percent. These are typical scenarios used by the traditional asset liability management approach.

In contrast to scenario analysis, Monte-Carlo simulations cover a wide range of possible values in financial variables and fully account for correlations. In brief, the method proceeds in two steps. First, the risk manager specifies a stochastic process for financial variables as well as process parameters; parameters such as risk and correlations can be derived from historical or optional data. Second, fictitious price paths are simulated for all variables of interest. At each horizon considered, which can go from one day to many months ahead, the portfolio is marked to market using full valuation. Each of these "pseudo" realizations is then used to compile a distribution of returns, from which a VaR figure can be measured. The MC method is similar to the historical simulation method, except that the hypothetical changes in prices for an asset are created by random draws from a stochastic process. This approach analysis is by far the most powerful method to compute VaR. It can account for a wide range of risks, and even model risk. It can incorporate time variation in volatility, fat tails, and extreme scenarios.

Most models in the literature focus on the VaR for negative returns as mentioned by Jorion (2000) [3]. Indeed, it is assumed that traders or portfolio managers have long trading positions, i.e. they bought the traded asset and are concerned when the price of the asset falls. Giot and Laurent (2003) [4], focus on modeling VaR for portfolios defined on long and short trading positions. Thus they model VaR for traders having either bought the asset (long position) or short-sold it (short positions). Correspondingly, one focuses in the first case on the left side of the distribution of returns, and on the right side of the distribution in the second case.

Black (1976) [5] first noted that often, changes in stock returns display a tendency to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to "bad news" and to fall in response to "good news" asymmetrically. This phenomenon is termed the "leverage effect" and can only be partially interpreted by fixed costs such as financial and operating leverage (see Gewe (1982) [6]). The asymmetry present in the volatility of stock returns is too large to be fully explained by leverage effect. According to the leverage effect, a reduction in the equity value would raise the debt-to-equity ratio, hence raising the riskiness of the firm as manifested by an increase in future volatility. As a result, the future volatility will be negatively related to the current return on the stock. Discussion of the leverage effect can also be found in Kupiec (1990) [7] where the leverage effect is tested within the context of a linear GARCH(p,q) model by introducing a stock price level in the variance equation. The coefficient is insignificant though this may be a result of a failure to adjust for strong trend in the price level. It is also worth noting that the leverage effect can only partially explain the strong negative correlation between current return and current volatility in the stock market. In contrast to the causal linkage of current return and future volatility explained by the leverage effect, the fundamental risk-return relation predicts a positive correlation between future returns and current volatilities in stock prices. However, an alternative explanation is the volatility feedback effect, studied in French, Schwert and Stambaugh (1987) [8] and Cambell and Hentschel (1990) [9].

3 Data and Research Methodology

For the purposes of this paper, eight main indices are used from the US, EU and Asia on a daily basis for a period of time which spans from 1928 to 2005. Precisely, the data refer to the Dow Jones, Nasdaq, S&P

500, FTSE/ATHEX, CAC40, DAX, FTSE 100 and Nikkei 225. Daily returns (log differences) often exhibit serial autocorrelation, and an AR(p) filter is applied:

$$r_t = \rho_0 + \rho_1 \cdot r_{t-1} + ... + \rho_p \cdot r_{t-p} + \varepsilon_t, \tag{1}$$

where ε_t are the residuals of the corresponding AR(p) filter. The residuals of the first moment models (ε_t), are ready for further analysis under the second moment models, the ARCH models. The first model adopted in the analysis is the Engle's (1982) Autoregressive conditional Heteroskedasticity Model:

$$\varepsilon_t = z_t \cdot \sqrt{h_t} \tag{2}$$

where z is a zero-mean Gaussian process with unit variance and h represents the qth order ARCH process:

$$h_t = a_0 + a_1 \cdot \varepsilon_{t-1}^2 \quad \text{or} \quad h_t = a_0 + a_1 \cdot \varepsilon_{t-1}^2 + ... + a_q \cdot \varepsilon_{t-q}^2 \tag{3}$$

The next volatility model is that of Bollerslev (1986) [10], the Generalized Autoregressive Conditional Heteroskedastic (GARCH), which allows a much more flexible lag structure. It is argued that a simple GARCH model provides a marginally better fit and a more plausible learning mechanism than the ARCH model with an eight-order linear declining lag structure as in Engle and Kraft (1983). The variance function of the GARCH model is given by the following equation with respect to the residual process ε_t of the first moment models:

$$h_t = a_0 + \sum_{i=1}^{q} a_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \cdot h_{t-i} \tag{4}$$

Furthermore, the logarithm ARCH model is applied which was proposed by Geweke (1986) [12], Pantula (1986) [13] and Milhoj (1987a) [14]:

$$log(h_t) = a_0 + a_1 \cdot log(\varepsilon_{t-1}^2) + ... + a_q \cdot log(\varepsilon_{t-q}^2) \tag{5}$$

Another extension is that of Higgins and Bera (1992) [15], the non-linear ARCH (NARCH) model:

$$h_t = \left[\phi_0 \cdot (\sigma^2)^\delta + \phi_1 \cdot (\varepsilon_{t-1}^2)^\delta + ... + \phi_q \cdot (\varepsilon_{t-q}^2)^\delta \right]^{1/\delta}, \tag{6}$$

, where $\sigma^2 > 0$, $\varphi_i \geq 0$, $\delta > 0$ and the φ_i's are such that $\sum_{i=0}^{q} \phi_i = 1$.

Furthermore, the Nelson's (1991) [16] EGARCH model is adopted in the paper which accounts for the leverage effect:

$$ln(h_t) = a_0 + \gamma \cdot \left(\left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - E \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| \right) + \theta \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta \cdot ln(h_{t-1}) \tag{7}$$

Particularly, the parameter θ captures the leverage effect (when $\theta < 0$ the leverage effect is taken place).

The next model which accounts for asymmetries in the volatility, is the Threshold GARCH (TGARCH) model of Zakoian (1991a) [17]:

$$h_t = \left(a_0 + a_1^+ \cdot \varepsilon_{t-1}^+ - a_1^- \cdot \varepsilon_{t-1}^- + ... + a_q^+ \cdot \varepsilon_{t-q}^+ - a_q^- \cdot \varepsilon_{t-q}^- + \right.$$
$$\left. + \beta_1 \cdot h_{t-1}^{1/2} + ... + \beta_p \cdot h_{t-p}^{1/2} \right)^{1/2} \tag{8}$$

where $\varepsilon_t^+ = max(\varepsilon_t, 0)$ and $\varepsilon_t^- = min(\varepsilon_t, 0)$

Another model which accounts for asymmetries and is applied at this paper is the GJR-GARCH model of Glosten et. al. (1993) [18]:

$$h_t = a_0 + \sum_{i=1}^{q} \left(a_i \cdot \varepsilon_{t-i}^2 \right) + \sum_{i=1}^{q} \left(\gamma_i \cdot d \left(\varepsilon_{t-i} < 0 \right) \cdot \varepsilon_{t-i}^2 \right) + \sum_{j=1}^{p} \left(\beta_j \cdot h_{t-j} \right) \tag{9}$$

where γ_i, for i=1,…,q, are parameters for estimation, d(.) denotes the indicator function (i.e. d($\varepsilon_{t-i} < 0$) = 1 if $\varepsilon_{t-i} < 0$, and d($\varepsilon_{t-i} < 0$) = 0 otherwise). The GJR model allows good news, ($\varepsilon_{t-i} > 0$), and bad news, ($\varepsilon_{t-i} < 0$), to have different effects on the conditional variance. Therefore, in the case of the GJR(0,1) model, good news has an impact of α_1, while bad news has an impact of $\alpha_1 + \gamma_1$. For $\gamma_1 > 0$, the "leverage effect" exists.

Finally, the paper adopts the Asymmetric Power ARCH or AP-ARCH(p,q) model of Ding et al. (1993) [19]:

$$h_t^{\frac{\delta}{2}} = a_0 + \sum_{i=1}^{q} a_i \cdot \left(|\varepsilon_{t-1}| - \gamma_i \cdot \varepsilon_{t-i} \right)^{\delta} + \sum_{j=1}^{p} \beta_j \cdot h_{t-j}^{\frac{\delta}{2}} \tag{10}$$

where $\alpha_0 > 0$, $\delta \geq 0$, $b_j \geq 0$, j=1,…,p , $\alpha_i \geq 0$ and $-1 < \gamma_i < 1$, i=1,…,q. The model imposes a Box and Cox (1964) [20] power transformation of the conditional standard deviation process and the asymmetric absolute innovations. The functional form for the conditional standard deviation is familiar to economists as the constant elasticity of substitution (CES) production function. In the case of the APGARCH modelling the leverage effect is captured by the parameter γ.

For the estimation of the volatility models I apply several symmetric and asymmetric distributions maximizing the likelihood accordingly. The symmetric distributions are the normal, the t-student and the GED, while the asymmetric ones are the Hansen's (1994) [21] and the Giot and Laurent's (2001) [22] skewed t distributions.

Hansen (1994) considered a skewed T distribution, which is applied in the residuals ε_t of the first moment models, yielding:

$$g\left(\varepsilon_t/n,\lambda\right)=\begin{cases} b\cdot c\cdot\left(1+\frac{1}{n-2}\cdot\left(\frac{b\cdot\left(\varepsilon_t\big/\sqrt{h_t}\right)_t+a}{1-\lambda}\right)^2\right)^{-\frac{n+1}{2}} & ,\ \varepsilon_t\big/\sqrt{h_t}<-\frac{a}{b} \\[2em] b\cdot c\cdot\left(1+\frac{1}{n-2}\cdot\left(\frac{b\cdot\left(\varepsilon_t\big/\sqrt{h_t}\right)+a}{1+\lambda}\right)^2\right)^{-\frac{n+1}{2}} & ,\ \varepsilon_t\big/\sqrt{h_t}\geq-\frac{a}{b} \end{cases} \quad (11)$$

where $2 < n < \infty$, and $-1 < \lambda < 1$. For computation purposes I adopt the logistic transformation to the degrees of freedom and the coefficient of skeweness: $x_t^* = L + \frac{(U-L)}{1+exp(-x_t)}$. Even if x is allowed to vary over the entire real line, x^* will be constrained to lie in the region [L, U].

The constants α, b and c are given by:

$$a = 4\cdot\lambda\cdot c\cdot\left(\frac{n-2}{n-1}\right) \quad,\quad b^2 = 1+3\cdot\lambda^2-a^2 \qquad \text{and } c = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi\cdot(n-2)}\cdot\Gamma\left(\frac{n}{2}\right)} \qquad (12)$$

This skewed student-t distribution specializes to the student-t distribution by setting $\lambda = 0$.

Along with this skewed distribution this paper also adopts the Giot and Laurent (2001) skewed distribution:

$$f\left(\varepsilon_t/n,\xi\right)=\begin{cases} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\cdot\sqrt{\pi\times(n-2)}}\cdot\frac{2\cdot s}{\xi+\frac{1}{\xi}}\cdot\left(1+\frac{\left(s\cdot\left(\varepsilon_t\big/\sqrt{h_t}\right)+m\right)^2}{n-2}\cdot\xi\right)^{-\frac{n+1}{2}} & ,\ \varepsilon_t\big/\sqrt{h_t}<-\frac{m}{s} \\[2em] \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\cdot\sqrt{\pi\times(n-2)}}\cdot\frac{2\cdot s}{\xi+\frac{1}{\xi}}\cdot\left(1+\frac{\left(s\cdot\left(\varepsilon_t\big/\sqrt{h_t}\right)+m\right)^2}{n-2}\cdot\xi^{-1}\right)^{-\frac{n+1}{2}} & ,\ \varepsilon_t\big/\sqrt{h_t}\geq-\frac{m}{s} \end{cases} \quad (13)$$

where ξ (ξ>0) is the asymmetry coefficient, and m and s^2 are respectively the mean and the variance of the non-standardized skewed student defined as:

$$m = \frac{\Gamma\left(\frac{n-1}{2}\right) \cdot \sqrt{n-2}}{\Gamma\left(\frac{n}{2}\right) \cdot \sqrt{\pi}} \cdot \left(\xi - \frac{1}{\xi}\right) \qquad \text{and} \quad s^2 = \xi^2 + \frac{1}{\xi^2} - m^2 - 1 \qquad (14)$$

Lambert and Laurent (2000) show that the quantile function $st^*_{a,n,\xi}$ of a non- standardized skewed student density is:

$$st^*_{a,n,\xi} = \begin{cases} \dfrac{1}{\xi} \cdot t\left(\dfrac{a}{2} \cdot \left(1 + \xi^2\right), n\right) & \text{if} \quad a < \dfrac{1}{1+\xi^2} \\[4mm] -\xi \cdot t\left(\dfrac{1-a}{2} \cdot \left(1 + \dfrac{1}{\xi^2}\right), n\right) & \text{if} \quad a \geq \dfrac{1}{1+\xi^2} \end{cases} \qquad (15)$$

where $t_{\alpha,n}$ is the quantile function of the (unit variance) Student-t density.

One of the most widely used methods for estimating the volatility models is the Maximum Likelihood Estimator and the Quasi-Maximum Likelihood Estimator if the normal distribution is used.

The one-step-ahead VaR computed at t-1 for long trading positions is given by $\mu_t + z_a \cdot \sqrt{h_t}$ when for short trading positions it is equal to $\mu_t + z_{1-a} \cdot \sqrt{h_t}$ with z_α being the left quantile at $\alpha\%$ for the normal distribution and $z_{1-\alpha}$ is the right quantile at $\alpha\%$.

For the t-student APARCH model, the VaR for long and short positions is given by $\mu_t + st_{a,n} \cdot \sqrt{h_t}$, and $\mu_t + st_{1-a,n} \cdot \sqrt{h_t}$ with $st_{a,n}$ being the left quantile at $\alpha\%$ for the (standardized) student distribution with (estimated) number of degrees of freedom n and $st_{1-a,n}$ is the right quantile at $\alpha\%$ for this same distribution. For the skewed Student APARCH model, the VaR for long and short positions is given by $\mu_t + skst_{a,n,\xi} \cdot \sqrt{h_t}$ and $\mu_t + skst_{1-a,n,\xi} \cdot \sqrt{h_t}$ with $skst_{a,n,\xi}$ being the left quantile at $\alpha\%$ for the skewed student distribution with n degrees of freedom and asymmetry coefficient ξ, while, $skst_{1-a,n,\xi}$ is the corresponding right quantile. If $\log(\xi)$ is smaller than zero (or $\xi < 1$), $|skst_{a,n,\xi}| > |skst_{1-a,n,\xi}|$ and the VaR for long trading positions will be larger (for the same conditional variance) than the VaR for short trading positions. When $\log(\xi)$ is positive, we have the opposite result. Therefore, the skewed student density distribution allows for asymmetric VaR forecasts and fully takes into account the fact that the density distribution of asset returns can be substantially skewed.

All models are tested with one-step ahead VaR level α and their performance is then assessed by computing the failure rate of returns. By definition, the failure rate is the number of times returns exceed (in absolute value) the forecasted VaR. If the VaR model is correctly specified, the failure rate should be equal to the pre-specified VaR level. Notice also, that since the normal distribution of the returns is under question, the aforementioned tests might overdo for small values of α.

According to the Kupiec test (unconditional coverage) let $N^a = \sum_{t=0}^{T-1} I_{t+1}^a$ be the number of days over a T

period that the time series of the residuals of the first moment model was larger (in absolute values) than the VaR_t^a estimate, where:

$$I_{t+1}^a = \begin{cases} 1, & if \quad \varepsilon_{t+1} < VaR(long)_{t+1}^a \\ 0, & if \quad \varepsilon_{t+1} \geq VaR(long)_{t+1}^a \end{cases} \quad \text{for long trading positions and} \tag{16}$$

$$I_{t+1}^a = \begin{cases} 0, & if \quad \varepsilon_{t+1} > VaR(short)_{t+1}^a \\ 1, & if \quad \varepsilon_{t+1} \leq VaR(short)_{t+1}^a \end{cases} \quad \text{for short trading position} \tag{17}$$

Hence, N^α is the observed number of exceptions in the sample. As argued in Kupiec (1995) [23], the failure number (failure is the event where the financial time series is lower than the VaR for long trading position and larger than the VaR for short trading position) follows a binomial distribution, $N^\alpha \sim$ Binomial $(T, fr^\alpha = N^\alpha/T)$. At the 5% level and if T yes/no observations are available, a confidence interval for fr (failure rate) is given by:

$$\left[fr - 1.96 \cdot \sqrt{fr \cdot (1 - fr)/T} \; , \; fr + 1.96 \cdot \sqrt{fr \cdot (1 - fr)/T} \right] \tag{18}$$

The pair of the null and alternative hypothesis for the following test is H$_0$: $fr = \alpha$ and H$_1$: $fr \neq \alpha$

If we assume independence for the Bernoulli sequence I$_1$,…I$_T$, the likelihood under the null hypothesis is simply $L(\alpha; I_1, ..., I_T) = (1 - \alpha)^{N^\alpha} \cdot \alpha^{T - N^\alpha}$ and the likelihood under the alternative is:

$$L(f; I_1, ..., I_T) = (1 - f)^{N^\alpha} \cdot f^{T - N^\alpha} = \left(1 - \frac{N^a}{T}\right)^{N^a} \cdot \left(\frac{N^a}{T}\right)^{T - N^a} \tag{19}$$

and consequently, the appropriate likelihood ratio statistic is:

$$LR = -2\ln\left(\frac{likelihood\,(under\ the\ H_0)}{likelihood\,(under\ the\ H_1)}\right) \tag{20}$$

$$LR = -2 \cdot \ln\left(a^{T - N^a} \cdot (1 - a)^{N^a}\right) + 2 \cdot \ln\left(\left(\frac{N^a}{T}\right)^{N^a} \cdot \left(1 - \left(\frac{N^a}{T}\right)\right)^{T - N^a}\right) \tag{21}$$

Asymptotically, this test is distributed as a χ^2 with one degree of freedom[2]. This test can reject a model for both high and low failures but, as stated in Kupiec (1995), its power is generally poor.

The unconditional coverage proposed by Kupiec (1995) tests the coverage of the interval but it does not have any power against the alternative that the zeros and ones come clustered together in a time-dependent fashion. In the test above the order of zeros and ones in the indicator sequence does not matter, only the total number of ones plays a role. In contrast, Christoffersen (1998) [24] introduced a conditional coverage test. Of course, simply testing for the correct unconditional coverage is insufficient when dynamics are present in the higher-order moments. The two tests presented below make up for this deficiency. The first, tests the independence assumption (LR_{ind}), and the second, jointly tests for independence and correct coverage (LR_{cc}), thus giving a complete test of correct conditional coverage.

The above tests, for unconditional coverage (LR_{uc}) and independence (LR_{in}) are now combined to form a complete test of conditional coverage:

$$LR_{cc} = -2 \cdot log \left[\frac{L\left(a; I_1, ..., I_T\right)}{L\left(\Pi_1; I_1, ..., I_T\right)} \right] \tag{22}$$

where $\Pi_1 = \begin{bmatrix} \dfrac{n_{00}}{n_{00} + n_{01}} & \dfrac{n_{01}}{n_{00} + n_{01}} \\[3mm] \dfrac{n_{10}}{n_{10} + n_{11}} & \dfrac{n_{11}}{n_{10} + n_{11}} \end{bmatrix} \tag{23}$

and n_{ij} indicates the number of transitions of i state to j state. The LR_{cc} is asymptotically χ^2 with degrees of freedom 2x(2-1)=2. As pointed out from Christoffersen (1998) conditioning on the first observation, the three LR tests are numerically related by the following identity,

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{24}$$

Its main advantage over the previous statistic is that it takes account of any conditionality in our forecast: if volatilities are low in some period and high in others, the forecast should respond to this clustering event. The Christoffersen procedure enables us to separate clustering effects from distributional assumption effects. Another illustration of the LR_{cc} is given below:

$$LR_{cc} = -2 \cdot \ln\left[\left(1-a\right)^{T-N^\alpha} \cdot a^{N^\alpha} \right] + 2 \cdot \ln\left[\left(1-\pi_{01}\right)^{n_{00}^\alpha} \cdot \pi_{01}^{n_{01}^\alpha} \cdot \left(1-\pi_{11}\right)^{n_{10}^\alpha} \cdot \pi_{11}^{n_{11}^\alpha} \right] \sim x_2^2 \tag{25}$$

where n_{ij} is the number of observations with value i followed by j for i, j=0, 1 and $\pi_{ij}^a = \dfrac{n_{ij}^a}{\sum_j n_{ij}^a}$ are the corresponding probabilities. If the sequence of I_t^a is independent, then the probabilities to observe or not observe a VaR^α violation in the next period must be equal, which can be written more formally as $\pi_{01} = \pi_{11} = \alpha$. The main advantage of this test is that it can reject a VaR model that generates either too many or too few clustered violations, although it needs several hundred observations in order the test to be accurate.

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[2] Two outcomes -1 = 1 degrees of freedom

5 Empirical Findings

From the empirical results, the parameters tend to be significant at 5% statistical significance level, in the cases where asymmetry volatility specifications and asymmetry distributions are recruited. Thus, the asymmetry in both volatility and distribution specifications, seems to be a decisive factor for modeling financial time series.

Another useful result, is that the parameter 'β' of the volatility functions seems to be close to unity, indicating a long memory process with high persistence to shocks. In this direction the parameter 'δ' of the AP-GARCH volatility function is close to two, and below two when the skewed T distribution is considered, indicating that there is substantially larger correlation between absolute returns than between squared returns. This is a stylized fact of high-frequency financial results (often called 'long memory'), according to Giot and Laurent (2003).

According to Kupiec and Christoffersen tests (as shown on Table 1 of the appendix), under the Normal Distribution the AP-GARCH model captures better[3] the characteristics of the data, since it gives more accurate results. Note also that for long trading positions the above tests overdo with respect to short trading positions, and this flows out, probably, from the existence of skewness in the data.

The same conclusions are derived when we use other symmetric distributions (such as the T-student Distribution or the GED), or the skewed T distribution. The models that capture the asymmetry in volatility seem to be preferable. By comparing the distributions' performance it is shown that the Skewed T distribution is better in capturing the structure of the data, since it considers their asymmetry. By applying skewed distributions, there is no relative efficiency for long trading positions compared with short trading positions.

The leverage effect (bad news at present tend to have bad impacts on the future, i.e. negative returns today tend to increase the volatility of the next day) seems to be a significant factor in determining the time-varying volatility processes.

The T-GARCH specification has the advantage of the time varying leverage effect, since it does not capture the phenomenon with a single fixed parameter. The leverage effect is explained as a time varying phenomenon and seems to be statistical significant at 5% significance level for all of the eight Indices considered in this dissertation.

On the other hand, the E-GARCH specification suggests that the leverage effect is statistical significance at 5% significance level for the Normal case except from the ASE Index where it is not significant. Furthermore, it is statistical significant for the T-Student distribution for all of the examined Indices and finally, it is statistical significant for the Generalized Error Distribution for all of the examined Indices

On the other hand, although that the GJR specification tracks out the leverage effect, it is not significant at 5% significance level. Furthermore, the AP-GARCH specification captures the leverage effect approximately in the whole study. Particularly, the leverage effect is captured in the case of the Conditional Normal distribution, with insignificant parameters only in the case of the ASE, is present in the case of the Conditional T-student distribution, is present in the case of the Conditional Skewed T-student distribution of Hansen and finally, is present in

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[3] Note that it is not defined always, because in some cases the Likelihood Ratio Statistic of the Kupiec or Christoffersen test is difficult to be computed

the case of the Conditional Skewed T-student distribution of Giot and Laurent, except for the case of the FTSE 100 Index.

For all of the estimated models, the degrees of freedom of the T-student and the Skewed T-student of Hansen and of Giot and Laurent, seems to be statistical significant, indicating that the phenomenon of leptokurtosis is present and is successfully captured by these models. In the case of the GED the parameter v is below two and is statistically significant, indicating that the conditional distribution of the returns is more leptokurtic than the Normal.

The skewed T distributions that have been considered in this thesis, suggest a statistically significant parameter of skewness, emphasizing the importance of Skewed Distributions in the analysis of financial data. The exception comes from the Nikkei 225 Index in the case of the Hansen Conditional Distribution approach and under the GARCH specification, where the skewness parameter is not significant.

In this analysis, the second moment autocorrelations have been successfully modelled. This is an indication of success in applying Volatility models, since they have captured the serious non-linear dependences of the second moments in many various ways.

6 Conclusion

Volatility is a key factor which permeates most financial instruments and plays a central role in many areas of finance. The ARCH model and its various extensions have proven very effective tools along these lines. Indeed, by any yardstick, the literature on ARCH has expanded dramatically since the seminal paper by **Engle** (1982). However, many interesting research topics remain to be examined.

This paper examines empirically the importance of accounting for potential asymmetries on both the volatility process and the distributional form of financial series. The empirical findings suggest that modeling market risk adopting skewed distributions provide more accurate estimations than when assuming symmetric distributions. This finding could be considered by institutional investors when quantifying the VaR estimations internally and most importantly by the regulative authorities which request from banks to meet specific standards and requirements for a more efficient functioning of the financial system.

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Appendix

Table 1. Panel A. p-values of the Christoffersen test with Normal distribution.

p-value of Christoffersen test for long trading positions using Normal ARCH(1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,22911	0,75000	-	-	0,10224	-
2.5%	-	-	0,20833	0,81605	0,08992	0,00444	0,04925	0,62657	-
1%	-	0,00000	0,00000	0,45648	0,00001	0,00000	0,00000	0,00625	0,00000
0.5%	-	0,00000	0,00000	0,19283	0,00000	0,00000	0,00000	0,00009	0,00000
0.25%	-	0,00000	0,00000	0,44685	0,00000	0,00000	0,00000	0,00000	0,00000

p-value of Christoffersen test for short trading positions using Normal ARCH(1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,02542	0,08224	0,00551	-	0,00001	-
2.5%	-	0,07348	0,26721	0,02010	0,87644	0,89880	0,64519	0,05004	0,90000
1%	-	0,03979	0,00095	0,44518	0,16696	0,01633	0,19240	0,81551	0,00179
0.5%	-	0,00002	0,00141	0,24085	0,00338	0,00035	0,03792	0,21743	0,00000
0.25%	-	0,00000	0,00000	0,49121	0,00038	0,00001	0,00368	0,04455	0,00000

p-value of Christoffersen test for short trading positions using Normal GARCH(1,1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,01703	0,00417	0,00019	-	0,00200	-
2.5%	-	0,00011	0,91545	0,02092	0,01848	0,00010	0,00013	0,08649	0,00050
1%	-	0,00949	0,28266	0,85717	0,65584	0,11366	0,13573	0,81578	0,68848
0.5%	0,09216	0,06205	0,24830	0,24085	0,21553	0,88254	0,70167	0,49543	0,07343
0.25%	0,00282	0,93495	0,45233	0,49121	0,86746	0,79606	0,73024	0,10060	0,00012

p-value of Christoffersen test for long trading positions using Normal GARCH(1,1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,32205	-	0,88392	-	0,06743	-
2.5%	-	-	0,02114	0,56630	0,01707	0,16430	0,12385	0,72115	-
1%	-	-	-	-	-	-	-	-	-
0.5%	-	-	-	-	-	-	-	-	-
0.25%	-	-	-	-	-	-	-	-	-

p-value of Christoffersen test for long trading positions using Normal TGARCH(1,1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,22911	-	0,32738	-	0,04143	-
2.5%	-	-	0,64680	0,81605	0,03081	0,00320	0,01581	0,92397	-
1%	-	-	-	-	-	-	-	-	-
0.5%	-	-	-	-	-	-	-	-	-
0.25%	-	-	-	-	-	-	-	-	-

p-value of Christoffersen test for short trading positions using Normal TGARCH(1,1) Model								
α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
5%	-	-	-	0,02151	0,07347	0,01708	-	0,00001	-
2.5%	-	0,46851	0,94637	0,01866	0,25690	0,79259	0,41122	0,13100	0,09800
1%	-	0,01009	0,00025	0,44518	0,01181	0,06483	0,03278	0,48531	0,00009
0.5%	-	0,00005	0,00003	0,24085	0,00046	0,00036	0,03792	0,00895	0,00000
0.25%	-	0,00000	0,00000	0,49121	0,00003	0,00000	0,00006	0,00073	0,00000

α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
p-value of Christoffersen test for long trading positions using Normal GJRGARCH(1,1) Model									
5%	-	-	-	0,32205	-	0,88392	-	0,00301	-
2.5%	-	-	0,23132	0,72788	0,09004	0,16430	0,31989	0,36996	-
1%	-	-	-	-	-	-	-	-	-
0.5%	-	-	-	-	-	-	-	-	-
0.25%	-	-	-	-	-	-	-	-	-

α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
p-value of Christoffersen test for short trading positions using Normal GJRGARCH(1,1) Model									
5%	-	-	-	0,02452	0,01731	0,00034	-	0,00002	-
2.5%	-	0,00036	0,93072	0,02092	0,01096	0,00024	0,00014	0,00543	0,00050
1%	-	0,02303	0,07994	0,96416	0,08998	0,15996	0,17992	0,77067	0,68848
0.5%	0,09216	0,10341	0,03086	0,92943	0,18828	0,88254	0,70167	0,49543	0,07343
0.25%	0,00282	0,93495	0,07444	0,49121	0,54107	0,79606	0,73024	0,20519	0,00012

α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
p-value of Christoffersen test for long trading positions using Normal AP-GARCH(1,1) Model									
5%	-	-	-	0,32205	-	0,73737	-	0,03169	-
2.5%	-	-	0,47053	0,72788	0,09004	0,58262	0,21048	0,87282	0,23931
1%	-	-	-	-	-	-	-	-	-
0.5%	-	-	-	-	-	-	-	-	-
0.25%	-	-	-	-	-	-	-	-	-

α level	Dow Jones	Nasdaq	S&P 500	ASE	CAC 40	DAX	FTSE 100	Mos Times	Nikkei 225
p-value of Christoffersen test for short trading positions using Normal AP-GARCH(1,1) Model									
5%	-	-	-	0,02452	0,03460	0,00447	-	0,00167	-
2.5%	-	0,00036	0,81739	0,02092	0,00795	0,00754	0,00036	0,11121	0,00908
1%	-	0,02303	0,29202	0,85717	0,11025	0,07753	0,04857	0,81551	0,06962
0.5%	-	0,16179	0,03545	0,92943	0,18828	0,69875	0,86336	0,66648	0,00068
0.25%	0,00009	0,96539	0,07444	0,49121	0,54107	0,44933	0,73024	0,20519	0,00000

Table 2. Panel B. p-values of the Christoffersen test with T-student distribution.

	p-value of Christoffersen test for long trading positions using T-student GARCH(1,1) Model			
α level	Dow Jones	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-	-
2,50%	-	0,00079	0,33870	0,00000
1%	-	-	-	-
0,50%	-	-	-	-
0%	-	-	-	-

	p-value of Christoffersen test for short trading positions using T-student GARCH(1,1) Model			
α level	Dow Jones	Nasdaq	FTSE 100	Nikkei 225
5%	-	0,00000	0,00000	0,00000
2,50%	-	0,00000	0,00000	0,00000
1%	0,00000	0,00000	0,00000	0,00000
0,50%	0,00000	0,00000	0,01332	0,00015
0%	0,00000	0,00072	0,03321	0,00007

p-value of Christoffersen test for long trading positions using T-student E-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	0,00012	0,08529	0,00000
1%	-	-	-
0.5%	-	-	-
0.25%	-	-	-

p-value of Christoffersen test for short trading positions using T-student E-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	0,00000	0,00000	0,00000
2.5%	0,00000	0,00000	0,00000
1%	0,00000	0,00008	0,00003
0.5%	0,00000	0,04768	0,00051
0.25%	0,00072	0,03321	0,01254

p-value of Christoffersen test for long trading positions using T-student AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	0,00023	0,18665	0,00000
1%	-	-	-
0.5%	-	-	-
0.25%	-	-	-

p-value of Christoffersen test for short trading positions using T-student AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	0,00000	0,00000	0,00000
2.5%	0,00000	0,00000	0,00000
1%	0,00000	0,00008	0,00003
0.5%	0,00000	0,00628	0,00131
0.25%	0,00072	0,03321	0,00333

Table 3. Panel C. p-values of the Christoffersen test with GED distribution.

p-value of Christoffersen test for long trading positions using GED GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2,50%	0,00000	0,00000	0,00000
1%	0,00000	0,00000	0,00000
0,50%	0,00000	0,00000	0,00000
0%	0,00000	0,00000	0,00000

p-value of Christoffersen test for short trading positions using GED GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	0,00000	0,00000	0,00000
2,50%	0,00000	0,00000	0,00000
1%	0,00000	0,00000	0,00000
0,50%	0,00000	0,00000	0,00000
0%	0,00000	0,00000	0,00000

p-value of Christoffersen test for long trading positions using GED E-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	0,00217	0,52134	0,71034
1%	-	-	-
0.5%	-	-	-
0.25%	-	-	-

p-value of Christoffersen test for short trading positions using GED E-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	0,00000	-	-
2.5%	0,00000	0,00000	0,00000
1%	0,00000	0,00350	0,12245
0.5%	0,00058	0,59684	0,31093
0.25%	0,01391	0,85559	0,08832

p-value of Christoffersen test for long trading positions using GED AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	0,01422	0,72575	0,31762
1%	-	-	-
0.5%	-	-	-
0.25%	-	-	-

p-value of Christoffersen test for short trading positions using GED AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	0,00000	-	-
2.5%	0,00000	0,00001	0,00002
1%	0,00000	0,00584	0,07745
0.5%	0,00389	0,48695	0,31093
0.25%	0,01391	0,90923	0,92198

Table4. Panel D. p-values of the Christoffersen test with skewed T-student distribution.

p-value of Christoffersen test for long trading positions using Skewed T-student Hansen AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	-	0,03467	0,26691
1%	-	-	-
0.5%	-	-	-
0.25%	-	-	-

p-value of Christoffersen test for short trading positions using Skewed T-student Hansen AP-GARCH(1,1) Model			
α level	Nasdaq	FTSE 100	Nikkei 225
5%	-	-	-
2.5%	-	0,00000	-
1%	-	0,00584	-
0.5%	0,00000	0,69862	-
0.25%	0,00000	0,90923	-