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Relationship between Crude Oil Prices and the U.S. Dollar Exchange Rates: Constant or Time-varying?

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Abstract

This paper aims to analyze the intertemporal interaction between crude oil prices and the U.S. dollar trade-weighted exchange rates from January 1997 through December 2012. To this end, the study assumes that the conditional covariance matrix between crude oil and the dollar exchange rate returns follows a bivariate GARCH process. Using daily data, I find strong evidence of a time-varying conditional covariance and correlation between crude oil prices and the U.S. dollar exchange rates. If on one day the change in the dollar price of oil is largely due to a change in the dollar's value, there is a tendency for the next day's change in oil prices to be primarily caused by changes in the dollar's value as well. On the other hand, if one day the change in the dollar price of oil is caused primarily by factors other than the dollar's value, there is a tendency for those to be primarily causes of changes in the dollar price of oil on subsequent days.

JEL classification numbers: E31, F31
Keywords: Garch process, crude oil prices, exchange rate

1 Introduction

This paper examines the intertemporal interaction between crude oil prices and the U.S. dollar exchange rates from January 1997 through December 2012. Crude oil is one of the most essential energy sources in the U.S., accounting for about 40% of the nation's energy consumption. Since OPEC's 1973 decision to regulate its oil price independently of large oil companies, crude oil prices have been subject to dramatic volatility. Oil prices increased from less than $11 per barrel in the beginning of 1999 to $38 per barrel in September 2000, decreased to $18 per barrel in January 2002 and went up to $77 per barrel in July 2006. The crude oil market experienced an unprecedented dramatic volatility in 2008 as crude oil prices reached an all-time high level of $145 per barrel in July and then fell sharply to $30 per barrel in December. This large oil price fluctuation tendency has continued in recent

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years. From less than $68 per barrel in May 2010, oil prices increased to $112 in April 2011 and then fell to $77 in October 2011.

The high volatility in crude oil prices is likely due to actual and anticipated fluctuations in supply and the short-term inelasticity of demand. Given that crude oil is one of the most essential energy sources, it is very difficult for most oil users to reduce their consumption within a short period of time following a price increase. On the other hand, there is considerable fluctuation in oil supply which depends on a variety of factors, such as changes in supply conditions, responses to geopolitics, institutional arrangements, and the dynamics of the financial markets. One of the most important factors impacting crude oil supply and demand is the value of the U.S. dollar whose appreciation or depreciation is often accompanied by a decrease or increase in oil prices. For example, in June-July 2008, a combination of supply uncertainties in oil producing countries and a falling dollar caused an unprecedented oil price spike. On the reverse, an appreciation of the dollar and signs of worldwide economic slowdown led to a sharp decrease in oil price in the end of 2008 and also in October 2014.

It is often argued that because oil prices are denominated in U.S. dollar, oil prices and the value of the dollar should be negatively correlated. An appreciation of the U.S. dollar would tend to make oil more expensive in non-dollar currencies and would reduce demand for crude oil thereby possibly lowering oil prices in dollars. On the contrary, a depreciation of the dollar is associated with a decline in the purchasing power of oil revenues for oil exporting countries and therefore, these countries have an incentive to counterbalance the adverse effect of the dollar depreciation by raising oil prices. Moreover, a depreciation of the dollar makes oil priced in dollar an attractive financial asset for foreign investors and therefore, a large amount of money would tend to flow to the oil market, thus oil price will be driven up. A causal relationship between oil price movements and changes in the U.S. dollar value has been commonly referred to in the financial market commentary as the following quotes illustrate: “Weak dollar central to oil price boom”[2] or “Oil prices fall on dollar strength”[3].

[2]http://www.reuters.com/article/2007/09/26/us-energy-oil-dollar-idUSL2576484820070926
[3]WSJ, September 4, 2014

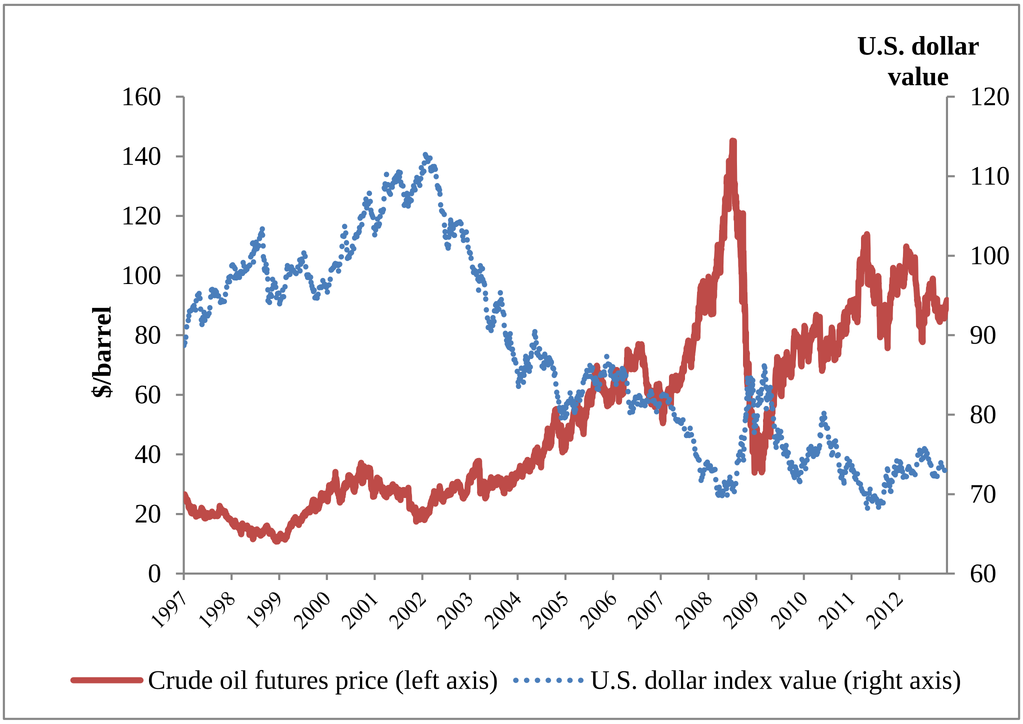

Figure 1: Crude oil prices and the U.S. dollar exchange rates

Figure 1 plots crude oil prices against the U.S. dollar exchange rates from January 1997 through December 2012. While apparently there is no clear relationship from 1997 to 2001, crude oil prices and the U.S. dollar value appear to be negatively correlated in recent years. As the dollar lost 35.40% of its value against the Trade Weighted Major Currencies Exchange Index between January 2002 and July 2008[4], oil prices soared from $18 to $145 per barrel during the same time period. On the contrary, during the financial crisis, oil prices collapsed from $145 to $50 per barrel from July 2008 to March 2009 while the dollar appreciated 23% in that 9-month period.

[4]Constructed by the author using data from the Federal Reserve Statistical releases, downloaded from http://www.federalreserve.gov/releases/h10/summary/indexn96_b.htm.

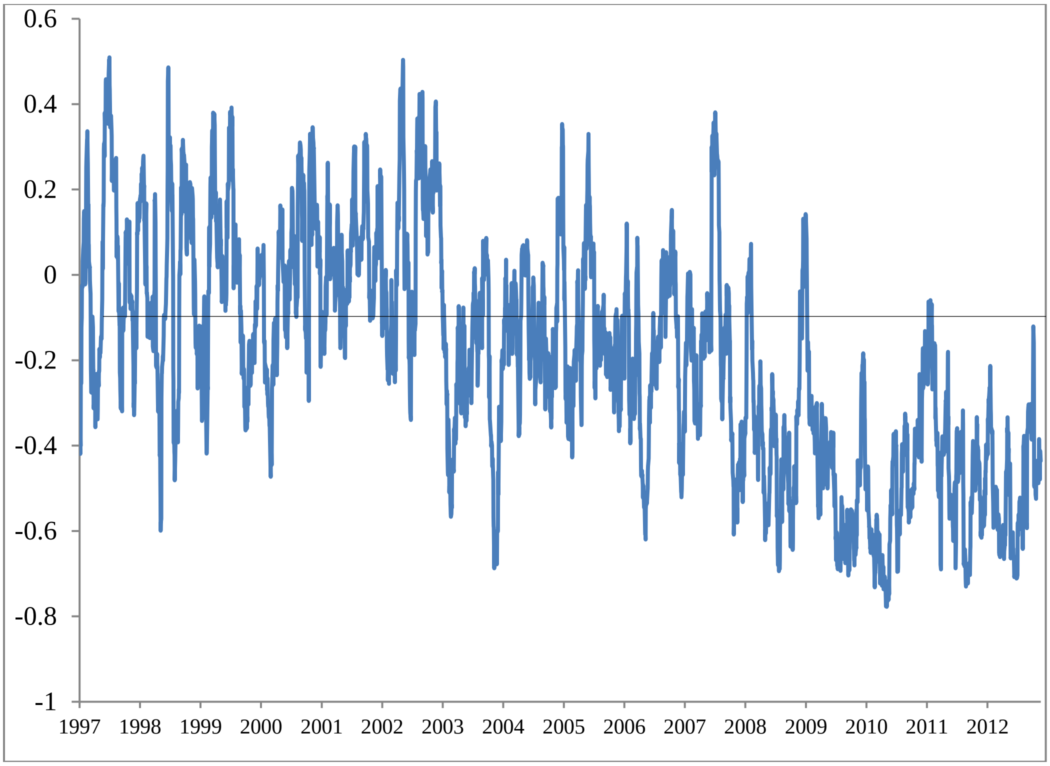

Figure 2: Oil prices and U.S. dollar exchange rates correlation

Figure 2 depicts the correlation between oil prices and the U.S. dollar exchange rates, computed over 30-day moving windows. While the correlation fluctuates between negative and positive values during most of the sample period, it tends to be more persistently negative after 2002.

While the negative relationship between oil prices and the U.S. dollar value is widely discussed in the popular economic press and among market practitioners, the academic literature documents mixed evidence regarding whether a change in the dollar exchange rate drives changes in oil prices or vice versa. Prior academic studies generally find that oil prices Granger-cause exchange rates, but not vice versa. For example, Zhou (1995) [1], Chaudhuri and Daniel (1998) [2], Amano and Norden (1998a,b)[3] [4], and Chen and Chen (2007) [5] find that oil price is the dominant source of real exchange rate movements. They argue that if an oil importing country is dependent on imported oil, an oil price rise may increase the prices of tradable goods in that country by a greater proportion than in other countries, and thereby cause a real depreciation of that currency and an appreciation of the U.S. dollar. On the contrary, Lizardo and Mollick (2010) [6] argue that as oil price goes up, the supply of U.S. dollars relative to the oil exporter's currency goes up which would lead to a depreciation of the U.S. dollar. However, Zhang, Fan, Tsai and Wei (2008) [7] documents that a change in the U.S. dollar exchange rate does Granger cause oil price changes, but a change in oil price does not significantly Granger cause U.S. dollar exchange rate changes. They further prove that the U.S. dollar depreciation is a crucial reason for the recent soaring oil price. On the contrary, Coudert, Mignon, and Penot (2008) [8] document that an oil price increase is linked to a dollar appreciation in the long run and therefore, the situation of a weakening dollar and an increasing oil price is atypical.

However, while there is a considerable amount of literature on the correlation between oil prices and the U.S. dollar values, none of the previous research have attempted to examine whether that correlation is constant or varying over time.

An understanding of the conditional correlation between crude oil prices and the U.S. dollar exchange rates is essential for risk management and asset allocation since oil is one of the dollar-denominated assets often included in the commodity portfolios of most serious individual and institutional investors. Furthermore, oil prices and exchange rates are among asset prices which are likely to respond instantly to economic news and developments in financial markets. The purpose of this study is to analyze the intertemporal interactions of crude oil prices and the U.S. dollar trade-weighted exchange rates. To this end I allow the conditional covariance matrix of oil and the dollar exchange rate returns to vary over time, according to a bivariate GARCH model since the evidence of heteroskedastic covariances among other financial assets[5] has been well documented in numerous previous studies.

This study hypothesizes that the conditional correlation between crude oil prices and the U.S. dollar exchange rates is time-varying rather than constant over time. Generally, oil price and the value of the dollar tend to have a negative relationship. An increase in the value of the dollar causes more expensive oil prices in non-dollar currencies which should result in a downward pressure on oil demand and hence lower oil prices in dollar and a decrease in the dollar value implies cheaper oil prices which would result in more money flowing to the oil market and hence drive up oil prices. However, oil prices are also impacted by international supply-demand shifts, such as changes in the global oil demand and supply conditions, responses to geopolitics, institutional arrangements (such as the OPEC), and the dynamics of the financial markets, which are not caused by changes in the value of the U.S. dollar. In those cases, there should be no correlation between oil prices and the U.S. dollar exchange rates. Occasionally, there may be forces, such as interest rate movements or the global growth outlook, that simultaneously increase or decrease oil prices and the value of the dollar, resulting in a positive correlation between the two. Hence, I test the hypothesis of a time-varying correlation between crude oil prices and the U.S. dollar exchange rates.

To the best of my knowledge, this is the first study to examine whether the conditional covariance and correlation between crude oil prices and the U.S. dollar exchange rates vary over time. Although the relationship between oil prices and the dollar value has been investigated previously, no study has yet focused on the time-varying correlation between the two and this paper is a first step toward filling this gap.

The paper is organized as follows. The data is presented in Section 2. Section 3 analyzes the bivariate GARCH model for the conditional covariance between crude oil prices and the U.S. dollar exchange rates. Section 4 presents the results and Section 5 concludes the paper.

[5]See, for example, Bollerslev et al. (1988) [9], Harvey (1989) [10], Bodurtha and Mark (1991) [11], Fleming, Kirby, and Ostdiek (1998) [12] and Goeij and Marquering (2004) [13].

2 Data

The crude oil data used in this study consist of daily closing prices of futures contracts traded on the New York Mercantile Exchange (NYMEX). Crude oil futures contracts, which began trading on the NYMEX on March 30, 1983, trade in units of 1,000 U.S. barrels. The sample period is January 1, 1997 to December 31, 2012 totaling 4,010 daily observations. This sample period includes both the fall and rise of oil prices and U.S. dollar value where the years 2002 and 2008 are supposedly the principal turning points. Crude oil prices are from the Energy Information Administration, downloaded from http://www.eia.gov/dnav/pet/pet_pri_fut_s1_d.htm.

Futures prices are used in place of spot prices for the following reasons. First, futures prices are the major prices in the crude oil market. The NYMEX crude oil futures contract is the world's most liquid forum for crude oil trading and is used as a principal international pricing benchmark. Crude oil futures prices are also the prices reported in newspapers. Second, the futures market for crude oil is liquid and centralized while spot markets are localized and illiquid. Third, futures prices are the prices normally used in most oil risk management contracts such as swaps and options.

To examine volatility in a GARCH type framework, I utilize daily log returns[6] defined as $r_{1,t}=ln(P_t/P_{t-1})$ wherein P_t is the price of the futures contract on day t and P_{t-1} is the price of the same contract the previous day. As traders often cover their positions on the last trading day of a contract’s life, trading volume and open interest decline and price volatility increases substantially. To avoid this “thin market” problem, in constructing the r_t series I replace the return of the nearest contract on its last trading day of each month with that of the second nearest contract.

As a measure of the exchange value of the dollar, I use the trade-weighted average of the foreign exchange value of the U.S. dollar against a subset of the broad index currencies that circulate widely outside the country of issue, including the Euro Area, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden. The exchange index data is from the Federal Reserve Statistical releases, downloaded from the following website (daily h10 reports): http://www.federalreserve.gov/releases/h10/summary/indexn96_b.htm. The index value is set 100 in March 1973 and calculated using the formula: $EI_t = EI_{t-1} \times \prod_{j=1}^{N(t)} (e_{j,t}/e_{j,t-1})^{w_{j,t}}$, where EI_t is the value of the index at time t, $e_{j,t}$ and $e_{j,t-1}$ are the prices of the U.S. dollar in terms of foreign currency j at times t and $t-1$, $w_{j,t}$ is the weight of currency j in the index at time t (based on annual data on international trade), $N(t)$ is the number of foreign currencies in the index at time t, and $\sum_j w_{j,t} = 1$. I also utilize daily log returns for the U.S. dollar exchange rate which is defined as $r_{2,t}=ln(EI_t/EI_{t-1})$ wherein EI_t is the index value on day t and EI_{t-1} is the index value the previous day.

[6]The daily crude oil “returns” are used to measure price changes only. These “returns” are not investment returns since no money is actually invested.

Table 1: Descriptive statistics for daily log changes of crude oil prices and U.S. dollar exchange rates

	Crude oil	U.S. dollar's value
Mean (x10^2)	0.0318	-0.0047
Minimum	-0.1654	-0.0411
Maximum	0.1641	0.0216
Standard deviation	0.0247	0.0046
Annualized Standard deviation	0.4710	0.0883
Skewness	-0.1134	-0.3151
Kurtosis	4.0796	10.4830

This table gives descriptive statistics for the daily log changes of crude oil prices and U.S. dollar's exchange index value for the period from January 01, 1997 to December 31, 2012.

Table 1 provides a summary of the descriptive statistics at the daily frequency. Both log changes in crude oil prices and U.S. dollar exchange rates are characterized by excess kurtosis, indicating that their empirical distributions have fatter tails than a normal distribution. Moreover, as illustrated in Figure 1, crude oil prices and the dollar exchange rates exhibit volatility clustering in that large price changes tend to be followed by large price changes of either sign. GARCH models are attractive and empirically successful in that they are, to a large extent, able to explain both the volatility clustering behavior and the excess kurtosis of the empirical distribution of returns.

3 Model Specification

I utilize a multivariate generalized autoregressive conditional heteroskedasticity (GARCH) model to test for a time-varying covariance between crude oil prices and the U.S. dollar exchange rates. While the GARCH specification does not follow any economic theory, it provides a good approximation to the heteroskedasticity typically found in financial time-series data. The development of multivariate GARCH models represents a major step forward in the modeling of volatility since these models allow for time-varying conditional variances as well as covariances. Among various multivariate GARCH models in the literature[7], the Diagonal VECH model introduced by Bollerslev, Engle and Wooldridge (1988) [9] is one of the most popular since it is a natural extension of the univariate GARCH model and is easy to understand. Moreover, recent studies by Ferreira and Lopez (2005) [23] and Bauwens, Laurent and Rombouts (2006) [24] show that among the most popular multivariate models, the diagonal VECH seems to provide the best out-of-sample (co)variance forecasts. In the general Diagonal VECH model, the conditional covariance follows a multivariate GARCH (1,1) process:

[7]Some examples include Harvey (1989) [10], Bollerslev (1990) [14], Bodurtha and Mark (1991) [11], Ng (1991) [15], Ng, Engle, and Rothschild (1992) [16], Braun, Nelson, and Sunier (1995) [17], Engle and Kroner (1995) [18], Nijman and Sentana (1996) [19], Ng and Kroner (1998) [20], Ding and Engle (2001) [21], and Engle (2002) [22].

$$H_t = \Omega + A \otimes \boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{'} + B \otimes H_{t-1} \quad (1)$$

where H_t is the conditional covariance matrix at time t, the coefficient matrices A, B and Ω are NxN symmetric matrices, and the operator $\otimes$ denotes the Hadamard product (element by element matrix multiplication). Since H_t must be symmetric, so must be the parameter matrices, and only the lower portions of these matrices need to be parameterized and estimated.

I hypothesize that the conditional covariance matrix of crude oil and exchange rate returns follows a bivariate GARCH process and estimate the following Diagonal VECH model:

$$\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t$$
$$\boldsymbol{\varepsilon}_t \sim N(0, H_t) \quad (2)$$

$$H_t = \Omega + A \otimes \boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{'} + B \otimes H_{t-1} \quad (1)$$

where $\mathbf{r}_t = (r_{1,t}, r_{2,t})'$ is a (2x1) vector containing crude oil and exchange rate returns and H_t is a (2x2) conditional covariance matrix. Let H_t follow the most unrestricted process among all Diagonal VECH models where the parameters in the matrices Ω, A, and B are allowed to vary without any restriction, the model may be written in single equation format as:

$$(H_t)_{ij} = (\Omega)_{ij} + (A_{ij})\varepsilon_{j,t-1}\varepsilon_{i,t-1} + (B)_{i,j}(H_{t-1})_{ij} \quad (3)$$

where, for instance, $(H_t)_{ij}$ is the *i*-th row and *j*-th column of matrix H_t. Ω is a (3x1) parameter vector; A and B are (3x3) diagonal parameter matrices. The covariance equations are estimated by maximum likelihood.

4 Results

This section presents the estimation results of the intertemporal correlation between crude oil prices and the U.S. dollar exchange rates.

Table 2: The Diagonal VECH model of the crude oil and U.S. dollar exchange rate covariance matrix

	Univariate GARCH (1,1)		Diagonal VECH
	U.S. dollar's value	Crude oil price	
Ω(1,1)	0.0015**		0.0015**
	(0.0005)		(0.0005)
Ω(1,2)			-0.0035*
			(0.0020)
Ω(2,2)		0.2051***	0.1935***
		(0.0437)	(0.0421)
A(1,1)	0.0376***		0.0362***
	(0.0053)		(0.0050)
A(1,2)			0.0268***
			(0.0074)
A(2,2)		0.0693***	0.0651***
		(0.0067)	(0.0064)
B(1,1)	0.9549***		0.9563***
	(0.0074)		(0.0071)
B(1,2)			0.9290***
			(0.0275)
B(2,2)		0.8968***	0.9026***
		(0.0123)	(0.0118)

This table reports the maximum-likelihood estimation results of Equations (1-2) using data from January 1, 1997 to December 31, 2012 (T=4,010). Standard errors are shown in parentheses. (***), (**) and (*) designate estimates significantly different from zero at the 0.001, 0.01 and 0.05 levels, respectively.

Estimation results from specification (1-2) are presented in the fourth column of Table 2. In order to provide some intuition on the bivariate model parameters, I present the estimates of the univariate GARCH(1,1) specification for exchange rate and crude oil volatilities in the second and third columns of Table 2. Results from Table 2 indicate that the bivariate GARCH estimates of volatility persistence for exchange rate and crude oil returns are close to, and not significantly different from, the univariate GARCH (1,1) estimates.

The estimate of Ω(1,2), the unconditional mean of the covariance between crude oil and exchange rate returns, is negative and significant at the 0.05 level, which is consistent with the observation of a negative correlation between crude oil prices and the value of the dollar.

The estimates of $A(1,2)$ and $B(1,2)$ (the ARCH and GARCH terms in the covariance equation) are both positive and significant at the 0.01 level, implying that the covariance between crude oil prices and the value of the dollar tends to cluster over time.

The positive estimate for $A(1,2)$, the ARCH term, means that shocks to oil prices and the U.S. dollar exchange rates of the same sign affect the conditional covariance positively, while shocks of opposite signs affect the forecasted covariance negatively. Apparently two negative (or positive) shocks lead to a significant increase in next period's covariance. Given that the unconditional mean of the covariance, $\Omega(1,2)$, is significantly negative, two shocks of the same sign would decrease and two shocks of opposite signs would increase the predicted covariance in absolute value terms. A significantly positive estimate of $A(1,2)$ also indicates that causes of the correlation between oil prices and the dollar's value tend to persist. If on one day the change in the dollar price of oil is largely due to a change in the dollar's value, there is a tendency for the next day's change in oil prices to be primarily caused by changes in the dollar's value as well. On the other hand, if on one day the change in the dollar price of oil is caused primarily by factors other than the dollar's value, there is a tendency for those to be the primary causes of changes in the dollar price of oil on subsequent days.

To examine whether the time variability in the covariance of crude oil and exchange rate returns is solely due to variation in the two variances, I calculate the conditional correlation coefficient at time *t+1*, $\rho_{12,t+1}$:

$$\rho_{12,t+1} = \frac{Cov_t\{r_{1,t+1}, r_{2,t+1})}{\sqrt{Var_t(r_{1,t+1})Var_t(r_{2,t+1})}} \tag{4}$$

If $\rho_{12,t+1}$ is constant over time, the variability in covariance is solely due to variation in variances. In that case, modeling of time-varying covariances is not very interesting, as all the dynamics are captured in variances. To test the null hypothesis of a constant correlation coefficient, I estimate the Constant Conditional Correlation (CCC) model and test the Diagonal VECH model against the CCC model. The likelihood ratio test statistics is 9.8 with 2 degrees of freedom and significant at the 0.01 level. Therefore, the Constant Conditional Correlation hypothesis is rejected, clearly showing that the correlation between oil prices and the value of the dollar is not constant over time. Consequently the variability in covariances is not solely due to time-varying variances, and modeling time-varying covariances is important.

Figures 3 and 4 present the plots of the conditional covariance forecasts and the estimated correlation coefficient over time, based on the estimation results of the diagonal VECH model as presented in Table 2. The figures show that the conditional covariance and the correlation coefficient vary considerably over time.

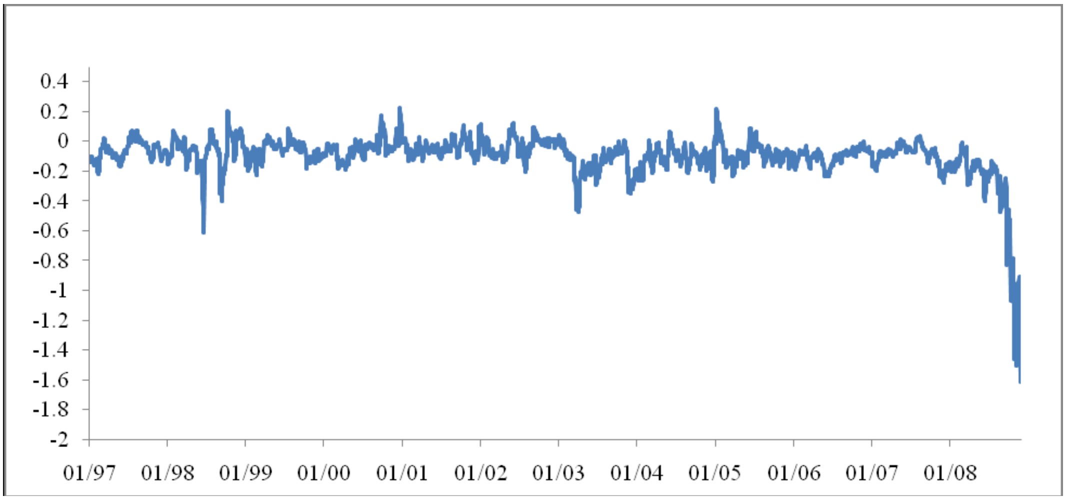

Figure 3: Conditional covariance between crude oil prices and the value of the dollar

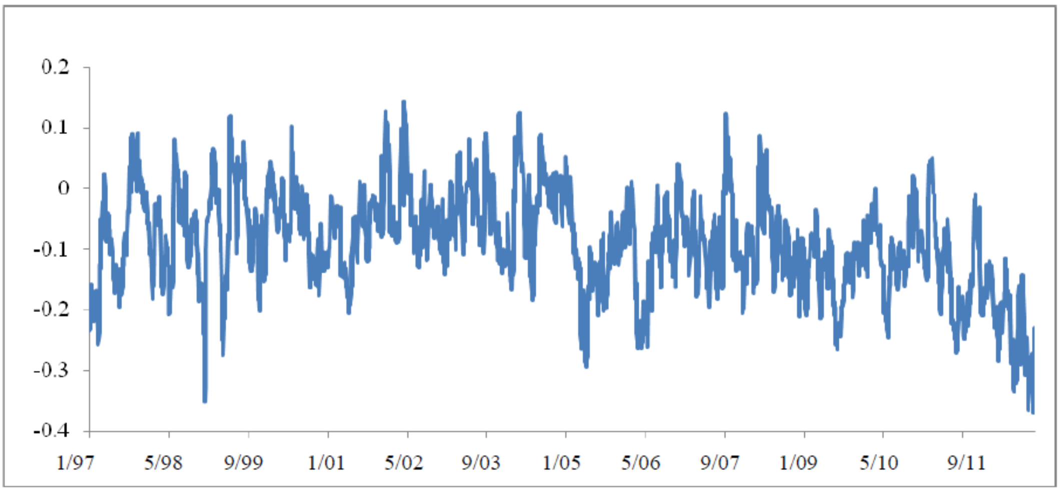

Figure 4: Conditional correlation between crude oil prices and the value of the dollar

5 Summary and Conclusions

Crude oil prices and the U.S. dollar value tend to move together, and appear to have been negatively correlated in recent years. A dollar appreciation (depreciation) is typically associated with lower (higher) oil prices. This paper aims to analyze the intertemporal interaction between crude oil prices and the U.S. dollar exchange rates from January 1997 through December 2012. To this end, the study assumes that the conditional covariance matrix follows a bivariate GARCH process. The contribution this paper makes is to provide strong evidence of a time-varying conditional covariance and correlation between oil prices and the U.S. dollar exchange value.

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