Estimation of Murabaha Margin

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Abstract

In the Last decade, the Islamic Finance market has grown significantly, and attracted several investors. However, the profitability of Islamic financial products remains one of the main concerns of portfolio managers as well as investors. Among the products that are mostly offered by the Islamic banks is the Murabaha in which return can not only be known but also guaranteed owing to the low efforts and costs invested. Indeed, the profit margin has a great importance in the elaboration of the Murabaha contract, and stands out as one of the main components of profitability from a commercial viewpoint. In this article, we propose a new approach to estimate the profit margin of Murabaha using the stochastic process and portfolio techniques. The aim is to determine an interval allowing Islamic banks to anticipate and check their profit margin in order to ensure the profitability of the Murabaha investment. This research comes up with a model which describes the profit margin of the Murabaha investment with meaningful benchmarks that inform the bank on the eventual ROI (Return on Investment).

1 Introduction

Islamic finance has rapidly gained worldwide renown. The number of Islamic financial institutions has increased considerably around the world. The Moroccan authorities have recently reached an important stage in the process of adopting Islamic finance through issuing a law defining the framework, regulations and arrangements that outline and frame the activities of Islamic banks in Morocco. However, financial risk remains a major concern

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for investors in project evaluation, as Islamic financing contracts must not go against the fundamental principles of the Charia, which requires that any profit must come from a contract allowing sharing both profit and loss risks. Therefore, the assessment of investment projects remains a challenge for financials and a crucial phase in analysis that should precede any investment. For a selection of Islamic finance to comply with Charia, the investment must take on some of the risks related to either the financed asset or the activity. The Murabaha contract is the most prevalent product in the market of Islamic banks , but the estimation of the profit margin(the key factor in the marketing of this contract) is based on the interest rate, which is prohibited by Charia. Therefore, it is suitable to determine a different approach which is likely to avoid the interest rate. In this study, the proposed approach undertakes to estimate the profit margin of Murabaha by avoiding the interest rate in order to allow an application of Islamic finance in its real regulatory framework. This research advances a model that describes the profit margin using signposts that inform banks, and allows them to avoid the classical tools in the assessment of the profit according to which managers would be allowed to take a reasonable decision based on a real simulation of the project. In other words, this approach constitutes a viable alternative to bypass the existing rules of classical financial tools prohibited by Charia.

Firstly, this research provides a theoretical review of the Murabaha product through a presentation of its concepts and foundations. Secondly, the difference between the conventional credit with interest and Murabaha contract will be shown. In the third part, we will deal with the profit margin and its evaluation. The final part will be devoted to giving a detailed description of our approach about the estimation of the profit margin of the Murabaha contract.

2 The Murabaha Product

2.1 Definition and Concept

The Murabaha Product is a profitable sales agreement between an Islamic bank and a customer. The customer asks the bank to buy on his/her behalf a cash commodity. Then, he /she commit to take back the property through a deferred payment, including the profit margin of the Murabaha contract. This payment can be made in one shot or spread over several maturity dates. As this transaction must respect the 'charia' principles, the bank must first buy the property and sell it later to the customer. Moreover, the property must be tangible, and the customer must be informed and agrees to pay the new price (purchase price plus profit margin).

Murabaha synoptic diagram

Murabaha synoptic diagram
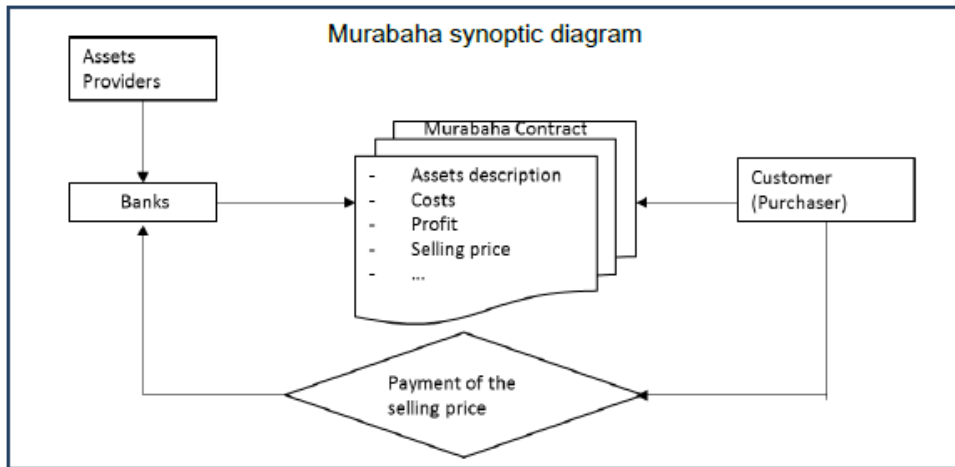
When all these conditions are met, the parties (financier and customer) sign "The Murabaha agreement" in which all terms and conditions are specified including the purchase price, profit margin, and date of payment

2.2 The Murabaha Product and Credit Interest

The credit with usual interest consists in borrowing an amount of money for a limited period with an additional payment added to the original amount. However, this additional sum of money, representing a 'Riba', is prohibited in the Islamic religion. The difference between Murabaha and credit with interest appears at several levels that can be described as follow:
- First, Murabaha is a contract requiring an exchange of one commodity against an amount of money which is the initial market price plus the fixed margin. Thus, Murabaha contributes to boosting the economy and keep it away from the ghost of inflation. On the contrary, credit with interest is an exchange of money against money since the surplus added to the original price is different depending on the time of borrowing, which leads to inflation.
- Moreover, the Murabaha contract is not a debt at the moment of signing the documents of the promise to purchase because the merchandise is not the customer's property who does not obtain it until the Islamic bank has obtained it first. The contract is signed subsequently. On the other hand, during the contract of credit interest, the debt is built up as of signature of the contract (either on delivery of the goods or before).
- On another side, the use of the Murabaha contract does not commit the customer to pay the amount to the Islamic bank in case the goods have been damaged before delivery. This amount remains bond to payment for traditional bank regardless of its safety or deterioration since it is customers who handled their purchase.
  In addition, the use of Murabaha contract does not bind the customer to additional interest to the Islamic bank in case of delay in payment while she /he must pay interest to the classical bank since interest is kept on the customer's own account.
Finally, the Murabaha contract makes the insurance of goods (at the time of the promise to purchase) a liability of the Islamic bank, so the customer takes no risk linked to the goods. However, the credit with interest contract makes the insurance of the goods a customer's responsibility. Thus, the insurance charges will be incurred by the customer.

2.3 The Profit Margin in the Murabaha Contract

The profit margin in the Murabaha contract must be known in advance, as well as it is not enough to content oneself with the overall price. In fact, benefit has to be determined at a definite value. The profit margin in the Murabaha contract may be estimated from the Libor indicator (considered as a world reference for the rate of short-term interests) before elaborating the contract during the promise period (which is not considered as a sale).

The profit margin will not be fixed but variable depending on the time period and will be calculated at the end of the payment period of credit. It follows that when the purchasing officer establishes the Murabaha contract, the profit margin cannot be based on the Libor indicator. Otherwise, the contract will not be in accordance with Charia bearing in mind that a condition of the legitimacy of Murabaha is the fact that the profit margin is fixed and known in advance.

Banks need to use another indicator that measures and assesses the profit margin of Murabaha contract beyond the usual Libor indicator used by conventional banks   in order to clear any doubts about the conformity of the Murabaha contract to 'Charia'. This can establish a serene climate among users of these Islamic products. Therefore, it helps to keep Islamic banks away from unbalanced competitiveness between them and conventional banks. Within this framework, the *Islamic Fiqh Group* has submitted a recommendation to accelerate the search for an indicator allowed by the Charia. This indicator is to be an alternative on which the value of the interest will be built in order to set the profit margin of the Murabaha contract.

One of the best proposals that have been advanced provided In this vein is Dr. Samir Kantakji 's assiduities. This approach consists mainly in using the annual profit distributions of eight financial institutions and utilizing their arithmetic mean to search the "price of sacrifice" suitable for all work areas and consider the minimum as the cost of the replacement opportunity. Still, Dr Kantakji's proposal remains ambiguous when it comes to its formulation and its application to particular situations. This limitation has constituted an impulse for us to search for an alternative for measuring the Murabaha margin.

3   Main Results: Estimation of the Murabaha margin

Before elaborating the approach concerning the estimation of the Murabaha margin further, it is suitable to check whether this approach, under current practices, complies with Charia. In other words, are the common practices to assess the Murabaha margin based on the interest rate?

For this purpose, we need to proceed to a modelization of the fluctuations of the Murabaha margin in relation to the margin of the conventional loan.

3.1 Relationship between Murabaha Margin and Interest Rate:

This section aims at checking the likelihood of a relationship existing between the Murabaha profit margin and the interest rate. In this perspective, statistical tools will be used to study and analyze this correlation.

A statistical tool that is used to study the relationship between two variables x and y is the simple regression. In the present study, we define y as the profit margin, and x as the interest rate. The corresponding model is written as follows:

$$y = \beta_0 + \beta_1 X + \varepsilon \tag{1}$$

Where

✓ β_0 and β_1 are the unknown model parameters

✓ ε is a random variable that refersto an error term satisfying the following conditions:

$E(\varepsilon) = 0$ and cov $(\varepsilon) = \sigma^2$

An empirical study was conducted on a sample of 112 observations with 2 variables on a financial institution database of alternative investment products for different investments of the Murabaha product over a period extending from 3/2012 to 2/2015. Then, the same study was carried out on conventional bank interest rates over the same period by comparing the same investments, which allowed us to generate a model of the Murabaha margin.

At this level, the distribution of the variables for modelization is as follow:

- The dependent variable   :   Murabaha margin
- The explanatory variable :   Classical Margin_credit

The hypothesis tested consists in checking whether classical Margin_credit explains variability (a significant part) in the model

3.1.1 Results of modeling the relationship between Murabaha margin and Margin_credit Classic

Model Summary

Model	R	R-square	Adjusted R-squared		Change in statistics	
					Change in R-two	Variation de F
1	,995a	,989	,989	48331,88246	,989	9979,966

The R^2 adjusted of this model is estimated to 98.9%, which is a high level. This means that the Classical Margin_credit explains 98% of the variability of the Murabaha margin.

3.1.2 Analysis of the Variance results:

ANOVA

Model		Sum of squares	Ddl	Average square	D	Sig.
1	Regression	23312908644363,766	1	23312908644363,766	9979,966	,000b
	Residue	256956794801,894	110	2335970861,835		
	Total	23569865439165,660	111			

The probability associated with F is lower than 0.0001.This means that the explanatory variable (Classical Margin_credit) makes a significant amount of information to the model. Statistics of residues

	Minimum	Maximum	Average	Standard deviation	N
Forecast	58137,9805	2987803,2500	329839,5536	458286,15952	112
Residue	-232903,15625	124565,27344	,00000	48113,67871	112
Forecast error	-,593	5,800	,000	1,000	112
Residual error	-4,819	2,577	,000	,995	112

Coefficients

Model		95.0%% confidence intervals for B	
		lower bound	upper limit
1	(Constant)	31652,602	53047,771
	Credit Margin Classic	,300	,312

Equation 3.1 : Equation of this model

$$Murabaha\ margin = 4250{,}186 + 0{,}306 * Conventional\ loan\ margin \qquad (2)$$

The results show that the Murabaha margin varies in the same direction as the margin of classical credit, which implies the existence of a relationship between the two variables.

Where:

$$Conventional\ loan\ margin = Initial\ investment\ amount * Interest\ rate \qquad (3)$$

Replacing the conventional credit margin in equation (1) yields:

$$Murabaha\ margin = 4250{,}186 + 0{,}306 * Initial\ investment\ amount * Interest\ rate \qquad (4)$$

This equation shows that there is a strong correlation between the two variables (Murabaha margin and Interest rate). According to this finding, we can deduce that there is a relationship between the Murabaha margin and the interest rate used in conventional banking. This result does not meet a key requirement of the Charia, which prohibits the credit based on interest rate 'Riba'. This leads us to seek a different estimation approach, avoiding interest rate.

3.2 Estimation Approach of the Murabaha Profit Margin

The assessment of the profit margin considered for this article is for investments in the portfolios of Islamic financial products. The added value of our approach is to allow the bank to estimate the Murabaha margin without resorting to the interest rate. Thus, the Islamic bank can anticipate the profit margin that can be made if the capital (C) is invested in an optimal portfolio (P) having positive returns. As these returns are following a stochastic process, they can be forecast in our approach according to Box& Jenkins' techniques of forecasting.

3.2.1 Portfolio elements

A financial portfolio is primarily a set of title values or stocks allowing making profitability in the form of dividends paid over a given period. The profitability of a share during a period T is as defined by the equation:

$$R = \frac{Div_1 + P_1 - P_0}{P_0}$$ (5)

where

✓ P_1: The course of action i at the end of the period T

✓ P_0: The share price i at the beginning of the period T at time t 0

✓ Div_1: The dividend at the end of the period T

Profitability is assumed to be a random variable in which the values are normally distributed: $R \sim N(r, \sigma)$ where r is the expectation and σ the standard deviation.

In the P portfolio, if the number of securities is N, the expected profitability is:

$$r_p = \sum_{i=1}^{N} x_i r_i$$ (6)

Risk is defined by portfolio volatility corresponding to its standard deviation:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sum_i x_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} x_i x_j \sigma_{ij}$$ (7)

3.2.2 The selection of the optimal portfolio theory:

According to Markowitz [4], portfolios characterized by rp maximum profitability expectancy (with a given variance σp2) are defined as efficient. The efficient frontier, which is the set of all efficient portfolios taken together, is found by solving the following program. The portfolios P combine any number n of risky securities, of which the optimization algorithm is written as follows:

$$Min \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \right\}$$ (8)

Under the constraints:

$$\sum_{j=1}^{n} xE[r_j] \geq \rho$$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_j \geq 0 \qquad j=1,\ldots,n$$

We must find the vector of weights x_i which minimizes portfolio variance, while providing a given expectancy r_p, and taking into consideration the fact that the sum of the weights of the securities is equal to one. Thus, for a given r_p, we find the portfolio (weights) of

minimum variance, and by varying r_p the entire efficient frontier is obtained.

Despite its importance in the development of modern finance and its contribution in the field of portfolio management, Markowitz's [4] model has several limitations raised by several practitioners of financial theory.

To address these limitations, and improve the quality of optimization, other approaches have been put forward, the most important of which is Hamza and Janssen's (2). This approach has retained much of Markowitz's[4] model, while being enhanced with interesting adjustments. In the same vein, our approach allows estimating the profit margin using portfolio techniques and stochastic processes.

3.2.3 Elements of a stochastic process

A stochastic process is a series of random variables indexed by time, allowing the modeling of random evolution of a variable over time. Four elements of this process are presented below.

3. 2. 3. 1 Delay Operator:

We Consider a stochastic process $\left(R_T \right), R_T = R_1, R_2 .... R_k$, with R_k represents Return of the portfolio P

The operator Delay denoted L is defined as:

$$LX_t = X_{t-1} \ \forall \ t \in Z \tag{9}$$

3. 2. 2. 2 Delay polynomial:

Given **L** as the Delay operator, a Q order delay polynomial is a polynomial that is written as:

$$\Psi(L) = \sum_{j=0}^{q} \Psi_j L^j = \Psi_0 + \Psi_1 L^1 + \ldots + \Psi_q L^q \tag{10}$$

The application of the delay polynomial $\Psi(L)$ to the stochastic process $\left(R_T \right)$ yields:

$$\Psi(L) R_T = \sum_{j=0}^{q} \Psi_j L^j R_T = \sum_{j=0}^{q} \Psi_j R_{T-j} \tag{11}$$

Using the delay polynomials, Wold's decomposition is written simply as:

$$R_T = d_T + \Psi(L) \varepsilon_T \tag{12}$$

Where $\Psi(L) = \sum_{j=0}^{\infty} \Psi_j L^j$ is an infinite polynomial; with $\Psi_0 = 1$

3. 2. 2. 3 Auto-regressive Representations and moving average
1. Autoregressive Representation AR (p):

A stochastic process (R_T) satisfies an autoregressive representation of p order, denoted AR (p), if:

$$\Phi(L)R_T = c + \varepsilon_T \tag{13}$$

With

$$\Phi(L) = \sum_{j=0}^{p} \Phi_j L^j, \Phi_0 = 1, \Phi_p \neq 0 \quad \text{and} \quad c \in IR \text{ and } (\varepsilon_T) \text{ is a white noise with}$$

$$E(\varepsilon_T) = 0 \text{ and. } Var(\varepsilon_T) = \sigma_\varepsilon$$

2. Moving average Representation MA (q):

A stochastic process satisfies a moving average representation of q order, denoted MA (q), if:

$$R_T = c + \Psi(L)\varepsilon_T \tag{14}$$

With $\Psi(L) = \sum_{j=0}^{q} \Psi_j L^j$, $\Psi_0 = 1$, $\Psi_q \neq 0$, $c \in IR$ and (ε_T) is a white noise with

$$E(\varepsilon_T) = 0 \text{ and } Var(\varepsilon_T) = \sigma_\varepsilon$$

If we set $E(R_T) = m$, we then obtain $c = m$

3. 2. 2. 4 Autoregressive Representation and Moving Average ARMA (p, q)

A stochastic process satisfies a moving average and autoregressive representation

of p and q orders, denoted ARMA (p, q), if:

$$\Phi(L)R_T = c + \psi(L)\varepsilon_T \tag{15}$$

With

$$\Phi(L) = \sum_{j=0}^{p} \Phi_j L^j, \Psi(L) = \sum_{j=0}^{q} \Psi_j L^j, \quad \Phi_0 = 1, \quad \Phi_p \neq 0, \quad \Psi_0 = 1, \quad \Psi_q \neq 0, c \in IR \quad \text{and}$$

(ε_T) is a white noise with $E(\varepsilon_T) = 0$ and $Var(\varepsilon_T) = \sigma_\varepsilon$

The following section describes the empirical research that has been carried out to

apprehend these basic concepts.

3.3 Modelization of the Murabaha Margin

In order to formulate the approach for the estimation of the profit margin of the contract Murabaha, the following hypotheses are posited:
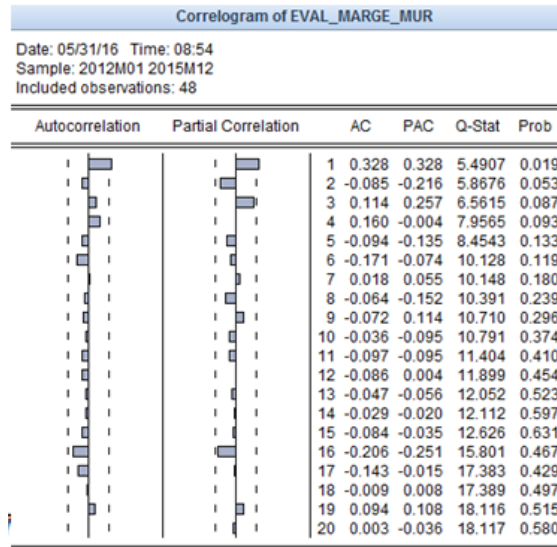
-     Firstly, the Islamic bank invests only in the Islamic products portfolio designed by P (as in Mudaraba, Murabaha, Musharaka, Salam and so on)

-     Secondly, for any demand of Capital C we can find an approximated amount C_p (close to it) that has been invested in an optimal portfolio P during various prior periods $T_1...T_n$ characterized by their respective returns $R_1…R_n$.

Both assumptions stated above consider investment in projects through Mudaraba,Musharaka,and Salam, as the main engine of the financial activity of Islamic banks.Hence, the presence of Islamic products portfolios in which the amount Cp has been invested during previous periods, represents a key objective of Islamic banks.

In the Murabaha contract, the revenue of the Islamic bank is compound of the initial value (market price) of the goods added to the profit margin mentioned in the contract. When a customer makes a request to the Islamic bank for the acquisition of a property with the amount C through the Murabaha act, he or she will pay back this amount in instalments after a certain period T on which all parties agree in the Murabaha contract. Thus, the customer deprives the bank of the amount C during the period T (the time required for the payment of all installments).So, the Islamic bank misses the opportunity of investing the amount C in a portfolio P to generate revenue represented by the return R_T. Hence, when establishing the Murabaha contract, the profit margin must take into account the value of the return R_T that Islamic banking can make if it invests the amount C in the portfolio P for the duration T.

Accordingly, the approach we presented in this work is adopted which stipulates that the capital C is approximated by a Cp amount close to the value that has been invested in an optimal portfolio P during earlier periods $T_1 ... ..T_n$ and resulted in positive returns $R_1... R_n$ during this period. The advantage of this approach isthat it allows the Islamic bank to anticipate and predict the performance it can achieve if it invests this amount C in the optimal portfolio P during the period T when the customer makes a request for the acquisition of property with the amount C through the contract Murabaha. The idea is that an Islamic bank must anticipate the return R_T it can get if it invests this amount C in a portfolio P during this period T. Then, the Islamic bank evaluates the profit margin equivalent to the value of the return R_T using our approach.

From a technical perspective, the perceived return R_T depends not only on to the value of C amount that will be invested in an optimal portfolio P, but also on the duration T of the investment. Therefore, the return $R_T$is a sequence of random variables indexed bytime,thereby followinga Stochastic Process. So we can model the random trend of this variable in time to predict its future value at the duration T. The determination of our model of random series of the revenue R_T is based on previous values of this revenue $(R_1…R_k)$ that the optimal portfolio P (consisting of 45 stocks of Islamic products ,mainly through Mudaraba, Musharaka, Ijar, has performed. Building on Box and Jenkins' methodology for the time series R1…Rk, particularly by following practical steps of this method, we have generated the corellogram below:

Correlogram of EVAL_MARGE_MUR

Date: 05/31/16 Time: 08:54
Sample: 2012M01 2015M12
Included observations: 48

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.328	0.328	5.4907	0.019
		2	-0.085	-0.216	5.8676	0.053
		3	0.114	0.257	6.5615	0.087
		4	0.160	-0.004	7.9565	0.093
		5	-0.094	-0.135	8.4543	0.133
		6	-0.171	-0.074	10.128	0.119
		7	0.018	0.055	10.148	0.180
		8	-0.064	-0.152	10.391	0.239
		9	-0.072	0.114	10.710	0.296
		10	-0.036	-0.095	10.791	0.374
		11	-0.097	-0.095	11.404	0.410
		12	-0.086	0.004	11.899	0.454
		13	-0.047	-0.056	12.052	0.523
		14	-0.029	-0.020	12.112	0.597
		15	-0.084	-0.035	12.626	0.631
		16	-0.206	-0.251	15.801	0.467
		17	-0.143	-0.015	17.383	0.429
		18	-0.009	0.008	17.389	0.497
		19	0.094	0.108	18.116	0.515
		20	0.003	-0.036	18.117	0.580

Analysis of this correlogram shows the term « AR » in 1 and the term MA in 1. Accordingly, the series of returns follows a process ARMA (1,1)

Now we are able to estimate the Murabaha margin which must be determined within an interval [Rmin, Rmax] in order to guarantee profitability of the investment. Therefore, we can identify a valid model which enables us to forecast the (minimal and maximal) margin of return of the invested capital C in the optimal portfolio of Islamic products.

EViews - [Equation: UNTITLED Workfile: EVAL_MARGE_MUR::Untitled\]

Dependent Variable: C
Method: Least Squares
Date: 05/31/16 Time: 08:57
Sample (adjusted): 2012M02 2015M12
Included observations: 47 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 2012M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.000000	1.61E-12	6.23E+11	0.0000
MA(1)	0.598513	1.87E-05	32032.20	0.0000

Mean dependent var	1.000000	S.D. dependent var		0.000000
S.E. of regression	7.42E-13	Sum squared resid		2.47E-23
Durbin-Watson stat	0.004851			

Inverted AR Roots	1.00
Inverted MA Roots	-.60

Hence, the model of Return below: $\quad R(T) = R(T-1) + 0{,}6\varepsilon(T-1)$

This model allows the bank to forecast the return of the capital C invested in the portfolio of islamic products representing the profit margin of the Murabaha contract.
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4   Conclusion

As is commonly known, the profit margin of the Murabaha contract has a great importance in its marketing. Since Islamic banks currently use the international interest average (Libor) to determine the value of this margin, it was necessary to find an indicator and a way to measure the profit margin as recommended by the Islamic Fiqh. In so doing, we have proposed an *eventual* approach in order to measure the profit margin of the contract Murabaha. This approach has been built on hypothesis, and it is based on previous returns of optimal portfolios of Islamic products in which the same amount C of merchandise has been invested in a previous period for a determined duration. This approach is also based on the formulation of a forecasting model ARMA that allows predicting the return interval which represents the profit margin of the Murabaha contract. Thus, this research has contributed with a visionary pre-econometric modelization showing to the Islamic bank that Murabaha margin will be determined through the ARMA time series. The advanced model will be based on previous data of returns used in products of an optimal portfolio (made up of various other Islamic Murabaha products) of a known investment during past periods, thereby allowing the bank to be informed on the margin interval. Ultimately, the model can be used by Islamic banks to set the forecast range of the income R_T. These banks can also choose to check their margin and ensure the profitability of the Murabaha investment product.

Finally, the above findings represent an impetus to think about an approach around the modelization of the revenue for an estimation of the risk. In this respect, subsequent research may focus on effective management of risk for a good evaluation of an investment project and better decision making.

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