

Modelling and Assessment of Product Musharaka Islamic Financial Risk

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Abstract

In this paper, we present a new approach to modeling and risk assessment of Musharaka Islamic financial product. Also we will present the basic principles of Musharaka, its features, its various forms and its risks.

The objective of this approach is to provide a decision-making support tool for managers to manage risks related to the product of Musharaka in Islamic banks.

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1 Introduction

Islamic banking is a banking model that was able to prevail in the greatest country in the world, reflecting an inescapable success of products offered. These have been strongly in competition with those offered by conventional banking, given the benefits they have brought to the individual as a social interest. The Musharaka is one of the strongest products in Islamic finance because it represents a form of investment par excellence, to serve the individual but also society. Generally, the concept of investment is accompanied by that of risk.

Thus, this type of contract represents several risk types for the Islamic bank. Some of them are common to the entire banking sector; others are specific to Islamic banks.

We speak mainly of the risk of withdrawal from the contract partner called credit risk, incompetence of human resources or material failure called operational risk of fluctuations in market variables on the project called market risk, difficulties encountered compared the applied laws and regulations called legal risk, fiduciary risk and finally notoriety or if the bank does not really meet the expectations of its customers.

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The Musharaka contract traces a new form of investment that is introduced only recently in the banking sector, which explains the difficulty to date to determine appropriate methods and approaches for a better assessment of its risk.

Thus, this work will be organized as follows: In the first section we will present a definition of the product Musharaka and its different types of risks. The risks of the Musharaka will be defined in a second section. Then, a third section will present a mathematical evaluation of the product risk Musharaka. Finally, a digital implementation will be presented at the fifth section.

2 Moucharaka Product

Musharaka means any contract for the acquisition of holdings by a credit institution in the capital of an existing company or establishment in order to make a profit.

Both parties participate in losses in proportion to their participation and profits in a predetermined proportion.

- On sharing the profit, according to Ba (1993), two measures may be adopted:
 - The sharing is done according to an agreement Presets (thesis of the Hanbali school or Hanafi);
- The profit sharing depends on the prior interests of each party (thesis and Maalikis Chaafites).

Musharaka type Entries can only be made in limited companies where the responsibility of the credit institution is limited to his contribution to the company in question.

Contracts of this type of product, must precisely define the obligations and rights of each party and the terms and conditions governing their relations. They must contain clauses specifying in particular:

- The nature of Musharaka contract;
- The object of the Moucharaka operation;
- The amount of capital and the percentage held by each party;
- The nature of the contributions (cash or in kind) of the parties;
- The duration of the transaction;
- The procedure for distribution of profits, based on an agreed proportion;
- The guarantees possibly made in favor of the bank, the customer only ensuring the management of the company to cover losses due to negligence and other similar acts;

2.1 Types of Musharaka

The Musharaka may take the following forms:

- **Musharaka Tabita (fixed)**: the credit institution and the customer remain partners in the company until expiry;
- **Musharaka Moutanakissa (decreasing)** the credit institution withdraws from the company as to and when the progress of the funded project. Indeed, the Contractor agrees to gradually and periodically buy out the credit institution, which is divided into several units in advance. The terms and conditions of dissolution of the Musharaka Moutanakissa, the contract must specify in addition to the terms of its participation over by the credit institution.

The Musharaka Tabita contract must not contain any provision designed to ensure one party recovery of his contribution regardless of the results of the operation.

2.2 Musharaka Contract Specifications

The Musharaka contract is marked by several characteristics, namely: It is a contract between two or more people in the capital and management of a project; It is essentially based on a synergy of efforts to serve the individual interests as well as social, representing a point stressed by the Sharia; The signing of the contract should be preceded by a clear determination of capital contributed by each party of the latter in order to deal with any potential conflict when sharing the results; Additional compensation may be awarded to one or more partners to manage the project; The Musharaka can be achieved in partnership with non-Muslim individuals or entities, without adversely on prohibited items to the Sharia while entrusting the management to Muslims; This type of contract provides that the profit will be shared according to the agreement agreed in advance between the partners, but the loss of his hand, depends on the contributions made by its last.

3 The risk of Musharaka

The notion of risk is strongly present in the logic of Islamic banking. The latter is liable alongside the various types of risks to conventional banks, a set of risks specific to Islamic finance sector.

3.1 Risks shared between Islamic and Conventional Banks

In this light, we can highlight the following types of risk:

- **Credit risk:** this is the case where the bank's partner (debtor) is unable to honor its commitment to the bank on time already provided;
- **Liquidity risk:** this is the case where the bank is confronted with a liquidity shortage, it means that it is more liquid funds to supply its current operations and meet its customers;
- **Operational risk:** This risk relates mainly to failures relating to the human factor or techniques that a bank may incur in connection with the completion of its internal processes. Various examples can be given at this level: lack of competence, technological errors ...
- **Market risk:** it is risk associated with fluctuations that a market can know and also various changes to its instruments. We speak mainly of price volatility, changes in interest rates and also the exchange rate.
- **Legal risk:** this is different regulations and laws governing the exercise of the banking activity and thus to facilitate its process or represent a real obstacle to the implementation of the latter.

3.2 Risks Specific to the Product Musharaka

The product "Musharaka" offered by Islamic banking can be considered as a risk product par excellence. Indeed, the risk is present even before the conclusion of the contract,

because the bank is supposed to establish a good project feasibility study , which is the subject of the contract "Musharaka" which necessarily includes a risk margin. Also, several types of risk can be emphasized at this product, namely:

- **Credit risk:** as any other bank, Islamic bank may be faced with a partner of "Musharaka" unable to meet its commitments either for good personal reasons or to reasons related to the unsatisfactory level of performance;
- **Operational risk:** this risk is very present in the case of Musharaka, since this is a new concept launched on the investment market and requires a significant level of skill and the use of new technologies;
- **Market risk:** This risk can be explained by the strong dependence between the success of the project contract relates to various market fluctuations. They can promote and increase the business performance or slow down such price changes;
- **Legal risk:** the legal form of the company Musharaka is also a risk in the event that the shares of the latter are not negotiable.
- **Fiduciary risk:** if the Islamic bank would not be able to respect the principles of the Sharia in launching his product "Musharaka" at the profit sharing as an example, which would affect its image brand among its customers;

4 Mathematics Evaluation of Musharaka Product Risk

When investors decide to invest an amount M in a project P for a period T in collaboration with an Islamic bank using Musharaka Islamic financial product, an important question arises regarding the risk on this investment and return that can generate this project.

In this context we assume the following assumptions:

- The investment in any project is done in collaboration with Islamic bank.
- There is a basic historical returns similar or identical projects in Project P .
- The amount M may be invested in a portfolio of Islamic financial products.
- The value of the investment M in the project P for a period T following a general Wiener.

4.1 Investment Project Return

The value of the investment M in the project P during a period T is a random variable whose evolution over time can be modeled by a stochastic process.

We Suppose that the variation of S is modeled by a general Wiener process, i.e there are two parameters a and b such the process is written as follows:

$$dS = a.dt + b.dz$$

where:

- a is a constant called drift of S .
- b is a constant such $b > 0$ called standard deviation of S .
- dz is a standard Wiener process

z is a Markov of process such the expected increases is equal to zero and the variance of these increments equal to 1 and it satisfies the following two properties:

- The length variation during a short time interval is expressed by $dz = \varepsilon\sqrt{dt}$ where ε is a random variable that follows the normal distribution $N(0,1)$.
- The values of dz for two short length intervals are independent.

Let S_t the value of the investment in the project P on the date t that drift is μS_t which μ represents the expected rate of return on the value of this investment.

The average S_t for a short length of time period dt is equal to $\mu S_t dt$.

We consider that the standard deviation of S_t for an interval of length Δt is proportional to S_t , thus the following result:

$$dS_t = \mu S_t dt + \sigma S_t dz$$

Or

$$\frac{dS_t}{S_t} = \mu dt + \sigma dz \quad (1)$$

This is the Black & Scholes model of the evolution of investment values in the P project including:

- μ is a constant that indicates the expected return of the value of the investment in the project P;
- σ is a constant that indicates the volatility of the value of the investment in the project P.

The return on this investment in the project P is given by:

$$\rho_t = \frac{\Delta S_t}{S_t} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (2)$$

where:

- The term $\mu \Delta t$ represents the expected return;
- The term $\sigma \varepsilon \sqrt{\Delta t}$ represents the stochastic component of the output;
- The term $\sigma^2 \Delta t$ represents the variance of the return.

According to equation (1), in the discrete case the variable $\frac{\Delta S_t}{S_t}$ following the normal distribution $N(\mu \Delta t, \sigma \sqrt{\Delta t})$.

4.2 Portfolio Return of Islamic Financial Products

The value P_0 of the investment in the project P at time $t = 0$ can be invested in Islamic financial products portfolio, noted p_1, p_2, \dots, p_k .

So what is the maximum return can be generated by this investment in this portfolio during a time interval Δt ?

4.2.1 Portfolio return

The return of an Islamic financial product, or rate of return obtained by investments in an Islamic financial product, the ratio of the value v_{t-1} invested in financial products at the moment and the value v_t obtained by this investment at the moment t .

$$r_t = \frac{v_t - v_{t-1}}{v_{t-1}}$$

The expected return on an Islamic financial product over a period is given by:

$$\bar{r}_i = \frac{1}{T} \sum_{t=1}^T r_{it}$$

Let x_1, x_2, \dots, x_k the proportions of the capital invested in the portfolio of Islamic financial products p_1, p_2, \dots, p_k of returns $r_i, i = 1, \dots, k$.

The return of a portfolio is:

$$R = \sum_{i=1}^n x_i r_i$$

The variation of return will obey the following relationship:

$$\Delta R = \sum_{i=1}^n x_i \Delta r_i$$

For an n-asset portfolio in number n_1, n_2, \dots, n_k its value is given by:

$$V = \sum_{i=1}^n n_i V_i$$

where V_i is the price of i^{th} the asset.

4.2.2 Portfolio risk

The risk of a financial asset is the uncertainty as to the value of the asset at a future date. The variance, the average of the absolute deviations, semi-variance, Value at Risk are means for measuring this risk.

Portfolio risk is measured by one of the measuring elements mentioned above.

It depends on three factors, namely:

- The risk of each Islamic financial product in the Portfolio
- The degree of independence of changes in Islamic financial products together
- The number of Islamic financial products in the portfolio

4.2.3 Value at Risk portfolio of Islamic financial products

Value-at-Risk (VaR) is the most widely used risk measure in the financial markets to quantify the maximum loss on a portfolio for a horizon and a given confidence level. It depends on three elements:

- the distribution of profits and losses of the portfolio valid for the period of detention.
- the level of confidence.
- the holding period of the asset.

VaR at a time horizon and for the probability threshold is a number that:

$$P[\Delta V \leq VaR_\alpha] = \alpha$$

where:

- t : Horizon combined with VaR that is: one day or for more than one day.
- α : probability level is typically 95%, 98% or 99%.

We have:

$$P[\Delta V \leq VaR_\alpha] = \alpha \Rightarrow P\left[\frac{\Delta V}{V_0} V_0 \leq VaR_\alpha\right] = \alpha \Rightarrow P[\rho V_0 \leq VaR_\alpha] = \alpha \Rightarrow P\left[\rho \leq \frac{VaR_\alpha}{V_0}\right] = \alpha$$

Let $\rho_0 = \frac{VaR_\alpha}{V_0}$. If the distribution ρ follows the normal distribution with parameters m

and σ then the result is obtained:

$$P\left[\rho \leq \frac{VaR_\alpha}{V_0}\right] = \alpha \Rightarrow P[\rho \leq \rho_0] = \alpha \Rightarrow P\left[\frac{\rho - m}{\sigma} \leq \frac{\rho_0 - m}{\sigma}\right] = \alpha \Rightarrow P\left[X \leq \frac{\rho_0 - m}{\sigma}\right] = \alpha$$

where $X \sim N(0,1)$.

So $\rho_0 = m + z_\alpha \sigma$ with z_α is the quantile α of the variable X .

4.2.4 Return and risk portfolio Markowitz

The maximum return of Islamic financial products portfolio that may be generated by the investment in this portfolio during a time interval is obtained by the following optimization program:

$$Max \left\{ \sum_{i=1}^k x_i E(r_i) \right\}$$

Under the constraints:

$$\sigma_p \leq \sigma_{p_0}$$

$$\sum_{j=1}^n x_j = 1$$

$$x_j \geq 0 \quad j=1, \dots, n$$

Where σ_p is the risk in this portfolio.

In principle, if we varies the risk we will obtains another maximum return by the optimization program, this allows to generate a combination of expected returns- risk. All possible combinations of couples expected returns-risks portfolios are efficient, if among all portfolios of the same expected return as him, there is no strictly less risk. The efficient frontier is the set of efficient portfolios.

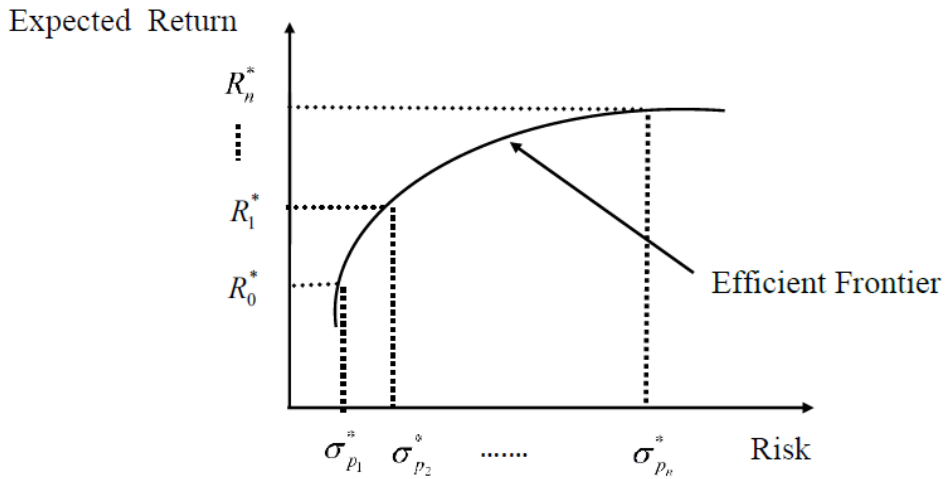


Figure 1: Efficient Frontier in the plan of Return- Risk

Note by $R_i^* (i = 1, 2, \dots, n)$ the return of efficient border. This is the highest possible efficiency which is the risk $\sigma_i^* (i = 1, 2, \dots, n)$ of investment of capital M in the portfolio Pf .

4.3 Risk of Islamic Musharaka Product

According to the behavior of bankers, the amount M invested in project P can generate multiple returns $R_i^* (i = 1, 2, \dots, n)$, as it can undergo a several risks $\sigma_i^* (i = 1, 2, \dots, n)$ that correspond these returns according to Markowitz's portfolio theory if it is invested in a portfolio of Islamic products.

Such that the variation of the investment of amount M in the project P for a period T follows a general process Winner and this investment can be achieved in a portfolio of Islamic financial products (PIFP) similar or identical to project P, then the return r of this investment can be expressed according to return of PIFP and risk corresponds to the return, that belonging to the combination of couples risks-expected returns (Efficient Frontier) of this portfolio.

As efficient frontier contains many couples risks-expected returns $(R_i^*, \sigma_{p_i}^*), i = 1, 2, \dots, n$,

so we can have multiple returns $r_i, i = 1, 2, \dots, n$ that can be expressed as follows:

$$r_1 = \frac{dS_1}{S_1} = R_1^* dt + \sigma_{p_1}^* dz$$

$$r_2 = \frac{dS_2}{S_2} = R_2^* dt + \sigma_{p_2}^* dz$$

.....

$$r_n = \frac{dS_n}{S_n} = R_n^* dt + \sigma_{p_n}^* dz$$

Let $\lambda = \frac{1}{n} \sum_{i=1}^n r_i$ the expected return of $r = (r_1, r_2, \dots, r_n)$ then

$$\lambda = \frac{1}{n} \sum_{i=1}^n r_i \Rightarrow \lambda = \frac{1}{n} \sum_{i=1}^n (R_i^* dt + \sigma_i^* dz) \Rightarrow \lambda = \left(\frac{1}{n} \sum_{i=1}^n R_i^* \right) dt + \left(\frac{1}{n} \sum_{i=1}^n \sigma_i^* \right) dz$$

Let $\tilde{R} = \frac{1}{n} \sum_{i=1}^n R_i^*$ and $\tilde{\sigma} = \frac{1}{n} \sum_{i=1}^n \sigma_i^*$ then $\lambda = \tilde{R}dt + \tilde{\sigma}dz$

The risk of return that can be considered like the Islamic Musharaka product is given by the following expression:

$$\gamma = V a R(\lambda)$$

We have

$$\lambda = \tilde{R}dt + \tilde{\sigma}dz$$

In the discrete case, it was

$$\lambda = \tilde{R}\Delta t + \tilde{\sigma}\sqrt{\Delta t}\varepsilon$$

We have $\varepsilon \sim N(0,1)$ then for a period T we will have $\lambda \sim N(\tilde{R}T, \tilde{\sigma}\sqrt{T})$

Also we have:

$$P(\lambda \leq \lambda_0) = P\left(\frac{\lambda - \tilde{R}T}{\tilde{\sigma}\sqrt{T}} \leq \frac{\lambda_0 - \tilde{R}T}{\tilde{\sigma}\sqrt{T}}\right) = P\left(X \leq \frac{\lambda_0 - \tilde{R}T}{\tilde{\sigma}\sqrt{T}}\right)$$

where $X \sim N(0,1)$.

Let Z_α the quartile of the normal variable $N(0,1)$ then

$$\frac{\lambda_0 - \tilde{R}T}{\tilde{\sigma}\sqrt{T}} = Z_\alpha \Rightarrow \lambda_0 = \tilde{R}T + Z_\alpha \tilde{\sigma}\sqrt{T}$$

As $\lambda_0 = \frac{VaR_\alpha}{S_0}$ so

$$\gamma = S_0 \tilde{R}T + Z_\alpha S_0 \tilde{\sigma}\sqrt{T} \tag{3}$$

5 Numeric Application

Let M an amount whose value is 500,000 DH that we want to invest at a project P for a period of 3 years, i.e: $T = [0,3]$. We Suppose there is a data base of projects that is similar or identical $\delta = (\delta_1, \dots, \delta_{n=100})$ to this project whose historic returns are

$\tau_1, \dots, \tau_{n=100}$. Its values are given in the following table:

Return	Value of return
τ_1	1.7%
τ_2	-0.1%
.	.
.	.
.	.
τ_{100}	0.6%

After determining the efficient frontier of portfolio vector $\delta = (\delta_1, \dots, \delta_{n=100})$

and the choice of different parameters $(R_i^*, \sigma_{\rho_i}^*), i = 1, 2, \dots, n$; we calculate returns

$\rho_i, i = 1, 2, \dots, n$.

In the end we calculate the risk of the return on this investment amount M which is given by the value of γ that is equal in this case 270,000 DH for significance $\alpha = 0.05$.

5 Conclusion

In this work, we presented a new mathematical modeling approach to risk assessment Musharaka Islamic financial product, the product's basic foundations Musharaka and also the various risks it entails.

This mathematical model allows for better decision making at the Islamic bank on the same product.

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