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Minimization of Value at Risk of Financial Assets Portfolio using Genetic Algorithms and Neural Networks

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Abstract

In this paper we have proposed an approach for minimization of a shares portfolio invested in a market which the fluctuations follow a normal distribution based in amathematical explicit formulae for calculating Value at Risk (VaR) for portfolios of linear financial assets invested using the Black-Scholes stochastic process and assuming that the portfolio structure remains constant over the considered time horizon.
We minimize this Value at Risk using neural networks and genetic algorithms.

JEL classification numbers: C45
Keywords: Value at Risk, Normal Market, Portfolio Risk, Black-Scholes stochastic process, Normal Distribution, Neural Networks, Genetic Algorithms.

1 Introduction

The optimization portfolio has long been a subject of major interest in the field of finance. Markowitz was the first to introduce a model based on the risk of choosing an optimal portfolio, offering the variance of returns observed around their average, as a measure of risk. But his model remains often used into practice because of the significant resources and it requires the character of the quadratic objective function and the calculation of the variance-covariance.

To simplify the difficulties associated to the design load of Markowitz model, several models have been proposed as alternative models to the mean-variance approach. Some

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authors have attempted to linearize the portfolio choice problem as Sharpe, Stone, Konno and Yamazaki, and Hamza and Janssen.

Rudd and Rosenberg, and Hamzaand Janssen showed that the Markowitz model in its classic formulation still far from meeting satisfying a professional investor and they proposed a realistic portfolio management.

Recently, Value at Risk (VaR) has been implemented to quantify the maximum loss that might occur with a certain probability, over a given period. This risk measure is easy to interpret.

Based on an explicit formula for calculating the VaR for a shares portfolio invested in a normal market, we minimize this VaR of portfolio formula by using neural network and genetic algorithms.

This work is organized as follows. In section 1, we deal with the presentation of some elements of the portfolio. Neural network and genetic algorithms are presented in section 2. In section 3, we present the VaR of shares portfolio under normal distribution and Black-Scholes stochastic process. Finally, we propose the portfolio minimization procedure.

2 Elements of portfolio theory

2.1 Return and Value Portfolio

We call return r_t of an action obtained by investing in an action, the ratio between the share price at the moment t and its course at the moment $t-1$ plus income (dividends) received during the period $[t-1,t]$:

$$r_t = \frac{c_t - c_{t-1} + d_t}{c_{t-1}} \tag{1}$$

where:

- c_t : The course of action i at the end of the period t.
- d_t : The dividend income at the end of the period t.

The expected return of a share for a period T is given by

$$\overline{r_i} = \frac{1}{T}\sum_{t=1}^{T} r_{it} \tag{2}$$

The profitability of a portfolio consisting of expected return of k shares $\overline{r_i}$, $i=1,...,k$:

$$\overline{R}_p(x) = \sum_{i=1}^{k} x_i \overline{r_i} \tag{3}$$

where $x=(x_1,x_2,...,x_n)$, $x_1,x_2,...,x_n$ are the proportions of wealth of the investor placed respectively in the shares i ($i=1,...,n$).

2.2 Risk Portfolio

The risk of a financial asset is the uncertainty about the value of this asset in an upcoming date. Variance, the average absolute deviation, the semi-variance, VaR and CVaR are means of measuring this risk. The portfolio risk is measured by one of the measuring elements mentioned above. It depends on three factors namely:

- The risk of each action included in the portfolio
- The degree of independence of changes in equity together
- The number of shares in the portfolio

The VaR is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon. In simple words, it is a number that indicates how much a financial institution can lose with some probability over a given time. It depends on three elements:

- Distribution of profits and losses of the portfolio that are valid for the period of detention.
- Level of confidence.
- The holding period of assets

Analytically, the VaR in time horizon *t* and the probability threshold α is a number $VaR(t,\alpha)$ such that:

$$\mathrm{P}\left[X \leq VaR(t,\alpha)\right] = \alpha \tag{4}$$

With

- Lh: represents the loss ("Loss"), is a random variable which might be positive or negative.
- t : is associated with the VaR horizon which is 1 day for RiskMetrics or more than a day
- α : The probability level is typically 95%, 98% or 99%.

If the distribution of the value of this portfolio is a multivariate normal, then:

$$VaR_{\alpha}(x) = -x'\mu + z_{\alpha}.\sqrt{x'\Omega x} \tag{5}$$

as:

- $\Delta V(x)$ is the value variation
- $\mu = E(\Delta V(x))$ is mean of values
- $\Omega = \sigma(\Delta V(x))$ is standard deviation
- z_{α} is the quantile of order of confidence α

3 The VaR of Shares Portfolio of Normal Distribution using Black-Scholes Stochastic Process

The price of a share S_t at time t is a random variable whose evolution over time can be modelled by a stochastic process $S=(S_t, t\geq 0)$ on a filtered probability space $(\Omega, \Im, (\Im_t), P)$ satisfying the Black-Scholes stochastic differential equation:

$$dS_t = \mu.S_t dt + \sigma.S_t dz \tag{6}$$

- The constant drift μ indicates the expected return of the share price per unit time;
- σ is a constant indicating the annual volatility of the share price.

The process z is a standard Wiener process so that z is a Markov process with expected increases which are zero and the variance of these increases is equal to 1 per unit time and it satisfies the following two properties:

- the process z is a standard Brownian motion so that for simulation, the variation dz during a short time interval and length dt is expressed by:

$$dz = \varepsilon\sqrt{dt}$$

where ε is a random variable that follows a reduced normal distribution $N(0,1)$.

- The dz 's values for two short intervals of time and length dt are independent.
- In discrete case we have $\Delta S_t = \mu.S_t\Delta t + \sigma.S_t\sqrt{\Delta t}\varepsilon \quad \Rightarrow \quad r_t = \dfrac{\Delta S_t}{S_t} = \mu.\Delta t + \sigma.\sqrt{\Delta t}\varepsilon$.

So for all $i=1,...,n$ we have :

$$r_i(t) = \mu_i.\Delta t + \sigma_i.\sqrt{\Delta t}\varepsilon_i$$

As $\varepsilon_i : N(0,1) \Rightarrow r_i(t) : N\left(\mu_i.\Delta t, \sigma_i.\sqrt{\Delta t}\right)$.

The Value at Risk of a portfolio for a horizon t is noted VaR, such as the loss on this portfolio during the $[0,t]$ not fall below VaR with a fixed probability α , i.e:

$$P\left[-\Delta V(t) \leq VaR\right] = \alpha \tag{7}$$

where:

$$\Delta V(t) = V(t) - V(0) \tag{8}$$

$V(0)$ and $V(t)$ are respectively the values of portfolio at the beginning and end of the period. More rigorously, the VaR can be defined as:

$$VaR_\alpha = \max\left\{B / P\left[-\Delta V(t) \leq B\right] \leq \alpha\right\} \tag{9}$$

When the random variable $\Delta V(T) = V(T) - V(0)$ is distributed according to a normal distribution $N\left(E\left[\Delta V(t)\right],\ \sigma\left[\Delta V(t)\right]\right)$, the VaR of probability level α is defined as follows:

$$P\left[\frac{\Delta V(t)-E\left(-\Delta V(t)\right)}{\sigma\left(\Delta V(t)\right)}\leq\frac{VaR_\alpha-E\left(-\Delta V(t)\right)}{\sigma\left(\Delta V(t)\right)}\right]=\alpha$$

If $\tau_\alpha=\dfrac{VaR_\alpha-E\left(-\Delta V(t)\right)}{\sigma\left(\Delta V(t)\right)}$ is the quantile of the distribution N (0,1), we obtain;

$$VaR_\alpha=-E\left(\Delta V(t)\right)+\tau_\alpha\sigma\left(\Delta V(t)\right) \tag{10}$$

Let $V(t)$ the value of the portfolio of n shares invested in a given market at time t.

We denote by x_i the number of shares in the portfolio. Let $S_i(t)$ the price of stock i at time t. It follows that:

$$V(t)=\sum_{i=1}^{n}x_iS_i(t) \tag{11}$$

The portfolio value to the horizon T is characterized by the following equations:

$$V(T)=\sum_{i=1}^{n}x_iS_i(T)=\sum_{i=1}^{n}x_i\left[S_i(0)+\Delta S_i(T)\right] \tag{12}$$

By the definition of return r_i of $i\ (i=1,...,n)$

$$r_i(T)=\frac{S_i(T)-S_i(0)}{S_i(0)}=\frac{\Delta S_i(T)}{S_i(0)} \tag{13}$$

The relation (13) becomes:

$$\begin{aligned}V(T)&=\sum_{i=1}^{n}x_i\left[S_i(0)+r_i(T)S_i(0)\right]\\&=\sum_{\mathrm{i=1}}^{\mathrm{n}}x_iS_i(0)\left[1+r_i(T)\right].\end{aligned}$$

The disadvantage of the equation(10) is that both parameters require knowledge of the univariate parameters $E\left(\Delta S_i\right)$ and $\mathrm{var}\left(\Delta S_i\right)$ for each title $i\ (i=1,...,n)$ and the bivariate parameters $\mathrm{cov}\left(\Delta S_i,\Delta S_j\right)$ for each pair of tracks , either in total $\dfrac{n(n+1)}{2}$ parameters.

Hence the suggestion of the use of Black-Scholes stochastic process which the simplest and most widely used.
We have:

$$r_i(t) = \mu_i.\Delta t + \sigma_i.\sqrt{\Delta t}\varepsilon_i \tag{14}$$

for all $i = 1,...,n$;

So

$$V(T) = \sum_{i=1}^{n} x_i S_i(0)\left[1 + r_i(T)\right] = \sum_{i=1}^{n} x_i S_i(0) + \sum_{i=1}^{n} x_i r_i(T) = V(0) + \sum_{i=1}^{n} x_i r_i(T)$$

It comes

$$V(T) - V(0) = \sum_{i=1}^{n} x_i r_i(T) \quad \Rightarrow \quad \Delta V(T) = \sum_{i=1}^{n} x_i r_i(T)$$

$$E\left[\Delta V(T)\right] = \sum_{i=1}^{n} x_i E\left[r_i(T)\right] = \sum_{i=1}^{n} x_i \mu_i T \tag{15}$$

And

$$\operatorname{var}\left[\Delta V(T)\right] = \operatorname{var}\left[\sum_{i=1}^{n} x_i r_i(t)\right] = v\operatorname{ar}\left[\sum_{i=1}^{n} x_i\left(\mu_i T + \sigma_i\sqrt{T}\varepsilon_i\right)\right] = T\left(\sum_{i=1}^{n} x_i^2 \sigma_i^2\right)$$

Or

$$VaR_\alpha = -E\left(\Delta V(t)\right) + \tau_\alpha \sigma\left(\Delta V(t)\right)$$

Then

$$VaR_\alpha = -\sum_{i=1}^{n} x_i \mu_i T + \tau_\alpha \sqrt{T}\left(\sum_{i=1}^{n} x_i^2 \sigma_i^2\right)$$

4 Minimization Procedure of the VaR of Shares Portfolio using Genetic Algorithms and Neural Network

4.1 Genetic Algorithms (GA)

A genetic algorithm was originally developed by John Holland. It is an algorithm
Iterative for finding optimum, it manipulates a population of constant size. This population is composed of candidate points called chromosomes.
The constant size of the population leads to a phenomenon of competition between chromosomes.

Each chromosome represents the encoding of a potential solution to the problem to besolved, it consists of a set of elements called genes, which can take several values belonging to an alphabet which is not necessarily digital.
At each iteration, called generation, a new population is created with the same number of chromosomes. This generation consists of chromosomes better "adapted" to their environment as represented by the selective function. As in generations, the chromosomes will tend towards the optimum of the selective function.
The creation of a new population base on the previous one is done by applying the genetic operators that are: selection, crossover and mutation. These operators are stochastic.
The selection of the best chromosomes is the first step in a genetic algorithm. During this operation the algorithm selects the most relevant factors that optimize the function.
Crossing permits two chromosomes to generate new chromosomes "children" from two "parents" chromosomes selected.
The mutation makes the inversion of one or more genes of a chromosome. Figure 1 illustrates the various operations involved in a basic genetic algorithm:

Random generation of initial population
Calculation of the selective function
Repeat
Selection
Crossing
Mutation
Calculation of the selective function
Until stopping criterion satisfaction

Figure 1: Basic genetic algorithm

4.2 Minimization of the VaR using Genetic Algorithms

$$VaR_\alpha = -\sum_{i=1}^{n} x_i \mu_i T + \tau_\alpha \sqrt{T}\left(\sum_{i=1}^{n} x_i^2 \sigma_i^2\right) \qquad (16)$$

The objective of this algorithm is to determine dynamically the proportions of the portfolio shares under certain constraints to minimize this measure. So we seeking to minimize the proportions using genetic algorithms (GA) as indicated by the following figure:

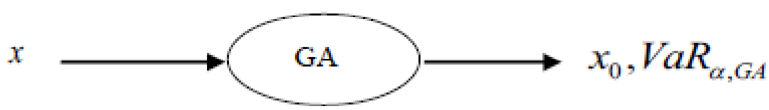

Figure 2: Structure of AG used in the algorithm Minimization of the VaR

under the following constraints:

$$\begin{aligned} & V(x) \geq \rho_0 \\ & x_i \geq 0 \\ & \sum_{i=1}^{n} x_i = 1 \\ & VaR_{\alpha,NN} \leq VaR_{\alpha,GA} \end{aligned}$$

Where

- ρ_0 : is the performance that determined by the investor.
- $VaR_{\alpha,GA}$: is the value at risk obtained by genetic algorithms.
- $VaR_{\alpha,NN}$: is the value at risk obtained by neural networks using an initial vector $x^0 = \left(x_1^0, x_2^0, ..., x_n^0\right)$ in first step or iteration.

At this level, the proportions are considered variables. The process of minimization followed by genetic algorithms is as follows:

a- Initialization

The population is a set of chromosomes which are composed of k genes representing $x_i\left(i=1,...,k\right)$ numbers, which x_i is the number of wealth invested in the action i. This population is initially randomly using real code.

x_1	x_2	…………………………..	x_k

Figure 3: Structure of chromosome

b- Evaluation Function

The following operation is the evaluation of chromosomes generated by the previous operation by an evaluation function (fitness function), while the design of this function is a crucial point in using GA. The fitness function used in this work is:

$$h = VaR_{\alpha}\left(x\right) \tag{17}$$

c- Operations of selection

After the operation of the assessment of the population, the best chromosomes are selected using the wheel selection that is associated with each chromosome a probability of selection, noted, P_i.

$$P_i = \frac{1}{N-1}\left(1 - \frac{h_i}{\sum_{i \in Pop} h_i}\right) \tag{18}$$

Each chromosome is reproduced with probability. Some chromosomes will be "more" reproduced and other "bad" eliminated.

d- Operations crossing

After using the selection method for the selection of two individuals, we apply the crossover operator to a point on this couple.

This operator divide each parent into two parts at the same position, chosen randomly.

The child 1 is made a part of the first parent and the second part of the second parent when the child 2 is composed of the second part of the first parent and the first part of the second parent.

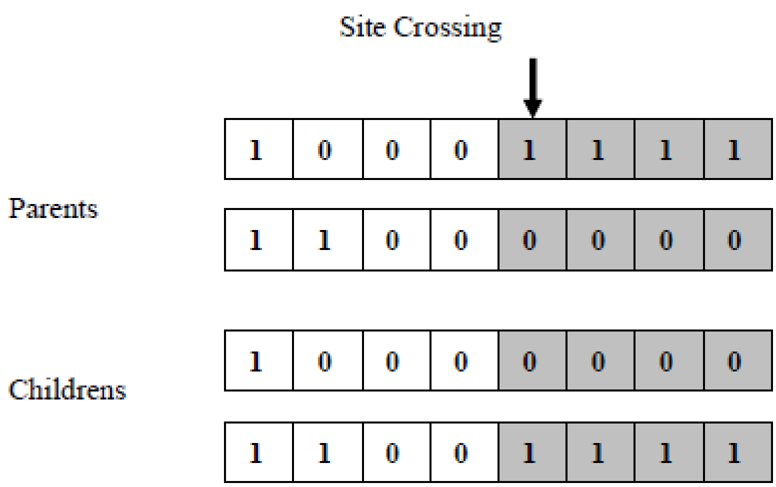

Figure 4: Operation at a crossing point

e- Operation of mutation

This operation gives to genetic algorithms property of ergodicity which indicates that it will be likely to reach all parts of the state-owned space, without the travel all in the resolution process. This is usually to draw a random gene in the chromosome and replace it with a random value.

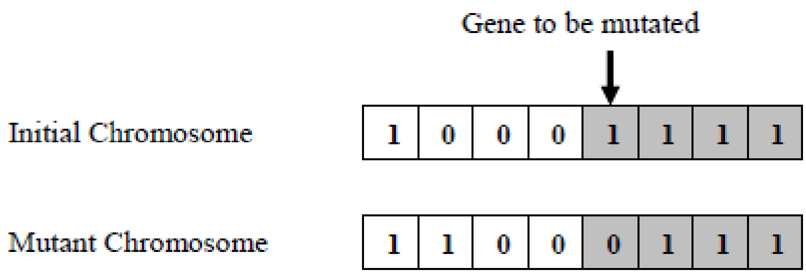

Figure 5: Mutation operation

f- Conditions for Convergence
At this level, the final generation is considered. If the result is favorable then the optimum chromosome is obtained. Otherwise the evaluation and reproduction steps are repeated until a certain number of generations, until a defined or until a convergence criterion of the population are reached.
After this step, we use neural networks to minimize dynamically further the VaR.

4.3 Neural Networks (NN)

4.3.1 Definition of neural networks

The neural networks (NN) are mathematical models inspired by the structure and behavior of biological neurons. They are composed of interconnected units called artificial neurons capable of performing specific and precise functions. Figure1 illustrates this situation.

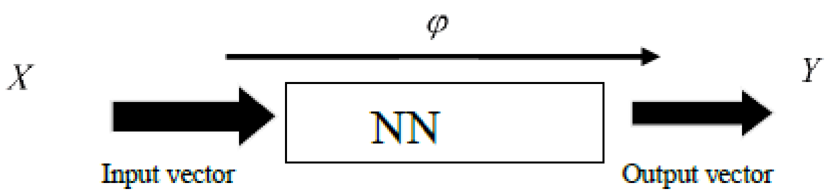

Figure 6: Black box of Neural Networks

For a neural network, each neuron is interconnected with other neurons to form layers in order to solve a specific problem concerning the input data on the network.
The input layer is responsible for entering data for the network. The role of neurons in this layer is to transmit the data to be processed on the network. The output layer can present the results calculated by the network on the input vector supplied to the network. Between network input and output, intermediate layers may occur; they are called hidden layers. The role of these layers is to transform input data to extract its features which will subsequently be more easily classified by the output layer.

4.3.2 Back-propagation algorithm

The objective of this algorithm is to approximate a function $y = f(X)$ where X is an input vector of returns (risk respectively) presented the input layer assigning each component of X to a neuron. These inputs are then propagated through the network until they reach the output layer. For each neuron, activation a_i is calculated using the formula:

$$a_i = F\left(\sum_j o_i w_{ij}\right) \tag{19}$$

where:

- o_j is the output of neuron j of the preceding layer,
- w_{ij} is the weight connecting neuron j to neuron i,
- *F*is the transfer function (or activation function) of the neuron *i*.

The output vector that the network is compared with the product of expected output.
An error E is calculated as follows:

$$E = \sum_i \left(o_i - t_i\right)^2 \tag{20}$$

- o_i is the value neuron output of i in the output layer
- t_i is the i th output target value

If the error value is not close to 0, the connection weights should be changed to reduce this error. Each weight is either increased or reduced by propagating the error back-calculated. The mathematical formula used by this algorithm is known as the Delta rule:

$$\Delta w_{ij} = \eta \delta_i o_j \tag{21}$$

where:

- Δw_{ij} is the variation weight w_{ij}
- η is the learning rate (set by user)
- δ_i is the error on the output of the neuron i of a layer.

The calculation depends on the type of neuron. If the neuron is a neuron output, then the error is:

$$\delta_i = F'\left(a_i\right)\left(t_i - o_i\right) \tag{22}$$

else (hidden neuron)

$$\delta_i = F'\left(a_i\right) s_k \delta_k w_k \tag{23}$$

where k neurons belonging to the next layer of the neuron *i*.
The algorithm is repeated for each pair of input / output and more passes are performed until the error has dropped below an acceptable threshold or a maximum number of passes is reached.

In our case, the neural network architecture used is an architecture containing a single input layer, one hidden layer composed of n neurons where n is the number of x_i where $i=1,...,n$ and a layer of containing a single output neuron representing the value of $VaR_{\alpha,GA}$.

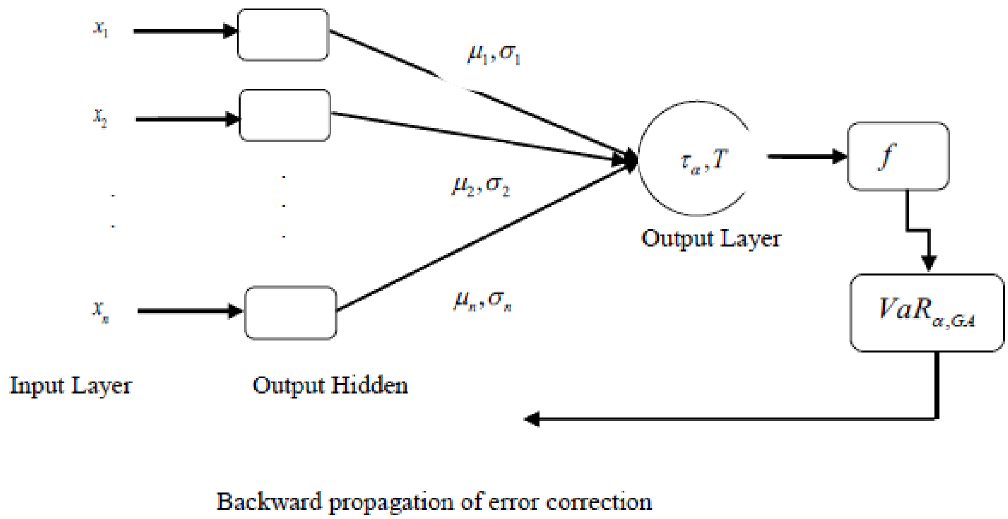

Figure 7: Neural network architecture

The learning algorithm used is the gradient back-propagation supervised.
The error between the current output (obtained by neural networks) and the desired output (observed) spreads, while adjusting the weights with the aim to correct the weights of the network to reduce the global error expressed by the following formula:

$$E = \sum_{i=1}^{n}(f_i - VaR_{\alpha,GA})^2 \tag{24}$$

where:

- f_i represents the estimated value of f in ith iteration
- E is the overall error,

The operation of the network illustrated as follows by the figure 7: Each neuron i ($i=1,...,n$) in the input layer receives a value of the x_i to be weighted by the proportions of μ_i,σ_i and the result transmitted to the output layer. In this case, the output f is given by the following formula:

$$f(x) = -\sum_{i=1}^{n} x_i\mu_i T + \tau_\alpha\sqrt{T}\left(\sum_{i=1}^{n} x_i^2\sigma_i^2\right) \tag{25}$$

The minimization procedure is based genetic algorithms and neural networks is shown in the following figure:

Initialization: $k=2$, $x^0=\left(x_1^0,x_2^0,...,x_n^0\right)$ and $VaR_{NN}=NN(x^0)$

While $k\geq 2$ **Do**

$VaR_{GA}=MinGA\left(x\right)$

If $VaR_{\alpha,GA}\leq VaR_{\alpha,NN}$ **Then**

$VaR_{\alpha,NN}\leftarrow VaR_{\alpha,GA}\times\dfrac{1}{k}$

$k\leftarrow k+1$

End

If No improvement of risk $VaR_{\alpha,NN}$ or After a certain number of iterations **Then**

$k=0$

End

End

Return $VaR_{\alpha,NN}$

Figure 8: Minimization algorithm of the VaR

5 Conclusion

In this paper we presented a new approach to minimize the VaR of a stock portfolio investing in a market whose fluctuations follow a normal stochastic process using Black-Scholes stochastic process developed in discrete time and assuming that the portfolio structure remains constant over the time horizon.
The minimization procedure is based genetic algorithms and neural networks.

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