Forecasting Stock Return Volatility: Evidence from the West African Regional Stock Market

N’dri Konan Léon

Abstract

This paper compares the out-of-sample forecasting performance of the GARCH, EGARCH, and GJR-GARCH models across the Normal distribution, Student-t distribution, and Generalized Error Distribution (GED) in the regional stock market of the West African Economic and Monetary Union called the BRVM. The study uses weekly returns ranging from 4 January 1999 to 10 March 2005 for in-sample estimation of conditional variance models, and the period from 11 March 2005 to 29 July 2005 for out-of-sample forecasting. Using the RMSE and MAE as measures of forecasting accuracy, I find that the EGARCH model outperforms both the GARCH and the GJR-GARCH models under the Student-t distribution and the GED.

JEL classification numbers: C22, C53
Keywords: Volatility models, Volatility forecasts, BRVM, forecasting performance.

1 Introduction

Forecasting the volatility of financial time series has been extensively examined over the past three decades. Theory predicts that accurate estimation and forecast of the volatilities of asset prices would have important implications for investments, security valuation, risk management, and monetary policy making. Volatility is a major input to investments decisions and portfolio selection since a good forecast of the volatility of asset prices over the investment holding period is the starting point for assessing investment risk. Volatility plays a central role in option pricing as the knowledge of the volatility of the underlying asset is mandatory to price such a derivative. Since the first Basle Accord in 1996, volatility forecasting in terms of value-at-risk has been compulsory for most financial institutions. Finally, policy makers rely on market estimates of volatility as a barometer for the

1 UFR des Sciences Economiques et de Gestion, Université Félix Houphouet Boigny, Cocody. BPV 43 Abidjan, Côte d’Ivoire. Tel.: +225 0761 0329.

Article Info: Received : August 2, 2015. Revised : August 31, 2015. Published online : November 1, 2015
vulnerability of financial markets and the economy in handling monetary policy. Despite their importance, empirical studies on forecasting and measuring the performance of volatility have largely been carried out on developed stock markets (Hansen, 1994; Gokcan, 2000; Awartini and Corradi, 2005; Bae et al., 2007; Bali, 2007; Chuang et al., 2007; Curto et al., 2009; Liu and Hung, 2010 amongst others), and on emerging stock markets (Sandoval, 2006; Komain, 2007; Hien, 2008; Kovacic, 2008; Lee, 2009; Shamiri and Isa, 2009; Su, 2010 for example). Studies on African stock markets have been however limited (Appiah and Menyah, 2003; Ogun et al., 2005; Eskandar, 2005; Alagidede and Panagiotidis, 2009), and inexistent for the regional stock market of West Africa. The present study fills this gap and contributes to the existing empirical literature by examining the forecasting of volatility and their performance in the BRVM using various conditional volatility models and different loss functions. My results show that the EGARCH model, in-sample and out-of-sample, outperforms both the GARCH and the GJR-GARCH models under the Student-t distribution and the GED.

The rest of the paper is organized as follows. After the introduction section, an overview of the BRVM is provided by section 2. Section 3 presents the research methodology. Section 4 exposes the empirical results and their analysis and section 5 concludes the paper.

2 The Regional Stock Market of West Africa

The regional stock exchange of West Africa, known as the Bourse Régionale des Valeurs Mobilières (hereafter, BRVM), was established on 18 December 1996, and began its operations on 16 September 1998. It is the stock exchange of the West African Economic and Monetary Union (WAEMU). The BRVM is responsible for organizing the securities market and distributing related information. To that end, it guarantees the listing of securities on the exchange, the quotation of securities, the publication of market prices and information, the promotion and development of the securities exchange.

The BRVM is headquartered in Abidjan, Côte d’Ivoire, and represented in every member state by a national branch office charged with: 1) overseeing public relations for the exchange and the central clearing house; 2) disseminating market information; 3) assisting brokerage firms and other market stakeholders; 4) organizing local promotion of the regional exchange. At its inception, the BRVM has two sections for stocks and a single section for bonds with eligibility conditions differing across sections. A company seeking to be listed on the BRVM must satisfy the following conditions: 1) be incorporated; 2) sign a written agreement to publishing the annual statements in the official newsletter, and to participate in the organization of the exchange; 3) sign a written agreement to obey the rules and regulations of the BRVM; 4) to apply for listing on the BRVM, the candidate company must mandate a brokerage firm to assist and advise it.

The BRVM is a centralized spot exchange driven by orders, that is, the price of a security is fixed by matching bid and ask orders. The quotation is done by fixing, that is, a single price is obtained by matching bid and ask orders. Currently, there are five sessions a week. Two BRVM market indices represent the activities of stock market shares: the BRVM Composite comprises all securities listed on the exchange, and the BRVM 10 is composed of the ten most active companies on the exchange. As of 17 March 2006, 39 companies are listed on the BRVM.

Formulation and selection criteria for the BRVM indexes are based on the leading global market indexes, especially, the FCG index of the International Finance Corporation, a
World Bank affiliate. The formulation of the indexes takes into account market capitalization, transaction per session, and transaction frequency. Only common shares are used to calculate the indexes. The indexes are automatically generated by the BRVM trading system and circulated after every session. The BRVM 10 is reviewed four times a year, and the BRVM Composite after every new listing.

Table 1 below shows some performance indicators of the BRVM from 31 December 2002 to 29 July 2005. This table reflects the high performance of the BRVM 10 and shows how volatile the regional stock market is.

### Table 1: Selected Figures Of The BRVM

<table>
<thead>
<tr>
<th>Capitalization</th>
<th>31/12/ 2002</th>
<th>31/12/ 2003</th>
<th>31/12/ 2004</th>
<th>29/07/ 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRVM 10</td>
<td>465 634 264</td>
<td>617 337 595</td>
<td>607 239 551</td>
<td>766 467 440</td>
</tr>
<tr>
<td>BRVM Composite</td>
<td>832 398 094</td>
<td>858 140 223</td>
<td>1 005 047 884</td>
<td>1 094 198 936</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRVM 10</td>
</tr>
<tr>
<td>BRVM Composite</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Some measures</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume traded of stocks</td>
<td>4823</td>
<td>2994</td>
<td>2031</td>
<td>1033</td>
</tr>
<tr>
<td>Value traded (CFAfr)</td>
<td>171 254 520</td>
<td>72 584 325</td>
<td>35 861 220</td>
<td>47 493 520</td>
</tr>
<tr>
<td># of transactions</td>
<td>30</td>
<td>85</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td># of securities traded</td>
<td>9</td>
<td>15</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

### 3 Research Methodology

This section describes the data, exposes the volatility models, and presents the forecasting accuracy measures.

#### 3.1 Sample Data and their Properties

The data set used in this study is weekly closing prices on the BRVM 10 index obtained from the Official Newsletter of the Regional Stock Market (BRVM). The choice of the BRVM 10 over the BRVM Composite is motivated by two reasons: first, it is composed of the ten most actively traded stocks in the BRVM and second, it accounts for about 70% of the total market capitalization of the BRVM as shown in Table 1 above. The study period ranges from 4 January 1999 to 29 July 2005. The period from 4 January 1999 to 10 March 2005 is used for estimation purposes and, the period from 11 March 2005 to 29 July 2005 is used for out-of-sample evaluation or forecasting. I compute the continuously compounded weekly stock market returns, $R_t$, as follows:

$$R_t = 100 \times \ln \left( \frac{P_t}{P_{t-1}} \right)$$

(1)
Where $P_t$ is the value of the BRVM 10 price index for the period $t$, and $t$ represents time in weeks. $P_{t-1}$ is BRVM 10 index price for period $t-1$. $\ln(.)$ is the logarithm operator. All returns are expressed in local currencies and are not adjusted for dividends. Table 2 below reports summary statistics of weekly stock market returns.

### Table 2: Summary statistics

<table>
<thead>
<tr>
<th>Series</th>
<th>T</th>
<th>Mean</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>J.B.</th>
<th>ARCH(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRVM10</td>
<td>741</td>
<td>0.027</td>
<td>0.810</td>
<td>-0.758</td>
<td>16.409</td>
<td>5621*</td>
<td>19.611*</td>
</tr>
</tbody>
</table>

*(Notes: T is the number of observations; SD is the standard deviation; * means significant at 5%. JB is the Jarque-Bera statistic; ARCH(5) is the ARCH test for homoskedasticity with 5 lags; p-values are in parentheses.)*

The mean and the standard deviation are 0.072 and 1.636 respectively. The skewness statistic of 0.930 shows that the distribution is positively skewed relative to the normal distribution (0 for the normal distribution). This is an indication of a non symmetric series. The kurtosis is very much larger than 3, the kurtosis for a normal distribution. This suggests that for the BRVM, large market surprises of either sign are more likely to be observed, at least unconditionally. The Ljung-Box test statistics $Q(.)$ and $Q^2(.)$ provide tests for the absence of autocorrelation and homoscedasticity, respectively. The significance values of $Q$ – statistics indicate significant serial correlation in the mean return series. This suggests that the inclusion of a lag dependent variable in the mean equation is appropriate. Strong autocorrelation is also detected in the squared mean returns as shown by the values of the $Q^2(.)$. It results in volatility clustering in the distribution of stock market returns. In addition, the Jarque-Bera normality test rejects the hypothesis of normality.

### 3.2 Volatility Models

Since the seminal of work of Engle (1982) on conditional volatility, conditional heteroskedastic models have become the main statistical instruments to estimate and forecast asset returns volatility. I present in this section, the three different models used to evaluate the predictive power of volatility models in the BRVM. The models presented here are all variant of the Autoregressive Conditional Heteroskedasticity (ARCH) of Engle (1982).

#### 3.2.1 The GARCH Model

The ARCH model developed by Engle (1982) estimates the variance of returns as a simple quadratic function of the lagged values of innovations but exhibits weaknesses as it requires many parameters and a high order of lagged values to capture the volatility process. Bollerslev (1986) and Taylor (1986) remedy to these weaknesses by proposing the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model that has an infinite ARCH specification and allows the reduction of the number of parameters to be estimated by imposing nonlinear restrictions. In the GARCH model, the conditional variance of a variable is dependent upon previous lags of the squared residual from the
mean equation and present news about the volatility from the previous period. The GARCH models just as the ARCH model, captures the two characteristics of financial time series: Volatility clustering that occurs when large changes tend to be followed by large changes and small changes tend to be followed by small changes (Mandelbrot, 1963); leptokurtosis which describes the fact that the distribution of their returns is fat-tailed, that is the kurtosis of the distribution exceeds the kurtosis of a standard Gaussian distribution.

In order to expose the GARCH model, I exploit the ARCH technology. I consider:

\[ R_t = \mu + \rho R_{t-1} + \epsilon_t \]  

(2)

Where \( R_t \) is the continuously compounded return defined in equation 1; \( \mu \) is the constant term; \( \rho \) is a coefficient; \( \epsilon_t \) is the disturbance or unpredictable part with mean 0 and variance \( \sigma_t^2 \); the conditional variance is defined as:

\[ \sigma_t^2 = \text{var}(R_t | \psi_{t-1}) \]  

(3)

Where \( R_t \) is defined as above; \( \psi_{t-1} \) is the set of all available information at time t-1; \( \text{var}(.) \) is the variance operator. The volatility measure defined by the conditional variance is in an expectation formulation. The Autoregressive Conditional Heteroskedasticity (ARCH) process of Engle (1982) is defined as:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \]  

(4)

q refers to the order of the lagged squared returns included in the model; \( \alpha_0 > 0; \alpha_i \geq 0 \).

The Generalized ARCH model, GARCH (p,q) of Bollerslev (1986) and Taylor (1986) is expressed as follows:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \]  

(5)

In this equation, \( \epsilon_{t-i}^2 \) refers to past innovations and \( \sigma_{t-j}^2 \) is about past variances. In order to have a positive value, a sufficient condition for the conditional variance bears on its parameters as follows: \( \alpha_0 > 0; \alpha_i \geq 0 \), and \( \beta_j \geq 0 \). The GARCH (p, q) is weakly stationary if and only if \( \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 0 \). The model keeps not only the volatility clustering and the leptokurtosis characteristics of the ARCH model but it is also a linear function of lagged conditional variances. The GARCH model is therefore an extension of the ARCH model.

3.2.2 EGARCH Model

The GARCH model takes into account the time series characteristics of volatility clustering and leptokurtosis but fails to consider the leverage effects first documented by Black (1976).
To remedy that, Nelson (1991) proposes the Exponential GARCH (EGARCH). This model, a refinement of the GARCH model imposes a non negativity constraint on market variance, and allows for the conditional variance to respond to asymmetrically to return innovations of different signs (leverage effect). I specify the EGARCH model as follows:

\[
\ln \sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \beta_j \ln \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \left( \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) - E \left( \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{i=1}^{p} \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}}
\]

(6)

Where \( \alpha_0, \beta_j, \alpha_i, \) and \( \gamma_i \) are parameters. \( \ln \sigma_t^2 \) is the one-period ahead volatility forecast. This implies that the leverage effect is exponential rather than quadratic and forecast of conditional variance are guaranteed to be non-negative; \( \alpha_0 \) is the mean level; \( \beta_j \) represents the persistence parameter; \( \ln \sigma_{t-j}^2 \) is the past period variance. Unlike the GARCH model, the EGARCH model allows for leverage. If \( \gamma_i \) is negative, leverage effect exists. That is an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude (Black, 1976; Christie, 1982). If \( \alpha_i \) is positive, then the conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller).

3.2.3 The GJR Model

A main limitation of the GARCH model is its failure to capture asymmetries in financial time series. It considers mainly that negative and positive shocks have the same effects on volatility. To remedy that and take into account the volatility clustering and the leptokurtosis characteristics of returns, Glosten, Jagannathan and Runkle (1993) extend the GARCH model with an additional term to account for asymmetries. This model known as the GJR-GARCH model is expressed as follows:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma_i \varepsilon_{t-i}^2 d_{t-i} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]

(7)

Where \( d_{t-i} \) is a dummy variable with:

\[
d_{t-i} = \begin{cases} 
1 & \text{if } \varepsilon_{t-i} < 0, \text{ bad news } \\
0 & \text{if } \varepsilon_{t-i} \geq 0, \text{ good news }
\end{cases}
\]

Good news is shown in the model by \( \alpha_i \) whereas bad news is shown by \( \alpha_i + \gamma_i \). In addition, when \( \gamma_i > 0 \), leverage effect exists. Non negativity condition is satisfied by \( \alpha_0 > 0, \alpha_i > 0, \beta_j \geq 0 \) and \( \alpha_i + \gamma_i \geq 0 \).

3.3 Out-of-Sample Forecasts Performance

In order to evaluate the forecasting performance of the above conditional variance models, I compute two measures: the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE). Assuming that the forecast sample is \( j = T + 1, T + 2, \ldots T + h \), and denote the actual and forecasted value in period \( t \) as \( y_t \) and \( \hat{y}_t \) respectively, the reported forecast
error statistics are computed as follows:

\[
RMSE = \sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}
\]

\[
MAE = \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| / h
\]

4 Empirical Results and Analysis

In order to assess the forecasting performance of equations (5), (6), and (7), I estimate each of them jointly with equation (2) upon specifying the assumptions about the distribution of error terms. With the aim to compare the models, I consider three density functions: the Normal distribution, the Student-t distribution, and Generalized Error Distribution (GED). The two later are expected to reduce the excess kurtosis and skewness displayed by the residuals of the conditional heteroskedasticity models. All models are estimated by maximum likelihood method. The nonlinearity in the equations imposes an iterative method to find the parameters. I adopt the Berndt, Hall, Hall, and Hausman (BHHH) method. The use of the BHHH algorithm requires the log-likelihood for a single observation. I estimate all models considering \(p=q=1\); the models become:

**GARCH (1, 1):**

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]

**EGARCH (1, 1):**

\[
\ln \sigma_t^2 = \alpha_0 + \beta_1 \ln \sigma_{t-1}^2 + \alpha_1 \left( \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}
\]

**GJR- GARCH (1, 1):**

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1}
\]

All parameters are defined as in section 3.2. Table 3 below shows Estimates of GARCH (1,1); EGARCH (1,1), and GJR-GARCH (1,1) under Normal Distribution, Student-t Distribution, and GED.
Table 3: Estimates of GARCH (1,1), EGARCH (1,1), and GJR-GARCH (1,1) under Normal Distribution, Student-t Distribution, and GED.

<table>
<thead>
<tr>
<th></th>
<th>Normal Distribution</th>
<th>Student-t Distribution</th>
<th>GED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GAR EGA GJR</td>
<td>GAR EGA GJR</td>
<td>GAR EGA GJR</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.014 0.045 0.060</td>
<td>-0.004 0.007 0.019</td>
<td>-0.046 -0.036 0.016</td>
</tr>
<tr>
<td>$\rho$</td>
<td>(0.789) (0.441) (0.326)</td>
<td>(0.940) (0.901) (0.727)</td>
<td>(0.308) (0.449) (0.871)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.643* -0.288* 0.497*</td>
<td>0.804* -0.313* 0.550*</td>
<td>0.662 -0.321* 1.442</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>(0.000) (0.006) (0.001)</td>
<td>(0.030) (0.002) (0.015)</td>
<td>(0.003) (0.000) (0.181)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.501* 0.585* 0.664*</td>
<td>0.667* 0.676* 0.780*</td>
<td>0.515* 0.599* 0.111*</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(0.002) (0.000) (0.010)</td>
<td>(0.039) (0.000) (0.027)</td>
<td>(0.003) (0.000) (0.056)</td>
</tr>
<tr>
<td></td>
<td>(0.041) (0.000) (0.000)</td>
<td>(0.077) (0.000) (0.000)</td>
<td>(0.132) (0.000) (0.212)</td>
</tr>
</tbody>
</table>

Notes: This table reports estimates of the three conditional variance models under the Normal and Student t distributions, and the GED. GAR, EGA, and GJR are the GARCH (1,1), EGARCH (1,1), and GJR-GARCH (1,1) respectively. AIC is the Akaike information criterion; SC is the Schwarz criterion, and Log lik. is the Log likelihood function. The p-values are in parentheses. * denotes significance at the 5 percent level.

The persistence parameters $\beta_1$ from Table 3 are only significant for the EGARCH. They are 0.700, 0.743, and 0.724 for the Normal and Student-t distributions, and GED respectively. This suggests that the degree of persistence is high and close to one. In other words, once volatility increases, it is likely to remain high over several periods. The positive and statistically significant coefficient $\alpha_i$ in all models confirms the presence of clustering, that is, conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). The $\gamma$ coefficients are all negative and significant in the GJR-GARCH model thereby implying that leverage effect exists, that is, an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude. On the other hand, the $\gamma$ coefficients are positive and statistically significant for the GARCH model thereby indicating the presence of asymmetries, that is, volatility is higher during market booms than when market declines. From this discussion, it appears that considering the models, the EGARCH seems to better estimates the series as shown by its high log-likelihood and low Akaike information criterion. When I consider the densities, the Student-t distribution outperforms the Gaussian and the GED for its log-likelihood is higher than that of other distributions.

I conduct a diagnostic check to ascertain whether the models are fairly specified. Table 4 below shows the diagnostic tests for the standardized residuals, squared standardized residuals, and the skewness and kurtosis of the standardized residuals.
The kurtosis is around 5 for all models across all densities except for the GJR-GARCH (1,1) under the GED, which is quite an improvement from the raw series (7.63). Furthermore, the skewness is close to zero for all models across all densities but GJR-GARCH (1,1) under the GED. The Q-statistics for the absence of autocorrelation in the standardized residuals have p-values ranging from 0.380 to 0.972 across all densities against 0.000 for the raw series. This is a seal of absence of autocorrelation. The p-values of the Q²-statistics for the absence of heteroskedasticity range from 0.380 to 1.00 across all densities relative to 0.00 in the original series. It suggests that there is absence of heteroskedasticity. The results above suggest that the models are fairly specified and can therefore be used for forecasting purposes. I conduct in the next paragraphs, the static out-of-sample forecast. The results are shown in Table 5 below.

Two measures of forecasting accuracy, the roots mean square error (RMSE) and the mean absolute error (MAE) are used. Apart from the Normal distribution, the EGARCH model systematically outperforms all the GARCH and GJR-GARCH under the Student-t...
distribution and the GED by yielding smaller RMSE and MAE. Furthermore, the improvement in the log-likelihood function shown in Table 3 corroborates this finding. The EGARCH model does a better job in describing the data under study and can therefore be used for forecasting purposes.

5 Conclusion

This study has addressed the issues of the forecasting performance of conditional volatility models in the regional stock market of the West African Economic and Monetary Union called the BRVM. I compare the forecasting performance of GARCH, EGARCH, and GJR-GARCH models across the Normal distribution, Student-t distribution, and Generalized Error Distribution (GED). My in-sample estimations reveals that the EGARCH model does a better job in describing the data than others; my out-of-sample forecasts using the RMSE and MAE as measures of forecasting accuracy, show that the EGARCH model outperforms both the GARCH and the GJR-GARCH models under the Student-t distribution and the GED. This finding could have implications for market analysts. They would find it useful to use EGARCH models and non-Normal densities to estimate securities’ volatilities with the aim to design volatility timing strategies.

References

Forecasting Stock Return Volatility in the West African Regional Stock Market


