Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson: A Case of a Zimbabwean Bank

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Abstract

The primary objective of the study was to determine the best parametric model that can be used for fitting yield curves for a bank between Nelson-Siegel model and Nelson-Siegel-Svensson. Nelson-Siegel and Nelson-Siegel-Svensson models using Ordinary Least Squares after fixing the shape parameters to make the models linear models. A t-test conducted is conducted on the adjusted $R^2$ of the two models and results showed that Nelson-Siegel-Svensson model fits better the yield curves of the Bank compared to Nelson-Siegel model. An analysis of the out-of-sample forecasting abilities of the two models using AR(1) conducted using E-views shows that the two parametric models have excellent out-of-sample forecasting abilities on all of their parameters. The time independent of Nelson-Siegel-Svensson model was found to be negative in most of the time and could not be interpreted as a long run yield of the Bank. It is also highlighted that the models produces very low levels of $R^2$ in many cases because of the high volatility that is found in the market interest rates of certificates of deposits. The estimated yield curve may be used as a reference curve for funds transfer pricing systems.

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Keywords: Fund transfer price, Nelson-Siegel, Nelson-Siegel-Svensson, yield curve, out-of-sample forecasting.

1 Introduction

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The term structure of interest rates is the relationship between yield to maturity of default-free zero coupon securities and their maturities (Sundaresan 1997 p. 176) and it is usually represented by means of a yield curve indicating different rates for different maturities. The yield curve can take many different shapes and thus can be upward sloping, downward sloping, flat or hump-shaped as explained by four main theories which happen to be not part of this study. Interest rate term structures have important applications in economics and finance especially for banks and governments. An understanding of the term structure is very important for appraising the interest rate risk of banks because all funding or investing decisions resulting from liquidity gaps have an impact on interest rate risk. Funding or investing decisions require a choice of maturity and a choice of interest rate which can only be made with the interest rate structure as the basic input. This study tries to contribute to the Asset and Liability Management (ALM) of banks in Zimbabwe by providing a solution to the estimation of bank specific term structures that are used in setting up of Fund Transfer Pricing systems. According to BIS (2005), all term structure estimation models can be broadly classified into parametric and spline based approaches. Nelson-Siegel and its extension the Nelson-Siegel-Svensson model are parametric model and are also known as function based models because they are specified as single-based functions that are defined over the entire maturity domain. Zimbabwe’s interbank market which is a key component of the money market and the starting point of the monetary transmission mechanism has suffered from the persistent liquidity challenges. The absence of money market instruments in form of government paper has been contributing to the poor functioning of the interbank market, because market players cannot trade without suitable and acceptable collateral instruments to cover counterparty risks. According RBZ (2013) the cost of capital has remained high as evidenced by high lending rates as well as high bank charges. The high lending rates have discouraged borrowing in the economy, while initiatives to meaningfully mobilize savings have been militated by high bank charges, thus undermining the intermediary role played by banks.

The estimations and calibrations in this study were focused only on two parametric models namely the Nelson-Siegel and Nelson-Siegel-Svensson. The study explored the estimation approaches that produce the best parameters and explored techniques for in-sample and out-of-sample forecasting that maintains goodness of fit and smoothness.

2 Literature on Estimation of Term Structures

The Nelson Siegel Model can express the yield curve at any point of time as a linear combination of the level, slope and curvature factors, the dynamics of which drive the dynamics of the entire yield curve as mentioned by Diebold et al (2004). The most important factor in determining the movement of term structures is the level factor according to Litterman and Scheinkman (1994). The second factor tends to have an effect on short-term rates that is opposite to its effect on long term rates. The third factor is the curvature factor, because it causes the short and long ends to increase, while decreasing medium-term rates. Diebold et al (2004) cited that the dynamics of the three factors drive the dynamics of the entire term structure. Nelson and Siegel in their initial model formulated the parameter...
such that it can change with time, but Diebold and Li (2003) argued that fixing the parameter for the entire time resulted in a very little loss of fit and therefore concluded that $\lambda t$ should be fixed at 0.0609 so that the estimation procedure is simplified economic intuition is sharpened.

Nelson and Siegel (1987) parameterised the Nelson Siegel model and computed the best fitting values of the coefficients using linear least squares. The procedure was repeated over a grid of values for $\lambda$ (time constant) to produce the overall best fitting values. Annaert et al (2012) referred to this procedure as a grid search. Nelson and Siegel (1987) found the best fitting values of $\lambda$ within the range of 50-100 and discovered that small values of $\lambda$ are able to fit the curvature at low maturities because they correspond to rapid decay in the regressors. Correspondingly large values of $\lambda$ were found to produce slow decay in the regressors and fitted curvature over long maturity ranges though unable to follow extreme curvature at short maturities.

Nelson and Siegel (1987) obtained the best fit for US T-bills to be given by $\lambda = 40$. Nelson and Siegel (1987) highlighted that the set values of parameters are not expected to fit the data perfectly because a highly parameterised model that could follow all the wiggles in the data is less likely to predict well than a parsimonious model that assumes more smoothness in the underlying relation than one observed in data. The Nelson Siegel model was able to account for a very large fraction of the yields with a median $R^2$ of 0.9159 and Nelson and Siegel (1987) empirically found that little is gained in practice by fitting $\lambda$ to each data set individually.

Aljinovic et al (2012) compared the performance of Nelson-Siegel and Nelson-Siegel-Svensson models using yield data from the Croatian market. The main objective of Aljinovic et al (2012) was to find the best fit model for yield curve estimation in Croatia. Yield data that was used was collected on weekly basis and Aljinovic et al (2012) used Excel in estimating the parameters of the two models using OLS with quasi Newton. It cases where it was difficult to estimate the parameters, Aljinovic et al (2012) resorted to using the Simplex method in statistic that is found in StatSoft. Nelson-Siegel model and Nelson-Siegel-Svensson model were compared using $R^2$ that gives information about goodness of fit of a model. Aljinovic et al (2012) compared the determination coefficient for the two models and performed t-tests at a 1% level of significance and found out that Nelson-Siegel-Svensson model produced a better fit for Croatian term structure.

Annaert et al. (2012) compared the different estimation methods and evaluated the estimation procedures based on the mean absolute error of their forecasting performance (Mean Absolute Prediction Error/ MAPE) and considered the estimation procedure that produce the lowest MAPE to be the best method. Ridge regression produced the minimum MAPE out of the estimation methods that where evaluated. Rezende and Ferreira (2011) compared the modeling and forecasting ability of a class of Nelson-Siegel models that included the three factor Nelson Siegel (1987) model, Bliss’s three factor Model (1997), Nelson-Siegel-Svensson Model (1994) and their Five Factor Model based on a Quantile Autoregression (QAR)forecasting approach and daily implicit yield data from the interbank market. Rezende and Ferreira (2011) used the same approach as used by Annaert et al (2012) of minimising the average of the root mean squared error in comparing the model fitness and found that Nelson-Siegel-Svensson QAR forecasts produce a smaller Root Mean Squared Error when compared with Nelson Siegel QAR forecasts.
Diebold and Li (2006) found out that Nelson Siegel Model produces term structure forecasts that appear much more accurate at long horizons than various standard benchmark forecasts. Nelson Siegel model is however inconsistent with the no-arbitrage property meaning that the consistency between the dynamic evolution of interest rates and the actual shape of the yield curve is not ensured at certain moments as argued by Bjork and Christensen (1999).

Elen (2010) empirically tested whether the Nelson Siegel parameters legitimately reflect the level, slope and curvature elements of a term structure by first constructing a level, slope and curvature from observed yield data and then comparing them with the estimated parameters of the model. 25 year yield was taken to be the level of the yield curve and the slope was defined as the difference between the 25 year and 3 month yields (straight line). The curvature was computed as 2 times 2 year yield minus the sum of the 3-month and 25 yields. Elen (2010) then created a time series of the three factors of Nelson Siegel found by ordinary least squares and found out that the estimated factors and the defined factors followed the same pattern thus concluding that based on Canadian yields, the three factors of Nelson Siegel were indeed level, curvature and slope.

The method of Svensson (1994) is more flexible and has a better fitting than the original method of Nelson& Siegel (1987) as noted by Laurini and Moura (2010). Gilli et al (2010) estimated the parameters of Nelson-Siegel-Svensson using the approach introduced by Diebold and Li (2003) of fixing $\lambda_1$ and $\lambda_2$ and then estimate the rest of the parameters using a least squares algorithm. Gilli et al (2010) pointed out that the need to have constraints when solving optimization problem of obtaining parameters to guarantee the getting reasonable values. Just like Nelson and Siegel (1987), Diebold and Li (2003) and others, Gilli et al (2003) used the least squares approach to obtain the parameters for Nelson-Siegel-Svensson model with constraints: $0<\beta_1<15; -15<\beta_2<30; -30<\beta_3<30; -30<\beta_4<30; 0<\lambda_1<30$ and $0<\lambda_2<30$. Gilli et al repeated the estimation procedure using other algorithms like MATLAB’s fmin search that uses Nelder-Mead in obtaining parameters and observed that the yield fit was better than the parameter fit and most errors where of 10 basis points and concluded that Nelson-Siegel-Svensson works well in interpolating observed yields.

To avoid potential challenges in numerical optimization, Molenaars et al (2013) followed Diebold and Li (2003)'s approach by fixing $\lambda_1=\lambda_2=0.0609$ and estimated the remaining parameters using ordinary least squares regression. The out of sample performance of forecasting procedures was evaluated using RMSE. The smaller the RMSE is, the better the forecasting ability of the model considered is. MATLAB was used in this study and Molenaars et al (2013) concluded that Nelson-Siegel model does not properly fit the yield curve at all dates because it imposes a functional form to the yield curve that can result in inferior yield estimates if the yield curve does not fit to the functional form.

Bliss (1996) demonstrated the dangers of using in-sample goodness of fit as the sole criterion for comparing term structure estimation method. Bliss used parametric and non-parametric tests in comparing five term structure estimation methods and concluded that the Unsmoothed Fama-Bliss is overall the best, but also recommended that users interested in fitting term structures parsimoniously need to consider either the Smoothed Fama-Bliss or the Extended Nelson Siegel methods. In-sample results give a distorted view of the performance of the term structure, because there is a danger of over-fitting the data and this can be eliminated by using out-of-sample tests evaluating estimation methods. Nelson and Siegel (1987) also argued that the criteria for a satisfactory yield
curve model is that it be able to predict yields beyond the maturity range of the sample
used to fit it because a function may be flexible to fit data over a specific interval, but may
have very poor properties when extrapolated outside that interval.

3 Methodology

The researcher employed a quantitative research design in an attempt to answer the
research questions of the study. The primary research question of this study was to find
the most suitable method of fitting term structures for the Bank and suitability implies
performing some statistical tests to compare the Nelson-Siegel model and the
Nelson-Siegel-Svensson model.

3.1 Research Data

The researcher used daily yield data on certificates of deposits for the period starting
March 2012 to March 2013. Yield data was annualized first before it was used in
estimating the parameters of the term structure fitting models to ensure that yield data was
based on the same temporal platform. The formula for calculating weighted annual
effective rates is given below:

\[
\text{Weighted AER}(M_j) = \frac{\sum_{i=1}^{n} \text{Cost}_i \cdot \text{AER}_i}{\sum_{i=1}^{n} \text{Cost}_i} \quad \text{for} \quad \sum_{i=1}^{n} \text{Cost}_i > 0
\]  

(1)

Where:
- \( j \) = maturity for certificate of deposits for \( \forall j > 0 \)
- \( n \) = the number of deposits with the same maturity,
- \( \text{Cost}_i \) is the size of deposit \( i \) with maturity \( M_j \),
- \( \text{AER}_i \) is the Annual Effective Rate for deposit \( i \) with maturity \( M_j \) and it is calculated as:

\[
\text{AER} = \left(1 + \frac{r_i}{360}\right)^{\frac{360}{1}} - 1
\]  

(2)

The weighted annual effective rates were regarded as the observed yields for the Bank
and they were then used in the estimation of parameters for Nelson-Siegel model and
Nelson-Siegel-Svensson models. Results from the two models were compared to
determine the best fit model.

3.2 Term Structure Relationships

Term structure estimation methods are designed for the purpose of approximating one of
the three equivalent representations of the term structure: spot rate curve, discount rate
curve and the forward rate curve (Pooter, 2007). Assuming that all rates are continuously
compounded, a forward rate \( f_t(\tau, \tau^*) \) is the interest rate of a forward contract on an
investment which is initiated \( \tau \) periods in the future and matures \( \tau^* \) periods after the
date of initiation. The instantaneous forward rate \( f_t(\tau) \) is obtained by letting the maturity
of such a forward contract go to zero;
lim_{\tau \to 0} f_t(\tau, \tau^*) = f_t(\tau) \tag{3}

The spot rate (yield) with \( \tau \) periods to maturity denoted by \( y_t(\tau) \) can be obtained from the instantaneous forward rate. When the instantaneous forward rates are continuously compounded up to time to maturity \( \tau \) and equally weighted, \( y_t(\tau) \) is obtained:

\[
y_t(\tau) = \frac{1}{\tau} \int_0^{\tau} f_t(m) dm \tag{4}
\]

The discount function \( P_t(\tau) \) denoting the present value of zero coupon bonds paying out a nominal amount of $ after \( \tau \) periods can be obtained from the spot rate \( y_t(\tau) \):

\[
P_t(\tau) = \exp[-\tau y_t(\tau)] \tag{5}
\]

equivalently:

\[
y_t(\tau) = -\frac{1}{\tau} \log P_t(\tau) \tag{6}
\]

Linking forward rates directly to spot rates:

\[
y_t(\tau) = \frac{1}{\tau} \int_0^{\tau} f_t(m) dm
\]

\[
\frac{d}{d\tau} y_t(\tau) = \frac{1}{\tau} f_t(\tau) \bigg|_0^\tau
\]

\[
\tau \frac{d}{d\tau} y_t(\tau) = f_t(\tau) - f_t(0)
\]

\[
f_t(0) = y_t(\tau) = \text{spot rate}
\]

\[
f_t(\tau) = \frac{d}{d\tau} y_t(\tau) + y_t(\tau) \tag{7}
\]

### 3.3 Nelson Siegel Model

Nelson and Siegel (1987) introduced a simple parsimonious model for approximating the forward rate curve with mathematical approximating functions in the form of Laguerre functions. Nelson and Siegel (1987) assumed that the instantaneous forward rate at maturity \( \tau \) denoted \( f_t(\tau) \) is given by the solution to a second order differential equation with real equal roots for spot rates:

\[
f_t(\tau) = \beta_{0,t} + \beta_{1,t} \left( \frac{\tau}{\lambda_t} \right) + \beta_{2,t} \left( \frac{\tau}{\lambda_t} \right) \exp \left( \frac{-\tau}{\lambda_t} \right) \tag{8}
\]

Averaging the forward rates

\[
y_t(\tau) = \frac{1}{\tau} \left[ \int_0^{\tau} \left[ \beta_{0,t} + \beta_{1,t} \exp \left( \frac{-m}{\lambda_t} \right) + \beta_{2,t} \left( \frac{m}{\lambda_t} \right) \exp \left( \frac{-m}{\lambda_t} \right) \right] dm \right]
\]
\[ y_t(\tau) = \beta_{0,t} + \beta_{1,t} \left[ \frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} \right] + \beta_{2,t} \left[ \frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} - \exp\left(-\frac{\tau}{\lambda_t}\right) \right] \] (9)

The Nelson-Siegel Model that is widely used is the one for estimating spot rates given by equation (3.7) above. The model has four parameters that had to be estimated in order to estimate a yield for a given maturity \((\tau)\) which are \(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}\) and \(\lambda_t\). The factors that make up the Nelson Siegel model are best interpreted by interpreting the factor loadings in the model because they exhibit different properties over a range of maturities. The first component in the Nelson-Siegel model is \(\beta_{0,t}\) and its factor loading is 1 which is a constant that is independent of maturity, thus it is often interpreted as the level factor representing the long run yield.

The second component \(\beta_{1,t}\) is interpreted as the slope and is weighted by a function of time to maturity \((\tau)\). The factor loading of \(\beta_{1,t}\) is \(\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}}\) and when \(\tau = 0\) it is unit and it exponentially decays to zero when \(\tau \to \infty\) primarily affecting \(\beta_{1,t}\) in the short run. The component can either have a downward \((\beta_{1} > 0)\) or upward \((\beta_{1} < 0)\) slope. The third component in the model \(\beta_{2,t}\) is interpreted as the curvature and its factor loading is \(\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} - \exp\left(-\frac{\tau}{\lambda_t}\right)\). The function represent the medium term component in a yield curve because it starts at zero gradually increasing and decreases back to zero as \(\tau \to \infty\) thereby adding a hump to the curve and is termed the medium term. The component can generate a hump if \(\beta_{2,t} > 0\) or a trough if \(\beta_{2,t} < 0\). The higher the absolute value of \(\beta_{2,t}\), the more pronounced the trough or hump is.

The limiting behavior of the Nelson-Siegel model:

\[
\lim_{\tau \to 0} y_t(\tau) = \beta_{0,t} + \beta_{1,t} \\
\lim_{\tau \to \infty} y_t(\tau) = \beta_{0,t} + \beta_{1,t}
\] (10) (11)

### 3.4 Nelson Siegel Svensson Model

Nelson-Siegel (1987) model has gone through several improvements to enhance flexibility and to capture a wider variety of curve shapes with the modifications mainly carried out by adding factors and decaying parameters. Svensson (1994) extended the Nelson Siegel model by incorporating for a second hump/trough in the model. Svensson (1994) added another exponential term more similar to the third term in Nelson-Siegel Model, but with a different decaying parameter \(\lambda_{2t}\).

The additional factor loading on the Nelson-Siegel-Svensson model is of \(\beta_{3,t}\) and is given by the same Laguerre function as the one in Nelson-Siegel model that determines the curvature of the yield curve, but with a different decay parameter \(\lambda_{2t}\). The additional factor loading is \(\frac{1-\exp\left(-\frac{\tau}{\lambda_{2t}}\right)}{\frac{\tau}{\lambda_{2t}}} - \exp\left(-\frac{\tau}{\lambda_{2t}}\right)\). The limiting behavior of the Nelson-Siegel-Svensson model is the same as the limiting behavior of Nelson-Siegel model provided in (8) and (9).
\[ y_t(\tau) = \beta_{0t} + \beta_{1t} \left[ \frac{1 - \exp\left(-\frac{\tau}{\lambda_{1t}}\right)}{\lambda_{1t}} \right] + \beta_{2t} \left[ \frac{1 - \exp\left(-\frac{\tau}{\lambda_{1t}}\right) - \exp\left(-\frac{\tau}{\lambda_{1t}}\right)}{\lambda_{1t}} \right] + \beta_{3t} \left[ \frac{1 - \exp\left(-\frac{\tau}{\lambda_{2t}}\right)}{\lambda_{2t}} \right] \]

The NSS model therefore incorporates two decaying parameter which are \( \lambda_1 \) and \( \lambda_2 \) which means it has 6 parameters that will have to be estimated \( \beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t}, \lambda_{1t}, \lambda_{2t} \) compared to the 4 parameters in NS model. Bolder and Streliski (1999) discussed the multi-collinearity problem that came within the model if \( \lambda_{1t} = \lambda_{2t} \). The multicolinearity problem is reduced by ensuring that \( \lambda_{1t} \neq \lambda_{2t} \).

### 3.5 Parameter Estimation

**Ordinary Least Squares**

The researcher fixed the \( \lambda_{1t} \) parameter as recommended by Diebold and Li (2004) and Elen (2010) to simplify the estimation procedure by reducing the model to a linear regression. The estimation of parameters was done by considering the following

\[ Y_t(\tau) = X_t \ast \beta_t + \epsilon_t \]  

(13)

functional form for a regression:

where \( Y_t(\tau) \) denotes the vector of interest rates (annual effective rates) at time \( t \) for \( n \) different times of maturity collected in vector \( \tau \), \( \epsilon_t \) is a vector of error terms at time \( t \) for \( n \) different estimates of interest rates, \( X_t \) is a vector of the factor loadings of the Nelson Siegel Model which can be estimated given maturity (\( \tau \)) and \( \lambda \). The vector of the factor loadings can be interpreted as the vector for independent variables that explain the dependent variable yield. Using equation (3.9) for Nelson-Siegel model the following vector is obtained:

\[ X_t = \begin{bmatrix}
1 & \frac{1 - e^{-\kappa_1 \tau_1}}{\kappa_1 \tau_1} & \frac{1 - e^{-\kappa_1 \tau_1}}{\kappa_1 \tau_1}
1 & \frac{1 - e^{-\kappa_1 \tau_2}}{\kappa_1 \tau_2} & \frac{1 - e^{-\kappa_1 \tau_2}}{\kappa_1 \tau_2}
1 & \frac{1 - e^{-\kappa_1 \tau_3}}{\kappa_1 \tau_3} & \frac{1 - e^{-\kappa_1 \tau_3}}{\kappa_1 \tau_3}
1 & \frac{1 - e^{-\kappa_1 \tau_n}}{\kappa_1 \tau_n} & \frac{1 - e^{-\kappa_1 \tau_n}}{\kappa_1 \tau_n}
\end{bmatrix} \]

(14)

where \( \frac{1}{\lambda_t} \) is replaced by \( \kappa_t \) for simplistic in writing the vector \( X_t \) and vector \( \beta_t = \beta_{0t}, \beta_{1t}, \beta_{2t} \).

Equation 3.14 was extended with the inclusion of another column of the fourth loadings of Nelson-Siegel-Svensson model. Estimations were done for 252 different dates and the researcher built a model that used macro for ordinary least squares (algorithm for ordinary least squares) in Excel using Visual Basics in Excel. The VBA code to perform
the ordinary least squares regression and the analysis of variance was written using
ordinary least squares formulas provided by Davidson and MacKinnon (1993) and the
steps found in Rouah and Vainberg (2007). The estimate of parameters using ordinary
least squares is given by:

$$\hat{\beta}_{OLS} = (X^T X)^{-1} X^T Y$$  \hspace{1cm} (15)

Where:
$$\hat{\beta}_{OLS}$$ = a vector of dimension k containing estimated parameters
$$X = (\mathbf{1} X_1 X_2 \ldots X_{k-1})$$ = a design matrix of dimension n x k containing independent
variable and the vector $$\mathbf{1}$$
$$\mathbf{1}$$ = a vector of dimension n containing ones
The fitted values can then be obtained using the estimated vector of parameters $$\hat{\beta}_{OLS}$$

$$\hat{Y} = X\hat{\beta}_{OLS}$$  \hspace{1cm} (16)

For the analysis, the total sum of squares (SSTO) is defined as the variability of the
dependent variable about it mean. SSTO can be expressed in terms of the error sum of
squares (SSE) and the regression sum of squares (SSR)

$$SSTO = SSE + SSR$$  \hspace{1cm} (17)

Where:
$$SSTO = \sum_{i=1}^{n} (y_i - \bar{y})^2$$
$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (Y - \hat{Y})^T (Y - \hat{Y})$$
$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

$$y_i$$ = elements of Y
$$\hat{y}_i$$ = elements of $$\hat{Y}$$ (i = 1, 2, ..., n)
$$\bar{y} = \frac{1}{n} \sum_{i=0}^{n} y_i$$ is the sample mean of the dependent variable

The mean square error (MSE) which is an estimate of the variance $$\sigma^2$$ is given as

$$\hat{\sigma}^2 = MSE = \frac{SSE}{n-k}$$  \hspace{1cm} (18)

and the estimate of the standard deviation is given by $$\hat{\sigma} = \sqrt{MSE}$$. The coefficient of
determination $$R^2$$ that measures the proportion of variability in the dependent variable that
can be attributed to a linear model is given by the proportion of SSTO
In order to incorporate for the increase in $R^2$ as a result of an increase in variables, adjusted $R^2$ is calculated and is given by

$$R_a^2 = 1 - \left(\frac{n-1}{n-k}\right)(1 - R^2)$$

(20)

The covariance matrix of the parameters is given by

$$\text{Cov}(\hat{\beta}_{\text{OLS}}) = \text{MSE}(X^T X)^{-1}$$

(21)

The t-statistic for each regression coefficient is given

$$t = \frac{\hat{\beta}_j}{\text{SE}(\hat{\beta}_j)}$$

(22)

where: $\hat{\beta}_j$ = j-th element of $\hat{\beta}$

$\text{SE}(\hat{\beta}_j)$ = standard error of $\hat{\beta}_j$ obtained as the square root of the j-th diagonal element of (3.21). Once a $t$-statistic is obtained for a parameter, the two-tailed $p$-value can then be obtained from a Student $t$-distribution with $(n-k)$ degrees of freedom to determine the statistical significance of the regression coefficient.

### 3.6 Model Comparison

The researcher compared the two models to determine which model best fits Zimbabwean data based on the yield data of certificates of deposits. The models were compared using the Coefficient of Determination ($R^2$) and the Root Mean Square Error (RMSE).

The formula for $R^2$ is given by:

$$R^2 = \frac{SS_R}{SS_T}$$

(23)

where $SS_R$ is the regression sum of squares calculated as $\Sigma_{i=1}^n (\hat{y}_i - \bar{y})^2$ and $SS_T$ is the total sum of squares calculated as $\Sigma_{i=1}^n (y_i - \bar{y})^2$. The t-test was conducted in Excel.

The formula for RMSE is given as:

$$\text{RMSE} = \sqrt{\frac{\Sigma_{i=1}^n (y_i - \bar{y}_i)^2}{n}}$$

(24)

### 3.7 Out-of-sample Forecasting

The researcher followed the works of Diebold and Li (2006) and De Pooter (2007) by specifying a first-order autoregressive process for forecasting the out-of-sample parameters for the Nelson-Siegel model and Nelson-Siegel-Svensson model. An autoregressive (AR) of order $p$ is a process where the realisation $\beta_t$ is a weighted sum of past realisations i.e.$\beta_{t-1}, \beta_{t-2}, \ldots, \beta_{t-p}$. The first-order autoregressive process (AR(1)) is
expressed as:

\[ \beta_t = \mu + \phi \beta_{t-1} + \nu_t \]  

where \( \mu \) is the intercept and \( \phi \) is the coefficient of the lagged value of \( \beta_t \) and \( \nu_t \) is the random error. The parameters of the AR (1) process were obtained using non-linear regression in E-views. The model was used in forecasting \( \beta_{0,t}, \beta_{1,t}, \beta_{2,t} \) parameters for Nelson-Siegel model and \( \beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t} \) for Nelson-Siegel-Svensson model. The researcher used both the Dynamic and Static methods available in E-views in forecasting in order to determine the best approach.

4 Main Results

4.1 Grid Search for \( \lambda \) for Nelson-Siegel Model

A Grid search for \( \lambda \) was done using data for 25 March 2013 to find a value of \( \lambda \) that could be fixed for all the dates while producing a good fit. Increasing \( \lambda \) resulted in fitting a yield curve that increased yield in the short maturity and minimised the yield in the longer maturity showing signs on high decay parameter that results in a strong negative slope in the long run. Decreasing the shape parameter resulted in lower short term yields that were compensated by higher yields in the longer maturity. The evolution of the fitted yield curve to changes in the shape parameter is shown below:

![Grid Search- NS fitted Yield curve](image1)

![Grid Search- NS fitted Yield curve](image2)

![Grid Search- NS fitted Yield curve](image3)

![Grid Search- NS fitted Yield curve](image4)

Figure 1: Fitted NS yield curves with different \( \lambda \)s
The value of \( \lambda \) that was chosen is the one that fits the both the short maturities and long maturities better. \( \lambda = 0.25 \) fitted the yield curve better and was considered as the shape parameter that was fixed for the whole year’s daily estimates.

### 4.2 Analysis of Time Series Results of Nelson-Siegel Model (\( \lambda = 0.25 \))

The researcher estimated and fitted the yield curve for 257 different days between March 2012 and March 2013. Analysis of variance was done on each day’s estimations with t-statistics, p-values and betas (s.e.)\(^3\) for the three parameters were calculated and analysed. R\(^2\), Adjusted R\(^2\), \( \sqrt{\text{MSE}} \) and \( \varepsilon^2 \) was also calculated for the 257 days and a time series of these statistics was generated. The time series of the parameters and statistics is presented below by making use of graphs:

\(^3\)Standard error of estimated parameters
The parameters of Nelson-Siegel exhibited an erratic behavior over the year that is mainly attributed to the erratic behavior in the economic environment in Zimbabwe. $\beta_0$ started at 0.0684 and increased steadily to reach a peak of 0.1202 in July 2012 before it started dropping and ended at 0.0752 in March 2013. $\beta_0$ is interpreted as the long run yield in literature (Diebold and Li, 2006) because the parameter remained positive over the year of study as expected in a market where there are no negative interest rates on long term loans data. $\beta_1$ was a little more volatile than $\beta_2$ and had a standard deviation of 2.41%. $\beta_1$ started at -0.0401 in March 2012 and increased gradually increased to reach a positive value of 0.00778 and started dropping to end at -0.00987 a higher value than the starting value. $\beta_2$ was the most volatile parameter of the Nelson-Siegel model with a standard deviation of 4.86%.

From the results and graphs presented above it was found the $R^2$ of the Nelson-Siegel model ranged from 1.9% to 72.9% during the year depending on the structure of the market data that was used in making the estimates. The best fit was obtained on 3 March 2012 and the worst performance of the Nelson-Siegel model was observed on 1 October 2012. The overall average $R^2$ of the model is 24.4%. As earlier on argues at the beginning of the chapter, this small value of $R^2$ does not mean that Nelson-Siegel Model was unable to fit well a smooth curve that can be used for fund transfer pricing. The small value of $R^2$ is largely attributed to the noise in the market data based on certificate of deposits.

A correlation analysis of the parameters of Nelson-Siegel and $R^2$ showed that $\beta_2$ has a strong positive correlation with $R^2$ which implies that whenever the model has trouble in obtaining a good fit, it is reflected by a drop in the value $\beta_2$ that determines the weight of the curvature component of the mode. Thus any trouble in determining the curvature of the yield curve implies that the data is too volatile and the model will only manage to account for less of the proportion of variability. Correlation results are shown in Appendix 1. The diagram below shows the movements between $\beta_2$ and $R^2$. 

Table 1: Measures of central tendency for parameters and statistics of Nelson-Siegel model

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$R^2$</th>
<th>Adj $R^2$</th>
<th>$\sqrt{\text{MSE}}$</th>
<th>$\varepsilon^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.067008</td>
<td>-0.068990</td>
<td>-0.063170</td>
<td>0.018918</td>
<td>-0.006565</td>
<td>0.009233</td>
<td>0.006530</td>
</tr>
<tr>
<td>Max</td>
<td>0.120198</td>
<td>0.007775</td>
<td>0.217751</td>
<td>0.737676</td>
<td>0.728631</td>
<td>0.021569</td>
<td>0.033962</td>
</tr>
<tr>
<td>Mean</td>
<td>0.087544</td>
<td>-0.027450</td>
<td>0.072067</td>
<td>0.244388</td>
<td>0.224150</td>
<td>0.015346</td>
<td>0.017770</td>
</tr>
<tr>
<td></td>
<td>0.012451</td>
<td>0.024142</td>
<td>0.048620</td>
<td>0.152127</td>
<td>0.155248</td>
<td>0.002456</td>
<td>0.005000</td>
</tr>
</tbody>
</table>
4.3 Nelson-Siegel-Svensson Model

Grid Search for $\lambda_1$ and $\lambda_2$ for Nelson-Siegel-Svensson Model

The Nelson-Siegel-Svensson required two shape parameters $\lambda_1$ and $\lambda_2$ so that the model can be estimated using OLS. The researcher conducted a grid search by varying $\lambda_1$ and $\lambda_2$ in the range (0, 5] and (0, 5] respectively. The grid search was done on data for 31 August 2013 that posted one of the smallest $R^2$ using Nelson-Siegel model. 2500 estimates were done in the grid (0, 5] with variations of 0.1 on the estimations of shape parameters and the best fits that had a high $R^2$ were plotted to determine the evolution of the fitted yield curves as $\lambda_1$ and $\lambda_2$ changes. The diagram below shows some of the yield curves that were estimated in the grid search:

The estimates were then further reduced making use of $R^2$ and observation and eliminated all the yield curves that did not produce a better fit and obtained the following 11 curves.
The 11 curves were then analysed to obtain the curve that produced a better fit at the end of the curve because one that produces a lower cost at longer maturities can be misleading and very costly for the bank. The researcher realized that increasing the value of $\lambda_1$ resulted in a trough at the shortest maturity but this was compensated by significant loss of fit at the longer maturity as shown in Diagram 8. Based on the analysis of the results from the grid search, the researcher finally obtained the best yield curve that was estimated using $\lambda_1 = 0.714$ and $\lambda_2 = 10$. The yield curve is shown in the diagram below:

![NSS curves-grid search](image)

Figure 5: Reduced estimates of yield curve using varying shape parameters

Analysis of time series results of Nelson-Siegel-Svensson Model ($\lambda_1 = 0.714$ and $\lambda_2 = 10$)

The researcher estimated and fitted the yield curves for the 257 different days between March 2012 and March 2013. Analysis of variance was done on each day’s estimations with t-statistics, p-values and betas (s.e.) for the four parameters were calculated and analysed just like what was done for Nelson-Siegel Model. $R^2$, Adjusted $R^2$, $\sqrt{MSE}$ and $\varepsilon^2$ was also calculated for the 257 days and a time series of these statistics was generated. The time series of the parameters and statistics of Nelson-Siegel-Svensson model is presented below by making use of graphs:

![NSS curve](image)

Figure 6: Best yield curve from the grid search
Figure 7: Time series of Nelson-Siegel-Svensson Parameters and Statistics
Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson

Table 2: Measures of central tendency for parameter and statistics of Nelson-Siegel-Svensson model

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$R^2$</th>
<th>Adj $R^2$</th>
<th>$\sqrt{MSE}$</th>
<th>$\epsilon^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-1.2168</td>
<td>-0.85409</td>
<td>-0.56155</td>
<td>-4.36781</td>
<td>0.03868</td>
<td>0.00073</td>
<td>0.00915</td>
<td>0.00670</td>
</tr>
<tr>
<td>Max</td>
<td>0.92950</td>
<td>1.25956</td>
<td>1.29225</td>
<td>5.90104</td>
<td>0.72439</td>
<td>0.70989</td>
<td>0.02135</td>
<td>0.03282</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.17147</td>
<td>0.23748</td>
<td>0.33615</td>
<td>1.00433</td>
<td>0.26425</td>
<td>0.23435</td>
<td>0.01520</td>
<td>0.01719</td>
</tr>
<tr>
<td></td>
<td>0.40473</td>
<td>0.40485</td>
<td>0.33236</td>
<td>1.96836</td>
<td>0.15327</td>
<td>0.15785</td>
<td>0.00225</td>
<td>0.00456</td>
</tr>
</tbody>
</table>

$\beta_0$ started at -1.2168 in March 2012 and increased steadily to reach a peak of 0.9295 in July 2012 before it started dropping and ended at -0.53452 in March 2013. This trend was also identified with $\beta_0$ of Nelson-Siegel model but the difference is that parameters of in Nelson-Siegel the parameter was never positive and was easily interpreted as representing the long run yield in the Zimbabwean market. $\beta_0$ is thus not interpreted as the long run yield for the market understudy as it is interpreted in literature (Diebold and Li, 2006) because of its negative values. The parameter $\beta_0$ also showed a very large standard deviation of 40.47% which is much higher that of Nelson-Siegel of 1.25%. $\beta_1$ of Nelson-Siegel-Svensson was unstable compared to that of Nelson-Siegel Model with a standard deviation of 40.48%. $\beta_1$ moved in a perfect opposite direction with $\beta_0$ as shown by a high negative correlation of 99.9%.

$\beta_2$ was the less volatile compared to the rest of Nelson-Siegel-Svensson parameters and moved in the same direction with $\beta_3$ and $\beta_1$ with correlations of 97.42% and 97.6% respectively. The fourth parameter of Nelson-Siegel-Svensson model that is responsible for fitting a second hump to a yield curve had a high negative correlation with $\beta_0$ of 99.9%. The p-values of the model parameters were below 0.05 in most of the estimates showing that they were statistically significant at a 95% significance level.

From the results and graphs presented above it was found the $R^2$ of the Nelson-Siegel model ranged from 3.8% to 72.4% during the year depending on the structure of the market data that was used in obtaining the estimates. The best fit was obtained on 3 March 2012 and the worst performance of the Nelson-Siegel-Svensson model was observed on 1 October 2012 and these dates correspond to the best and worst performance of Nelson-Siegel model. This shows that the performance of the two models depended on the structure of the market data. The overall average $R^2$ of the Nelson-Siegel-Svensson model is 26.4

4.4 Model Comparisons

Coefficient of Determination

A t-test was conducted on the adjusted coefficients of determination (adj $R^2$) of Nelson-Siegel and Nelson-Siegel-Svensson so that results that incorporated a penalty of additional variables are used in comparing the model. Results of a t-test at 5% significance level are shown in the Table 3 below.
Table 3: t-test for the means of adj R2 of Nelson-Siegel and Nelson-Siegel-Svensson models

t-Test: Two-Sample Assuming Unequal Variances

<table>
<thead>
<tr>
<th></th>
<th>NS Model</th>
<th>NSS Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.224149822</td>
<td>0.234357317</td>
</tr>
<tr>
<td>Variance</td>
<td>0.024102045</td>
<td>0.024919111</td>
</tr>
<tr>
<td>Observations</td>
<td>257</td>
<td>257</td>
</tr>
<tr>
<td>Hypothesized Mean Diff</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>df</td>
<td>512</td>
<td>512</td>
</tr>
<tr>
<td>t Stat</td>
<td>-0.739084265</td>
<td>-1.647835164</td>
</tr>
<tr>
<td>P(T&lt;=t) one-tail</td>
<td>0.230097259</td>
<td>0.460194518</td>
</tr>
<tr>
<td>t Critical one-tail</td>
<td>-1.647835164</td>
<td>1.964608113</td>
</tr>
<tr>
<td>P(T&lt;=t) two-tail</td>
<td>0.460194518</td>
<td>0.000118587</td>
</tr>
<tr>
<td>t Critical two-tail</td>
<td>1.964608113</td>
<td>1.647835164</td>
</tr>
</tbody>
</table>

A test statistic of -0.7391 was obtained against a 5% critical t-value of -1.6478. The test results failed to reject the notion that the proportion of variability of Nelson-Siegel-Svensson model was smaller than that of Nelson-Siegel model i.e. adj R^2 of Nelson-Siegel-Svensson model is statistically higher than that of Nelson-Siegel. The results therefore concludes that Nelson-Siegel-Svensson model is the better model of fitting a term structure for the Bank if the models are compared based on their proportion of variability. See Figure 15 below for graphical comparison of adj R^2.

Root Mean Squared Error

Daily comparisons of the RMSE from the two models were conducted to determine the model that fitted the term structure with precision on each day of the year. Summarized results are in the table below.
Table 4: RMSE comparisons

<table>
<thead>
<tr>
<th>Model</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nelson-Siegel</td>
<td>118</td>
</tr>
<tr>
<td>Nelson-Siegel-Svensson</td>
<td>139</td>
</tr>
<tr>
<td>Total</td>
<td>257</td>
</tr>
</tbody>
</table>

The assessment of the estimate of the standard deviation (RMSE) showed that Nelson-Siegel fitted the yield curve with more precision than its extension, Nelson-Siegel-Svensson model on 118 occasions out of the 257 days estimated. Nelson-Siegel-Svensson proved superior on 139 occasions which corresponds to 54% of the time.

![Comparison of Adj R² for NS and NSS](image1)

![Comparison of RMSE for NS and NSS](image2)

Figure 15: Comparisons of Adj R² and RMSE

4.5 In-sample and Out-of-sample forecasting
The researcher obtained the following summarized results for the AR(1) models for the different parameters of Nelson-Siegel model and Nelson-Siegel-Svensson model. Results were estimated using data observations obtained from daily estimations of the models for the period 01 Mar 2012 – 31 Oct 2012. The rest of the observations from 01 Nov 2012 – 26 Mar 2013 were excluded from parameter estimation and were treated as hold out sample. Results are presented in the format of the specification of an AR (1) model presented in equation (3.25):

\[ \beta_t = \mu + \phi \beta_{t-1} + \nu_t \]

Results for Nelson-Siegel model parameters:
\[
\begin{pmatrix}
\beta_{0t} \\
\beta_{1t} \\
\beta_{2t} \\
\end{pmatrix}
= 
\begin{pmatrix}
0.003283 \\
0.002245 \\
0.002697 \\
\end{pmatrix} + 
\begin{pmatrix}
0.965910\beta_{0t-1} \\
1.000530\beta_{1t-1} \\
0.937058\beta_{2t-1} \\
\end{pmatrix} + 
\begin{pmatrix}
\nu_t \\
\nu_t \\
\nu_t \\
\end{pmatrix}
\]

Results for Nelson-Siegel-Svensson parameters:

\[
\begin{pmatrix}
\beta_{0t} \\
\beta_{1t} \\
\beta_{2t} \\
\beta_{3t} \\
\end{pmatrix}
= 
\begin{pmatrix}
0.006538 \\
-0.0022582 \\
0.008314 \\
-0.021603 \\
\end{pmatrix} + 
\begin{pmatrix}
0.936134\beta_{0t-1} \\
0.937684\beta_{1t-1} \\
0.933942\beta_{2t-1} \\
0.937250\beta_{3t-1} \\
\end{pmatrix} + 
\begin{pmatrix}
\nu_t \\
\nu_t \\
\nu_t \\
\nu_t \\
\end{pmatrix}
\]

Detailed results for the estimation of parameters are found in Appendix 2 and 3. The in-sample and out-of-sample forecasts are plotted in the diagrams below:

![Diagram](image)

**Figure 9:** In-sample and out-of-sample forecasts for $\beta_0$ (NS)
Figure 10: In-sample and out-of sample forecasts for $\beta_1$ (NS)

Forecast: B1_NS
Actual: B1_NS
Forecast sample: 11/01/2012 3/26/2013
Included observations: 95
Root Mean Squared Error 0.002115
Mean Absolute Error 0.001530
Mean Abs. Percent Error 142.4685
Theil Inequality Coefficient 0.112411
Bias Proportion 0.020246
Variance Proportion 0.000000
Covariance Proportion 0.979754
Figure 11: In-sample and out-of sample forecasts for $\beta_2$ (NS)
Figure 12: In-sample and out-of sample forecasts for $\beta_0$ (NSS)

Forecast: B0F_NSS
Actual: B0
Forecast sample: 11/01/2012 3/26/2013
Included observations: 95
Root Mean Squared Error 0.083125
Mean Absolute Error 0.062235
Mean Abs. Percent Error 16.29405
Theil Inequality Coefficient 0.084343
Bias Proportion 0.228133
Variance Proportion 0.022526
Covariance Proportion 0.749340
Figure 13: In-sample and out-of sample forecasts for $\beta_1$ (NSS)
Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson

Figure 14: In-sample and out-of sample forecasts for $\beta_2$ (NSS)

Forecast: B2F_NSS
Actual: B2
Forecast sample: 11/01/2012 3/26/2013
Included observations: 95
Root Mean Squared Error 0.072147
Mean Absolute Error 0.052685
Mean Abs. Percent Error 10.70053
Theil Inequality Coefficient 0.065194
Bias Proportion 0.173213
Variance Proportion 0.028709
Covariance Proportion 0.798079
A graphical analysis of the in-sample (green trends) and out-of-sample (blue trends) forecasts shows that both Nelson-Siegel model and Nelson-Siegel-Svensson models have great out-of-sample forecasting capabilities as shown by the blue trends that are almost in line with the green trends. A statistical analysis of the results of each parameter that was forecasted using the Static method in E-views confirms that the models have very good out-of-sample forecasting abilities using AR(1). The variation proportion of the out-of-sample forecasts for all parameters (NS and NSS) measures the difference between variation in forecasts and variation of actual data and in all cases this value is very small confirming that the variation of out-of-sample forecasts is almost equal to the variation of actual data. The bias component that measures the extent to which the mean of the out-of-sample forecasts is different to the mean of the actual data is also low on all the forecasts. Root mean squared Error that is an estimate of the standard deviation of the out-of-sample is also low on most cases with the exception of forecasts for $\beta_3$ of NSS where it is 0.396.
5 Conclusion

The results presented in this chapter showed that the shape parameter for the Nelson-Siegel model should be fitted at 0.25 and the shape parameters for Nelson-Siegel-Svensson Model should be fitted at 0.714 and 10 respectively to obtain better fits. Analysis of time series parameters and statistics of the two models showed that the models are good in fitting the term structure of a Zimbabwean bank but the coefficients of variability were significantly affected by the noise that was prevalent in the market data of savings certificates used in the study. The t-test that was conducted to test the adjusted R² of the models showed that statistically Nelson-Siegel-Svensson model fits the market data better that Nelson-Siegel model. The average R² of Nelson-Siegel and Nelson-Siegel-Svensson model was 24.4% and 26.4% respectively. Despite the low levels of R² the researcher concludes that the models are fit for fitting the yield curves for a bank because a large proportion of variability of the model was lost due to the volatility nature of the market data used in the study. The two models showed that they have exceptional out-of-sample forecasting capabilities when using AR (1) as the forecasting model.

5.1 Prototype developed

The prototype (TeSffim) is a tool that can be used in the estimation and forecasting of daily term structures for a bank. The design of the prototype used the results obtained from the research titled “Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson Models: Case of a Zimbabwean Bank” The prototype is in the form of a Macro-Enabled Excel File (.xlsm) and it was designed using Visual Basics in Excel 2010. The Prototype uses data from an Exposure Limits report of a bank and the deliverables are a fitted smooth yield curve that can be used as a reference in building Fund Transfer Pricing System. The prototype should therefore be run on machines with Office 2010 or later versions for maximum performance and Macros must be enabled for the prototype to function.

References


**Appendix**

**A1: Correlations**

<table>
<thead>
<tr>
<th>Correlation Analysis-Nelson Siegel</th>
<th>( B_0 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_0 )</td>
<td>1</td>
<td>-0.69996</td>
<td>-0.78795</td>
<td>-0.01891</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>-0.69996</td>
<td>1</td>
<td>0.12006</td>
<td>-0.61416</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>-0.78795</td>
<td>0.12006</td>
<td>1</td>
<td>0.56956</td>
</tr>
<tr>
<td>( r^2 )</td>
<td>-0.01891</td>
<td>-0.61416</td>
<td>0.56956</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation Analysis-Nelson Siegel Svensson Model</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>1</td>
<td>-0.99921</td>
<td>-0.98388</td>
<td>-0.99886</td>
<td>-0.26194</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.99921</td>
<td>1</td>
<td>0.976083</td>
<td>0.99992</td>
<td>0.227363</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-0.98388</td>
<td>0.976083</td>
<td>1</td>
<td>0.974269</td>
<td>0.411925</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.99886</td>
<td>0.99992</td>
<td>0.974269</td>
<td>1</td>
<td>0.220185</td>
</tr>
<tr>
<td>( r^2 )</td>
<td>-0.26194</td>
<td>0.227363</td>
<td>0.411925</td>
<td>0.220185</td>
<td>1</td>
</tr>
</tbody>
</table>
### A2: Results for Nelson-Siegel Model parameters

#### i.) Estimation statistics for $\beta_0$

**Dependent Variable:** NS_B0  
**Method:** Least Squares  
**Date:** 04/23/14  **Time:** 19:27  
**Sample (adjusted):** 3/05/2012 10/31/2012  
**Included observations:** 161 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.003283</td>
<td>0.001559</td>
<td>2.105897</td>
<td>0.0368</td>
</tr>
<tr>
<td>NS_B0(-1)</td>
<td>0.965910</td>
<td>0.016476</td>
<td>58.62535</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

- **R-squared:** 0.955783  
- **Mean dependent var:** 0.094005  
- **S.D. dependent var:** 0.011258  
- **Akaike info criterion:** -9.235427  
- **Schwarz criterion:** -9.197149  
- **Log likelihood:** 745.4519  
- **Hannan-Quinn criter.** -9.219884  
- **Durbin-Watson stat:** 1.865832  
- **Prob(F-statistic):** 0.000000

#### Autocorrelation and Partial Correlation

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.977</td>
<td>1</td>
<td>0.979</td>
<td>249.39</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.959</td>
<td>2</td>
<td>-0.003</td>
<td>489.48</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.939</td>
<td>3</td>
<td>-0.015</td>
<td>720.31</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.918</td>
<td>4</td>
<td>-0.023</td>
<td>941.78</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.895</td>
<td>5</td>
<td>0.039</td>
<td>1155.1</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.883</td>
<td>6</td>
<td>0.082</td>
<td>1362.0</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.868</td>
<td>7</td>
<td>-0.005</td>
<td>1562.7</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson

ii.) Estimation statistics for $\beta_1$

Dependent Variable: NS_B1
Method: Least Squares
Date: 04/23/14   Time: 21:02
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

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<th>Prob.</th>
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R-squared           0.984831  Mean dependent var  -0.039740
Adjusted R-squared     0.984736  S.D. dependent var  0.022141
S.E. of regression     0.002736  Akaike info criterion -8.952638
Sum squared resid      0.001190  Schwarz criterion    -8.914360
Log likelihood        722.6874  Hannan-Quinn criter.  -8.937096
F-statistic           10322.89  Durbin-Watson stat    1.680162
Prob(F-statistic)     0.000000

Autocorrelation Partial Correlation AC PAC Q-Stat Prob
1 0.993 0.993 256.39 0.000
2 0.984 -0.113 509.39 0.000
3 0.977 0.063 759.40 0.000
4 0.988 -0.087 1003.1 0.000
5 0.961 0.071 1249.9 0.000
6 0.954 0.045 1491.3 0.000

Date: 04/23/14   Time: 19:58
Sample: 3/01/2012 3/26/2013
Included observations: 257
### Estimation statistics for $\beta_2$

Dependent Variable: NS_B2  
Method: Least Squares  
Date: 04/23/14   Time: 20:16  
Sample (adjusted): 3/05/2012 10/31/2012  
Included observations: 161 after adjustments

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<td>Mean dependent var</td>
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<td>Adjusted R-squared</td>
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<td>S.D. dependent var</td>
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<td>S.E. of regression</td>
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<td>Akaike info criterion</td>
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<td>Sum squared resid</td>
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<td>Schwarz criterion</td>
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<td>Log likelihood</td>
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<td>Hannan-Quinn criter.</td>
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<td>F-statistic</td>
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<td>Durbin-Watson stat</td>
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### Autocorrelation and Partial Correlation

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<th>PAC</th>
<th>Q-Stat</th>
<th>Prob.</th>
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</thead>
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<td>2 0.902 -0.028</td>
<td>3 0.858 0.036</td>
<td>4 0.814 -0.036</td>
<td>5 0.775 0.034</td>
<td>6 0.746 0.083</td>
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A3: Results for Nelson-Siegel-Svensson Model parameters

i.) Estimation statistics for $\beta_0$

Dependent Variable: B0
Method: Least Squares
Date: 04/23/14   Time: 15:39
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

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<th>t-Statistic</th>
<th>Prob.</th>
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<td>0.020042</td>
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</tbody>
</table>

R-squared    0.932073   Mean dependent var 0.006490
Adjusted R-squared 0.931646   S.D. dependent var 0.378895
S.E. of regression 0.099061   Akaike info criterion -1.773826
Sum squared resid 1.560266   Schwarz criterion -1.735548
Log likelihood 144.7930   Hannan-Quinn criter. -1.758283
F-statistic 2181.758   Durbin-Watson stat 1.799652
Prob(F-statistic) 0.000000

Date: 04/23/14   Time: 18:55
Sample: 3/01/2012 3/26/2013
Included observations: 257

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### ii.) Estimation statistics for $\beta_1$

Dependent Variable: B1  
Method: Least Squares  
Date: 04/23/14  Time: 18:47  
Sample (adjusted): 3/05/2012 10/31/2012  
Included observations: 161 after adjustments

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R-squared  0.934656  Mean dependent var  0.053246  
Adjusted R-squared  0.934245  S.D. dependent var  0.374162  
S.E. of regression  0.095945  Akaike info criterion  -1.837731  
Sum squared resid  1.463677  Schwarz criterion  -1.799452  
Log likelihood  149.9373  Hannan-Quinn criter.  -1.822188  
F-statistic  2274.273  Durbin-Watson stat  1.799141  
Prob(F-statistic)  0.000000

---

Date: 04/23/14  Time: 19:08  
Sample: 3/01/2012 3/26/2013  
Included observations: 257

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iii.) Estimation statistics for $\beta_2$
Dependent Variable: B2
Method: Least Squares
Date: 04/23/14   Time: 19:00
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

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<td>0.020321</td>
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R-squared: 0.929997
Adjusted R-squared: 0.929556
S.E. of regression: 0.087446
Sum squared resid: 1.215841
Log likelihood: 164.8714
F-statistic: 2112.319
Prob(F-statistic): 0.000000

Date: 04/23/14   Time: 19:08
Sample: 3/01/2012 3/26/2013
Included observations: 257

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<td>235.66</td>
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<tr>
<td>3 0.801 0.045</td>
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<td>1272.1 0.000</td>
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Estimation statistics for $\beta_3$

Dependent Variable: B3
Method: Least Squares
Date: 04/23/14   Time: 19:11
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

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R-squared   0.932560  Mean dependent var 0.105718
Adjusted R-squared   0.932135  S.D. dependent var 1.817027
S.E. of regression  0.473351  Akaike info criterion 1.354384
Sum squared resid   35.62566  Schwarz criterion 1.392662
Log likelihood    -107.0279  Hannan-Quinn criter. 1.369926
F-statistic        2198.636  Durbin-Watson stat 1.797856
Prob(F-statistic)  0.0000000

Date: 04/23/14   Time: 19:19
Sample: 3/01/2012 3/26/2013
Included observations: 257

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