The Ratio Model and its Application: A Revisit

Zhibai Zhang¹ and Xinyue Zou²

Abstract

In this paper, the ratio model, a simple currency valuation model proposed by Zhang (2012, International Research Journal of Finance and Economics, issue 97, pp. 55–59), is revisited. We use both the ratio and purchasing power parity (PPP) models to value the bilateral real exchange rates (RERs) of five Asian industrial countries and areas, namely, Japan, Korea, Taiwan, Hong Kong, and Singapore, against the United States. In the early 1950s to 2009, the RER misalignments of four new industrial countries and areas from the ratio model converged, but those from the PPP model did not, implying the competitiveness of the ratio model against the PPP model both in currency valuation and as an RER anchor. Based on the two models, from 2010 to 2011, the yen was shown to be overvalued by approximately 30%, whereas the Singapore dollar was undervalued by approximately 20%. However, the conclusions on the other three RERs were not consistent.

JEL classification numbers: F31, F41
Keywords: Equilibrium exchange rate, Absolute purchasing power parity, Penn effect, Ratio model

1 Introduction

Currency valuation, or the calculation of the equilibrium exchange rate of a specific currency, has been a popular topic in international finance since the 1910s because of the important role of exchange rate both in domestic economy and in global trade system. According to Isard (2007, p. 3), “Assessing the equilibrium levels of exchange rates is an important responsibility of macroeconomic policymakers. Exchange rates have a major influence on the prices faced by consumers and producers throughout the world, and the consequences of substantial misalignments can be extremely costly. The currency crises experienced by a number of emerging-market economies over the past decade testify to

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the large output contractions and extensive economic hardship that can be suffered when exchange rates become badly misaligned and subsequently change abruptly. Moreover, there is reasonably strong evidence that the alignment of exchange rates has a critical influence on the rate of growth of per capita output in low income countries.” Thus, many currency valuation models have been developed. These models are mainly classified as absolute or relative purchasing power parity (PPP) (Cassel, 1916a, 1916b; Yang and Bajeux-Besnainou, 2006; Isard, 2007), the Penn effect or Balassa–Samuelson (BS) regression or extended PPP (EPPP) (Frankel, 2006; Isard, 2007; Garroway et al., 2012) that will be defined and described in Section 2, the behavioral equilibrium exchange rate (BEER) (Clark and MacDonald, 1998; Wang et al., 2007; Lopez-Villavicencio et al., 2012), and the macroeconomic balance or fundamental equilibrium exchange rate model (Clark and MacDonald, 1998; Barisone et al., 2006; Lopez-Villavicencio et al., 2012).3 In a recently released short paper, Zhang (2012) proposes the ratio model, a new and simple currency valuation model based on the Penn effect. In his paper, the ratio model is used to value 11 main bilateral real exchange rates (RERs) against the US dollar from 1980 to 2010. In the present paper, the model is revisited from different perspectives. First, the ratio model is now compared to the PPP model, which has great theoretical and policy significances but was not done by Zhang (2012). Second, the concrete RERs examined in this paper are different from those analyzed by Zhang (2012). In this paper, we use the ratio model to value the bilateral RERs of five Asian industrial countries and areas, namely, Japan, Korea, Taiwan, Hong Kong, and Singapore, against the United States. All the five Asian economies experienced rapid economic growth stages, for example, Japan in 1950–1990 and the other four countries and areas from 1960s to the current times (except for some subsequent years after 1997). Therefore, they are good examples to illustrate and investigate the Penn effect, the basis of the ratio model. Third, the period examined (1950 to 2009) is considerably longer than that in the work of Zhang (2012), such that in the current paper, the misalignment changes from the ratio model in a long period can be investigated. Fourth, the misalignments of the five RERs in the latest years (2010–2011) are examined based on the ratio and PPP models. In Section 2, the ratio model is presented after a brief review on the relevant models. Section 3 provides the applications of the ratio and PPP models in the valuation of the five RERs from 1950 to 2009. Section 4 analyzes the findings from the ratio model. In Section 5, the ratio model is compared with the PPP model and the misalignment differences between the two models are presented. Section 6 gives the five RER misalignments not examined in Section 3 in the period of 2010–2011. Finally, Section 7 concludes.

3Additional relevant references regarding the PPP models are supplied by Ohno (1990), Chinn (2000), and Sidek et al. (2011). References on the Penn effect models are given by Takeuchi (2003), Chang and Shao (2004), and Cheung et al. (2010). References on the BEER models are given by Imam and Minoiu (2011), Sidek et al. (2011), and Alper and Civcir (2012). References on the macroeconomic balance models are given by Isard (2007), Imam and Minoiu (2011), and Sato et al. (2012).
2 PPP, Penn Effect, and Ratio Models

The ratio model is based on the Penn effect, and both are extensions of the PPP. Thus, the PPP and the Penn effect are discussed first. This section is not only obtained from the work of Zhang (2012), which is partly repeated in this paper for the sake of integrity, but also extends his work.

2.1 PPP Model, Penn Effect Model, and Their Faults

In its absolute version, the PPP is a basic model used in assessing a bilateral nominal exchange rate (NER). PPP uses Eq. (1), where \( P \) is the domestic price level of a country, \( P^* \) is the price level of a foreign country (in this paper, the United States), and \( NER \) is expressed as the national currency units per US dollar. In the PPP model, if the value of \( RER \) is equal to one, NER will be equal to its PPP rate and will be in equilibrium; otherwise, NER will be over- or undervalued. Alternatively, the PPP model can be viewed as a model used to determine the value of an RER. When the RER value is one, it is in equilibrium; otherwise, RER is over- or undervalued. The misalignment of an RER is obtained by "\( RER - 1 \)".

\[
RER_i = \frac{PPP_i}{NER_i} = \frac{P_i/P^*}{NER_i} = \frac{P_i}{NER_i \times P^*}
\]

According to Balassa (1964), PPP is not a proper model to value NER or RER in some cases. As Balassa (1964, p.596) states, “By incorporating non-traded goods in the model, the existence of a systematic relationship between purchasing power parities and (nominal) exchange rates is indicated in intercountry as well as in intertemporal comparisons.” Based on Balassa’s statement, the RERs (defined in Eq. (1)) of high-income countries become higher and those of low-income countries become lower. The “systematic relationship” called in Balassa (1964) is now a well-known empirical regularity (Rogoff, 1996; Takeuchi, 2003; Chang and Shao, 2004; Frankel, 2006; Isard, 2007; Cheung et al., 2010; Garroway et al., 2012), as illustrated in Fig. 1. This regularity is called the BS effect (Chang and Shao, 2004; Frankel, 2006), “Penn effect” (Samuelson, 1994; Isard, 2007; Cheung et al., 2010), “(long-run) deviations from PPP” (Rogoff, 1996), or others. Given that the Penn effect essentially refers to this empirical regularity (Samuelson, 1994) and that the BS effect is only one of its explanations (Rogoff, 1996), with the BS effect being an invalid explanation in some cases (Ito et al., 1997; Isard, 2007), the Penn effect is perhaps the more suitable name. Therefore, in this paper, we call this regularity the Penn effect.

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\(^4\)The \( RER \) (defined in Eq. (1)) also measures the relative price level between two countries in terms of a common currency. Thus, it is also called “the price level of the GDP of one country relative to that of the US” in the Penn World Tables (PWT) database.
Figure 1: RERs and income levels of 116 countries and areas in 2009
Source: Alan Heston, Robert Summers and Bettina Aten, Penn World Table (PWT) Version 7.0, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, June 2011.
Notes: RER (defined by Eq. (1)) and GDP per capita (PPP converted) are both normalized, with US = 100.

Given the invalidity of the PPP model between the United States and a poor country because of the existence of the Penn effect, in the early 2000s, some economists (Takeuchi, 2003; Chang and Shao, 2004; Frankel, 2006) used the Penn effect model, Eq. (2) or its log-linear form, to assess the value of the RER of the RMB. In Eq. (2), $RER$ is defined by Eq. (1), $GDPP$ (GDP per capita) represents the income level or the economic development stage, and the subscript $i$ denotes the cross-section data dimension. As Eq. (2) regresses the RERs on the income levels, the deviations from the regression line represent the over- or undervaluation of the RERs when the Penn effect is considered. Some recent application examples of this model are given by Isard (2007), Rodrik (2008), Cheung et al. (2010), and Garroway et al. (2012).

$$RER_i = \beta_0 + \beta_1 GDPP_i + u_i$$ (2)

However, the Penn effect model has a fault in that different econometric specifications in the model always give different misalignment results (Dunaway et al., 2009; Cheung et al., 2010). The different econometric specifications of the Penn effect model include the following: different function forms (double log or ordinary linear equation), different econometric methods (cross-section data or panel data), different proxies for GDPP (PPP converted or current priced), different databases (for a chosen variable proxy, use the PWT, WDI, or WEO to obtain data), and different observed values (sample point and time period). A different choice of econometric specifications always leads to different misalignment results. We use the same database to simulate the result of Frankel’s research (2006) and find the following. (1) The double-log function and the 118 sample...
countries give a RMB undervaluation of 36% in 2000, which is the same as the valuation of Frankel (2006). (2) However, when the number of the sample countries is reduced to 60 and the double-log is left unchanged, the RMB undervaluation changes to 22.4% in the same year. (3) When the double-log changes into an ordinary linear function and the 118 sample countries are left unchanged, the RMB undervaluation changes to 33.3%.

Aside from the Penn effect model, the fault also appears in all other models using an econometric method such as the BEER model and the macroeconomic balance model (see Dunaway et al. (2009) and Zhang (2010) for details).

### 2.2 Ratio Model

The new model called the ratio model is also based on the Penn effect which is used by the Penn effect model (although there is a difference in the two models in which the ratio model uses a simple algebraic calculation while the Penn effect model uses a regression estimate). Therefore, the model described in Eq. (2) can be called the “regression” Penn effect model, and the model presented in this paper and described in Eq. (3) can be called the “ratio” Penn effect model. We call the “ratio” Penn effect model the “ratio model” to simplify and differentiate the two terms. Using the classification given by Dunaway et al. (2009), the ratio and Penn effect models belong to the EPPP, as they are both developed under the line of PPP.

\[
\text{Ratio}_i = \frac{\text{RER}_i}{\text{GDPP}_i} \quad \text{misalignment} = \text{Ratio}_i - 1
\]

In Eq. (3), \textit{RER}_i is defined by Eq. (1). \textit{GDPP}_i is country \textit{i}’s per capita GDP relative to that of the United States, and \textit{Ratio}_i is the ratio of \textit{RER} and \textit{GDPP}. Therefore, the index \textit{Ratio}_i measures the difference in country \textit{i}’s RER (relative to the United States) and its GDPP (relative to the United States). The misalignment is equal to the value of “\textit{Ratio}_i - 1.”

According to the definition of RER, if the value of \textit{Ratio}_i is one or if the misalignment is equal to zero, the RER will be equal to the GDPP (both relative to the United States) and will be in equilibrium. If the value of \textit{Ratio}_i is more than one or if the misalignment is greater than zero, the RER will be greater than the GDPP and will be concluded as overvalued. Likewise, if the value of \textit{Ratio}_i is less than one or if the misalignment is less than zero, the RER will be undervalued.\(^5\)

According to the PWT 7.0, in 2009, the RER and GDPP of Japan (both relative to the United States, where US = 100) are 122.1 and 76.8, respectively. Thus, the \textit{Ratio}_i is 1.59 (= 122.2/76.8) and the misalignment is 59%, making the yen overvalued. The ratio model can be derived from the Penn effect model, which is not considered by Zhang (2012). When the coefficients \(\beta_0\) and \(\beta_1\) in Eq. (2) are specified as zero and one, simultaneously and respectively, the Penn effect model will be reduced as the ratio model.

We can imagine that, in Fig. 1, the fitted regression line from the Penn effect model is denoted by the bottom left line that does not cross the original point; however, the fitted line from the ratio model is constrained as a 45° (diagonal) line at the bottom left that must cross the original point.

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\(^5\)In the ratio model, each currency’s RER should remain at the same level of the country’s GDPP (both relative to the US). A poor country’s RER should remain as low as its GDPP, and a rich country’s RER should remain as high as its GDPP. Otherwise, the RER will be misaligned.
The uncertainty of econometric specification that the Penn effect model and all other econometric valuation models (e.g., the BEER) have is greatly reduced in the ratio model because the ratio model uses a simple algebraic calculation, not an econometric analysis. Compared with the Penn effect and BEER models, the ratio model is free from uncertainties in function form, econometric method, and observation. However, the uncertainties in variable proxy and database still remain. The Penn effect and BEER models require special software to conduct the econometric analysis. However, for the ratio model, hand computation is enough, making it easier to use. These two advantages of the ratio model are similar to those of the PPP model.

The following are brief discussions on the three closely relevant models, namely, PPP, Penn effect, and ratio models:

1. The PPP model is based on the PPP theory that holds only between a rich (similar income level) country and the United States (the specified foreign country), while the Penn effect and ratio models based on the Penn effect hold between an arbitrary country and the United States. Thus, the Penn effect and ratio models extend the application range of the PPP model.

2. The PPP model estimates the RER value of a country from the price level and does not consider the income level difference of that country with that of the United States. However, the Penn effect and ratio models estimate the RER value based on the income level difference between the country and the United States.

3. The equilibrium criteria are different among the three models. In the PPP model, whether an RER is in equilibrium is decided by whether the price level of the country is equal to that of the United States (measured in the common US dollar). In the Penn effect model, whether an RER is in equilibrium is decided by whether the RER is equal to the average RER of all the countries with same income levels given by a regression. In the ratio model, whether an RER is in equilibrium is decided by whether the RER is equal to the GDPP (both relative to the United States, where US = 100).

3 Application of the Ratio Model

In this section, the ratio model is used to valuate the bilateral RERs of the five Asian industrial countries and areas against the United States. The databases of the World Economic Outlook (WEO) of the International Monetary Fund (IMF) and the World Development Indicators (WDI) of the World Bank can supply more updated data. However, the data of WEO or WDI are only available after the year 1980, which is not long enough to illustrate the Penn effect because the five Asian economies took off in the 1950s and 1960s. Thus, the PWT 7.0 database is used to supply relevant data from the early 1950s to 2008 or 2009, as shown in Sections 3, 4, and 5. The RER (the variable p in the database) and the GDPP (PPP converted; the variable y in the database) can be directly obtained. In the PWT 7.0, the RER and GDPP are (already) both normalized, with US = 100 in each year. However, the periods for the different countries and areas obtained in the database are different. For Japan, Korea, and Taiwan, the period is from early 1950s to 2009. For Hong Kong and Singapore, the period is from 1960 to 2009 and

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6The updated data in 2010 to 2011 from the WEO will be used in Section 6.
from 1960 to 2008, respectively. The RER misalignment of each currency or area against the US dollar from the ratio model is obtained by using Eq. (3) (misalignment = \( \text{Ratio} - 1 \)). The RER misalignment from the PPP model is obtained by using the method introduced in Section 2.1 (misalignment = \( \text{RER} - 1 \)). The misalignments are presented in Fig. 2.

The X_MIS_RATIO and X_MIS_PPP represent the RER misalignments of country (or area) X from the ratio and PPP models, respectively (Fig. 2). A positive number indicates overvaluation, whereas a negative number indicates undervaluation. For example, in 1955, the TW_MIS_RATIO and TW_MIS_PPP values are 7.017 and −0.187, respectively, which indicates that, against the US dollar and in the same year, the new Taiwan dollar is overvalued by 701.7% on the basis of the ratio model, but undervalued by 18.7% on the basis of the PPP model.

![Figure 2: Five RER misalignments on the basis of the two models](image)

Source: Penn World Table (PWT) 7.0 and the authors’ calculations.
Notes: X_MIS_RATIO and X_MIS_PPP represent the misalignments from the ratio and the PPP models of country (or area) X, respectively. A positive (negative) number indicates overvaluation (undervaluation).

The misalignments vary greatly from more than 800% to −40% (Fig. 2). To distinguish...
these misalignments clearly, we classify each RER misalignment into five intervals, namely, highly overvalued, overvalued, near equilibrium, undervalued, and highly undervalued, which are denoted by Mis > 50%, $10\% \leq \text{Mis} \leq 50\%$, $-10\% < \text{Mis} < 10\%$, $-50\% \leq \text{Mis} \leq -10\%$, and Mis < $-50\%$, respectively. These interval classifications are listed in Table 1.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Taiwan</th>
<th>Korea</th>
<th>Hong Kong</th>
<th>Singapore</th>
<th>Japan</th>
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<tbody>
<tr>
<td></td>
<td>2004–2005</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>2006–2009</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Highly undervalued (Mis &lt; −50%)</td>
<td></td>
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</tbody>
</table>

Notes: For Japan, 1986–1992* means 1986–1992 except 1988, when the RER was highly overvalued (by 60.2%); 1993–2000* means 1993–2000 except 1998, when the RER was overvalued (by 46.7%); 2001–2007* means 2001–2007 except 2004, when the RER was highly overvalued (by 53.9%).

4 Analysis of Findings from the Ratio Model

The findings of the ratio model have been depicted in Fig. 2. The figure demonstrates that the RER misalignments of the four new industrial Asian countries and areas share a common characteristic, whereas the Japanese yen displays a different representation from the other four currencies. Thus, the RER misalignments of the four new industrial Asian countries and areas and those of Japan are discussed separately.

4.1 RER Misalignments of the Four New Industrial Asian Countries and Areas

In the RER misalignments of the four new industrial Asian countries and areas (Fig. 2), a common, apparent observation can be seen. Each RER is highly overvalued in the early period. Thereafter, a downward trend appears from the early period to 1985. Finally, the RER misalignment enters the narrow ±50% range (around equilibrium) from 1986 to 2009.

First, all the RERs of the four new industrial Asian countries and areas are highly overvalued in their early development stages. For example, the misalignments of the new Taiwan dollar are 830.4% in 1951 and 971.5% in 1952, and the misalignments of the Korean won are 407.3% in 1953 and 544.1% in 1954. By using the misalignment classification in Table 1, the new Taiwan dollar, Hong Kong dollar, and Singapore dollar
are highly overvalued in 1951 to 1984, 1960 to 1980, and 1960 to 1975, respectively. These three currencies are recorded from the first year to approximately early 1980s. The Korean won is highly overvalued from 1953 to 1996 (in a longer period). The high overvaluation of the four RERs is mainly caused by the high price levels relative to the low incomes of the countries and areas. For example, according to the PWT 7.0, in which the RER and GDPP are both normalized and the United States is equal to 100, from 1951 to 1955, the RER of Taiwan against the United States is greater than 75; however, the GDPP of Taiwan is less than 10. These two factors lead to a misalignment of more than 600%. Korea also has similar information. The RER of Korea is more than thrice its GDPP from 1953 to 1955, thus resulting in a misalignment of more than 200%. Besides the high price levels relative to the low incomes, the other reason that causes the unusually large misalignments such as more than 600% is the structure of the ratio model (Eq. 3). In the ratio model, the misalignment is mainly obtained from the quotient of the RER and GDPP of a country from which a result of more than 100% can be easily obtained when the difference between RER and GDPP is large. In other words, the ratio model lacks a mechanism that can make its misalignment smaller in whichever case. By contrast, the misalignment of a Peen effect model is obtained from the residual of a regression equation and the regression theory makes the residual value smaller. Therefore, its misalignment is always very small and approximately near zero. Similarly, in a time-series BEER model, the misalignment is obtained from the residual of a co-integration regression and is also minimal. For example, the RMB misalignment is 5% to 45% in 1980–2003 by using the Penn effect model by Chang and Shao (2004) and is no more than 5% in 1980–2004 by using the BEER model by Wang et al. (2007).

Second, all misalignments (in each RER from the 1950s or 1960s to mid-1980s) exhibit a downward trend before 1985 (Fig. 2). The misalignment of the new Taiwan dollar decreases from approximately 700% in the early 1950s to less than 50% in 1985. The misalignment of the Korean won declines from more than 400% in 1953–1954 to 85% in 1985. The misalignment of the Hong Kong dollar declines from about 250% in 1960–1961 to near zero in 1984–1985. The misalignment of the Singapore dollar declines from more than 110% in 1964–1965 to near zero in 1982–1985.

Why do all four RER misalignments exhibit a downward trend before 1985? Table 2 shows that the annual growth rates of RERs in Taiwan from 1953 to 1985, in Korea from 1954 to 1985, and in Hong Kong from 1961 to 1983 are −1.1%, −1.1%, and −0.5%, respectively. However, the corresponding annual growth rates of GDPPs in the same periods are 4.2%, 2.9%, and 5.1%, respectively. The decreasing RER (the numerator of Ratio in Eq. (3)) and the fast-increasing GDPP (the denominator of Ratio in Eq. (3)) make the value of Ratio fast decline, leading to the downward trend of the misalignment in each period. The RER in Singapore appreciates in its period unlike that in the aforementioned three countries and areas. However, the annual growth rate of RER appreciation (1.0%) is much less than that of the GDPP increase (5.6%). As a result, the misalignment in Singapore also declines during its period.
Table 2: RER and GDPP changes of the four new Asian industrial countries and areas in (or in part of) 1953–1985

<table>
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<tr>
<th></th>
<th>Taiwan</th>
<th>Korea</th>
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<tr>
<td></td>
<td>RER</td>
<td>GDPP</td>
</tr>
<tr>
<td>1953</td>
<td>76.1</td>
<td>9.4</td>
</tr>
<tr>
<td>1985</td>
<td>53</td>
<td>35.6</td>
</tr>
<tr>
<td>AGR</td>
<td>-1.1%</td>
<td>4.2%</td>
</tr>
<tr>
<td></td>
<td>Hong Kong</td>
<td>Singapore</td>
</tr>
<tr>
<td></td>
<td>RER</td>
<td>GDPP</td>
</tr>
<tr>
<td>1961</td>
<td>70.1</td>
<td>20</td>
</tr>
<tr>
<td>1983</td>
<td>62.6</td>
<td>59.4</td>
</tr>
<tr>
<td>AGR</td>
<td>-0.5%</td>
<td>5.1%</td>
</tr>
</tbody>
</table>

Source: PWT 7.0 and the authors’ calculation.
Notes: AGR represents the annual growth rate of a variable in its period. AGR is obtained from \( a \cdot (1+AGR)^n = b \), where \( a \) and \( b \) are values in the first and last years, respectively, and \( n \) is the number of years examined.

Third, after the mid-1980s, most misalignments of the four new Asian industrial countries and areas enter the ±50% range. Therefore, after twenty and thirty years of fast development and economic reform, these RERs tend to fluctuate around their equilibrium levels.

Finally, in economic policy, the evolution of the four RER misalignments confirms the following consensus: a country’s currency in its early development stage is often overvalued to exchange more foreign currency and buy more foreign goods because of the severe shortage of foreign exchange reserves (Xu, 2000, p.265). Shatz and Tarr (2000, p.2) state the following: “Experience shows that protection to defend an overvalued exchange rate will significantly retard the medium to long run growth prospects of the country and delay integration into the world trading community. In fact, an overvalued exchange rate is often the root cause of protection, and the country will be unable to return to the more liberal trade policies that allow growth and integration into the world trading community without exchange rate adjustment”. Based on the above arguments (Xu, 2000; Shatz and Tarr, 2000), the integration of the four countries and areas into the world economy causes the shortage of foreign exchange reserve to be replaced by trade liberation. Thereafter, the intentional, highly overvalued currency depreciates gradually to near equilibrium.

4.2 Misalignment of the Japanese Yen

Fig. 2 shows that the misalignment of the Japanese yen presents a different situation compared with the misalignment of the other four countries and areas. (1) All yen

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7 The evolution of the four RER misalignments in their whole periods also confirms the similar convergence phenomenon in 11 RER misalignments in 1980–2010 in Zhang (2012).
8 Shatz and Tarr (2000, p.13) also think that the Korea won is overvalued before 1960 and in the second half of the 1970s. This assumption partly confirms our result that the Korea won is highly overvalued (and that the degree of its overvaluation decreases) before 1985.
misalignments during its entire period (from 1950 to 2009) fluctuate in the range of –25% to 90%, and no unusual value more than 100% appears. (2) The yen misalignment mainly fluctuates around the zero line before 1985, and a downward trend doesn’t appear. Let us analyze the reason for this situation in the succeeding paragraphs.

Fig. 3 shows that the yen RER always fluctuates around and near the GDPP before 1985, giving the misalignment a narrow range of –25% to 50%. Moreover, it prevents the downward trend, which has emerged in the other four countries and areas, from appearing. The Plaza Accord takes effect in 1985, and the yen is pressured to appreciate. The RER increases from 81.0 to 173.3 (more than two times) from 1985 to 1995, but the GDPP only increases from 82.2 to 91.4. These increases make the misalignment in this period obviously greater than that before 1985, often overpassing 50%. The difference between the RER and GDPP is maximized in 1995, and the yen is overvalued by 89.6%, reaching the greatest misalignment in the whole period examined. Both the RER and GDPP decline after 1996, but the RER remains a bit greater than the GDPP, leading to the misalignment from 1996 to 2009 to be 30% to 75%.

Why does the yen RER misalignment have a different behavior from the misalignment of the other four? Three main reasons are possible. (1) The yen RER is too high. The RERs are all greater than 100 starting in 1986, with most of them more than 120, indicating that the price level of Japan is more than 120% of that in the United States. The continuous exchange rate appreciation pressure from the United States after the Plaza Accord may be a main factor. However, all other RERs of the four new industrial countries and areas are below 90 except those in Hong Kong from 1993 to 2001 and Singapore in 1996. (2) Japan developed very slowly or even stagnated in many years after 1986, as is well known. Fig. 3 shows that the GDPP decreases rather than increases from 1991 to 2009. By contrast, the GDPPs of the four new industrial countries and areas clearly increase from the mid-1980s to 2009. (3) Japan became an industrial country earlier than the other four countries and areas, and its economy has some special characters. For example, Japan’s GDPP reached 50 in 1964. The GDPPs of the other four countries and areas were less than 30 at that time. Hong Kong and Singapore reached a GDPP of 50 in about 1980, whereas Taiwan and Korea reached a GDPP of 50 in the 1990s. The frequent change in government could have had a negative effect on the economy.
4.3 More on Convergence

To better understand the changes in the five RER misalignments, we investigate in this section the nonlinearity in the convergence speed by using a time period classification method similar to that of Rodrik (2008), who analyzes nonlinearity in the relationship between undervaluation and economic growth. The time period classification method used in this paper is different from, but has the similar idea with, the panel nonlinear threshold method used by Bayoumi et al. (2005), who analyze the nonlinearity in the convergence speed for the exchange rates of 10 advanced economies.

According to previous sections (i.e., Sections 4.1 and 4.2), all five RER misalignments present different images before 1985 and after 1986. Thus, we divide each whole period into two sub-periods (i.e., before 1985 and after 1986).

The convergence is examined though the value of $\beta_1$ in Eq. (4), where Mis_ratio is the misalignment from the ratio model of the country or area X, GDPP is the country’s or area’s GDP per capita, and $t$ is the time-series dimension. The values of the two variables have been obtained in Section 3.

$$\text{Mis}_\text{ratio}_t = \beta_0 + \beta_1 \text{GDPP}_t + u_t$$ (4)

First, ADF and KPSS unit roots are used to determine whether the variables in different periods are I(0) or I(1). The results reveal that all Mis_ratios in all periods are I(0), and GDPPs are I(0) in some periods and I(1) in other periods. For the periods in which Mis_ratios are I(0) but GDPPs are I(1), Johansen cointegration tests indicate the existence of cointegration relationships. In theory, a cointegration relationship should not exist between an I(0) variable and an I(1) variable; this phenomenon occurs possibly because the unit root test lacks power. We solve this contradiction by treating all the variables in all periods as I(0). Thus, ordinary least square (OLS) estimations can be used in all cases in Table 3. The obtained values for $\beta_1$ in Eq. (4) in all periods (i.e., two sub-periods and the whole period) for each RER are listed in Table 3.

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<tbody>
<tr>
<td>Before 1985</td>
<td>-0.194***</td>
<td>-0.115***</td>
<td>-0.042***</td>
<td>-0.026**</td>
<td>-0.005*</td>
</tr>
<tr>
<td>Whole period</td>
<td>-0.075***</td>
<td>-0.056***</td>
<td>-0.025***</td>
<td>-0.017***</td>
<td>0.002</td>
</tr>
<tr>
<td>After 1986</td>
<td>-0.032***</td>
<td>-0.019***</td>
<td>-0.002</td>
<td>-0.007***</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Source: PWT 7.0 and the authors’ calculation.
Notes: “Before 1985” indicates 1985 (e.g., 1951–1985 for Taiwan), “After 1986” indicates 1986 (e.g., 1986–2009 for Taiwan), and “Whole period” indicates the two sub-periods “Before 1985” and “After 1986” (e.g., 1951–2009 for Taiwan). The superscripts *** and ** indicate that the values are significant at the 1% and 5% level, respectively. In the estimations, Newey–West robust standard errors are used.

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9 We also use a threshold autoregressive model and an ordinary autoregressive model to analyze the nonlinearity; see Appendix.

10 Considering the cointegration relationships existing in some cases, we also use fully modified OLS (FMOLS), but the estimated coefficients by FMOLS are very similar to those by OLS and the conclusions are the same, so the results obtained from FMOLS are omitted.
In Table 3, all the significant coefficients (at 0.01 or 0.05 level) are negative, indicating that the misalignments decrease as the GDPP increases. This result is expected when we trace back the changes in the misalignments and GDPPs of the four new industrial countries and areas before 1985 (Table 2). In Table 3, a regular phenomenon can be obtained except for cases in which the coefficients are not significant. First, the coefficients (in terms of the absolute values, as below) before 1985 are all greater than those after 1986 in each country or area, such as 0.194 before 1985 and 0.032 after 1986 in Taiwan. Second, compared with the coefficients in the whole period that can be viewed as average values, all coefficients before 1985 are great (e.g., 0.194 before 1985 is greater than 0.075 in the whole period in Taiwan), and all coefficients after 1986 are small (e.g., 0.032 after 1986 is smaller than 0.075 in the whole period in Taiwan). Thus, the greater the misalignment is, the faster the misalignment converges as the GDPP increases. In another word, the nonlinearity in the speed of misalignment convergence holds. Third, aside from the time-series dimension in each country or area, the significant coefficients also decline across countries and areas in the same period. For example, the coefficients before 1985 for the five RERs (on the second line) decline strictly from Taiwan to Japan (i.e., 0.194 in Taiwan, 0.115 in Korea, 0.042 in Hong Kong, 0.026 in Singapore, and 0.005 in Japan). Given that the misalignments also strictly decline from Taiwan to Japan before 1985 (with an average value of 271.5% in Taiwan, 226.0% in Korea, 94.08% in Hong Kong, 59.7% in Singapore, and 11.8% in Japan), the across-section comparison also provides the nonlinearity of the speed of convergence.

5 Combination with the PPP Model

After understanding the finding from the ratio model, we now combine it with the PPP model from which the difference between the two model findings can be analyzed and some inspirations can be obtained.

5.1 Difference between the Findings from the Two Models

The RER misalignments from the PPP model (alongside with those from the ratio model) have been shown in Fig. 2. Based on the PPP model and misalignment classification (Section 3), most of the RER misalignments of the four new industrial countries and areas are undervalued or highly undervalued in their entire periods. For example, the new Taiwan dollar is undervalued or highly undervalued from 1951 to 2009 with the misalignment ranging from −58% to −17% (negative numbers represent undervaluation), and the Korean won is also undervalued or highly undervalued from 1953 to 2009 with a misalignment ranging from −73% to −17%. By contrast, the Japanese yen fluctuates between high overvaluation and high undervaluation.

Thus, the following differences are observed when comparing the misalignment of the five RERs from the ratio model with that from the PPP model:

1. The difference between the findings of the two models is great for the four new industrial countries and areas before 1985. Most of the RERs are undervalued or highly undervalued in the PPP model. Conversely, most of the RERs in the ratio model are highly overvalued. For example, the RER misalignments of Taiwan and Korea in 1961–1973 are less than −50% in the PPP model but are greater than 100% in the ratio
model, giving the difference between the two misalignments in each same year in the period to be greater than 150% (in absolute value). This great difference reflects the ratio model’s adjustment in the PPP model. That is, when the PPP model is used and the income level difference is not considered, an RER of a low-income country against the United States is often considered undervalued. However, when the ratio model is used and the income level difference is considered, the RER may be overvalued.

2. The difference between the two model findings is clearly reduced for the four new industrial countries and areas after 1986. Concretely, the RERs in Taiwan and Hong Kong in 2003–2009 are undervalued in the PPP model. They are also undervalued or near equilibrium in the ratio model. The difference between the two model findings (in absolute value) for Korea is greater than 135% before 1985 but is reduced to approximately 100% in 1986–1993 and less than 80% in 1994–2009. The RER of Singapore in 1997–2008 is undervalued whether in the PPP or the ratio model. The GDPPs of the four countries and areas reach a relatively high level after 1986 compared with those before 1985, with most GDPPs being less than 40 before 1985 and greater than 60 after 1986. The reduced gap of the GDPPs between the four countries and areas and the United States reduces the Penn effect, making the difference between the two model findings smaller after 1986.

The similar phenomenon can also be observed in the five RER misalignments in 2010–2011 (see Table 4 in Section 6). Table 4 shows that the misalignment differences of Japan and Singapore (with their GDPPs near that of the United States) are smaller between the two models and that those of Korea and Taiwan (with GDPPs less than half of that of the United States) are greater between the two models. This finding gives the following impression: the nearer a GDPP is to that of the United States, the smaller the difference between the two model findings. When a GDPP is equal to that of the United States, the denominator of the Ratio in Eq. (3) is 1, and the ratio model is reduced to the PPP model.\(^\text{11}\)

3. The difference between the two model findings in Japan in 1950–1965 is in the range of 50% to 120%, with a mean of 81.7%. The difference between the two model findings in 1966–2009 is less than 50%, with a mean of 24.5%. For example, the misalignments are –60.2% in 1955 and 14.3% in 2005 in the PPP model and 38.3% in 1955 and 48.4% in 2005 in the ratio model. The difference is reduced from 98.5% in 1955 to 34.1% in 2005. Given that the GDPP increases from less than 50 (mostly less than 40) in 1950–1965 to more than 50 (mostly more than 70) in 1966–2009, the reduced difference between the two model findings in 1950–1965 and in 1966–2009 also reflects the ratio model’s adjustment in the PPP model: the nearer a GDPP is to that of the United States, the smaller the difference between the two model findings; although the reduced difference is not as obvious as that in the four new industrial countries and areas.

5.2 Ratio Model for the Completion of the PPP Model

Aside from the differences between the two model findings of demonstrating regularity just discussed above, the ratio model can also complete the PPP model.

\(^{11}\)For example, according to the same WEO database, the forecast GDPP of Singapore in 2014 will be 100.4% of that of the US; thus, undoubtedly, the misalignment in the same year in the ratio model (−19.6%) is also almost equal to that in the PPP model (−19.3%).
5.2.1 Complete the PPP model in measuring the currency misalignment

Just as the Penn effect completes the PPP theory, the ratio model can also complete the PPP model in currency valuation, as discussed to some extent in Section 2.2. The PPP model values an RER from the perspective of two price levels without considering the influence of income level difference between the two countries. Conversely, the ratio model values an RER based on the income level difference. The PPP model only holds well when a GDPP is similar to that of the United States. Conversely, the ratio model, as a corrected version of the PPP model, can be used in any case, especially in cases in which the PPP does not hold well. Thus, by combining the two models, we can obtain a more complete misalignment result.

The following are some examples. As shown in Fig. 2, the PPP model gives a misalignment of –30.8% for the Singapore dollar in 2008, whereas the ratio model gives a misalignment of –40.6%. As the two models provide the same result (the same misalignment classification, i.e. undervaluation), the obtained undervaluation should be more reliable than the one achieved by using only one model. The PPP model gives a misalignment of 26.8% (overvalued) for the yen in the same year, whereas the ratio model gives a misalignment of 65.7% (highly overvalued). Thus, the yen can be viewed as being overvalued when the income level difference is not considered and highly overvalued when it is considered. The PPP model gives a misalignment of –23.1% (undervalued) for the Hong Kong dollar in 2009, whereas the ratio model gives a misalignment of -9.6% (near equilibrium). Thus, the Hong Kong dollar can be viewed as being undervalued when the income level difference is not considered and near equilibrium when it is considered.

5.2.2 Complete the PPP model as an RER anchor

According to Rogoff (1996, p.647), “While few empirically literate economists take PPP seriously as a short-term proposition, most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates.” Taylor and Taylor (2004, p. 138–140) show that the bilateral NER between the sterling and the US dollar, whether measured by producer price indices or consumer price indices, moves closely to its PPP rate in a long period (i.e., 1820–2001).

The PPP model does not hold for the RERs of the four new industrial countries and areas in the entire periods examined. Fig. 2 shows that most of the RER misalignments from the PPP model are less than –20% in their entire periods, especially before 1985. The misalignments neither exhibit an upward trend near the equilibrium level (the zero line) nor fluctuate around the equilibrium level. By contrast, each RER misalignment of the four new industrial countries and areas from the ratio model exhibits a downward trend from the 1950s or 1960s to 1985, and then the misalignment mostly fluctuates in a narrow range of ±50% in 1985–2009, as discussed in Section 4.1. In other words, each RER misalignment first declines from high overvaluation to its equilibrium (the zero line) and then fluctuates around the equilibrium, which is an obvious convergence phenomenon (Zhang, 2012). This phenomenon indicates that the ratio model can be considered as an RER anchor. Furthermore, it seems to be a more suitable RER anchor than the PPP model.

However, in the case of the yen, the PPP model does give some indications as an RER anchor. Fig. 2 shows that the misalignment of the yen from the PPP model fluctuates around the zero line. Meanwhile, the misalignment from the ratio model is also around the
zero line, although it fluctuates in an opposite direction from that in the PPP model before 1970. The ratio model can also be considered as an RER anchor. In summary, the ratio model seems a more suitable RER anchor than the PPP model for the four new industrial countries and areas. The two models equally serve as RER anchors for Japan.

6 Updated Misalignments

In Sections 3, 4, and 5, the five RER misalignments are calculated and analyzed using the PWT 7.0 database, which is only until 2008 or 2009. The WEO database (April 2012) is used to calculate these RER misalignments in 2010–2011 in this section to obtain more updated results. The data in 2010–2011 are actual in the database; the data after 2012 are predicted by the IMF and not used. NER is calculated from two GDPs measured by national price and the US dollar (current price). Then, RER is calculated from NER and PPP according to Eq. (1). The GDPP is measured by the US dollar in its current price (not in converted PPP as in the PWT 7.0 used in earlier sections). The RERs and GDPPs are normalized with US=100 to be consistent with Eq. (3). Both the PPP and ratio models are used to obtain more complete results as analyzed in Section 5.2. The results are presented in Table 4.

| Table 4: Five RER misalignments in 2010–2011 in the PPP and ratio models |
|---------------------------------|---------|---------|---------|---------|
|                                 | Japan   | Singapore | Hong Kong | Korea   | Taiwan  |
| RER(US=100)                     |         |          |          |         |         |
| 2010                            | 125.3   | 77.4     | 68.4     | 69.1    | 52.2    |
| 2011                            | 132.2   | 82.5     | 69.3     | 71.8    | 53.3    |
| GDPP(US=100)                    |         |          |          |         |         |
| 2010                            | 91.7    | 93.5     | 67.3     | 44.3    | 39.6    |
| 2011                            | 94.9    | 101.8    | 70.4     | 47.1    | 41.5    |
| Misalignment (%)                |         |          |          |         |         |
| 2010                            | 25.3    | 36.6     | −22.6    | −17.2   | −31.6   |
|                                 | 1.6     | 56.0     | −30.9    | −47.8   | 31.8    |
| 2011                            | 32.2    | 39.3     | −17.5    | −19.0   | −30.7   |
|                                 | −1.6    | −28.2    | 52.4     | −46.7   | 28.4    |

Source: WEO database and the authors’ calculation. Notes: “PPP” and “Ratio” refer to the misalignments in the PPP and ratio models, respectively. A positive (negative) number indicates overvaluation (undervaluation).

Table 4 shows that the yen is overvalued by about 30% in 2010–2011 whether the PPP or the ratio model is used. The Singapore dollar is undervalued by about 20% in 2010–2011 regardless of which model is used. Therefore, the yen’s overvaluation and the Singapore dollar’s undervaluation can be confirmed. According to the WEO database, the annual compound GDP (constant price) growth rate of Singapore is 5.1% in 2000–2011, which is greater than that of the other four Asian countries and areas and the United States (from Japan’s 0.5% to Korea and Hong Kong’s 3.8%). Given this fast economic growth in Singapore, its RER is still undervalued in 2011, although it appreciates by 1.3% per year in the period. The PPP model gives a misalignment of about −30% for the Hong Kong dollar in 2010–2011, but the ratio model gives a misalignment of close to zero. Therefore, the Hong Kong dollar is undervalued by about 30% when the income level difference is not considered but is in equilibrium when it is considered.
The Korean won is undervalued by about 30% in 2010–2011 when the income level difference is not considered but is highly overvalued by about 55% when it is considered. The new Taiwan dollar is undervalued by nearly 50% in 2010–2011 when the income level difference is not considered but is overvalued by about 30% when it is considered. As the GDPPs of Korea and Taiwan are less than 50 in 2010–2011 (much smaller than that of the United States), the PPP model may not hold. Thus, we can mainly rely on the ratio model and consider that the Korean won is overvalued by about 55% and that the new Taiwan dollar is overvalued by about 30% in 2010 to 2011.

7 Conclusion

The ratio model proposed by Zhang (2012) is revisited in this paper from different perspectives. The misalignments of each RER of the five Asian industrial countries and areas against the United States are investigated by using this model and the PPP model from a long-term (from early 1950s to 2009 and through the PWT 7.0 database) and an updated (in 2010–2011 and through the WEO database) perspectives. The misalignments of the five RERs and the characteristics of the ratio model are determined from these applications.

The findings of the ratio model from the PWT database show that all RERs of the four new Asian industrial countries and areas are highly overvalued during their early development stages. This finding confirms the consensus that a country often overvalues its currency for the sake of imports (because of the shortage of foreign exchange) and trade protection. The main reason for the high overvaluation of the four RERs is the high price levels relative to the low incomes of the countries and areas at the time. Another reason lies in the model structure: the ratio model lacks a mechanism that can lessen misalignment. Our results also show that the RER misalignments of the four new Asian industrial countries and areas exhibit a downward trend before 1985. Then, after 1985, they fluctuate in a narrow range of ±50%. This demonstrates an obvious convergence (more formally, non-linear convergence) phenomenon. A similar convergence phenomenon cannot be found in the misalignments of the PPP model. This finding suggests that the ratio model can complete the PPP model as an RER anchor. However, the yen shows a different misalignment from that of the other four Asian countries and areas from the long-term perspective.

The ratio model can also complete the PPP model in currency valuation. The PPP model values an RER without considering the influence of income level difference between two countries, whereas the ratio model considers such income level difference. Thus, by combining the two models, we can obtain a more complete misalignment result. Based on this idea, we calculate the five RER misalignments in the latest years (2010–2011) in the WEO database. The yen is overvalued by about 30%, and the Singapore dollar is undervalued by about 20% whether the income level is considered or not. The Hong Kong dollar is undervalued by about 30% when the income level difference is not

12 Note that different databases or dimensions may give different values. Although the PPP-converted GDPPs of Korea and Taiwan from the PWT database used in Sections 3 to 5 are about 60 and 65 in 2008–2009, respectively, their current priced GDPPs from the WEO database used in this section are only about 45 and 40 in 2010–2011, respectively.
considered and is in equilibrium when it is considered. In the case of Korea and Taiwan, their GDPPs are too small (against that of the United States) for the PPP model to hold. Thus, we can mainly rely on the ratio model. Based on the ratio model, the RERs of Korea and Taiwan are highly overvalued by about 55% and overvalued by about 30%, respectively.

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References


Appendix

We report the results from a threshold autoregressive (TAR) model and an ordinary autoregressive (AR) model as below.

The self-exciting threshold autoregressive (SETAR) model (Tong, 1990) is specified in Eq. (A1), where $y$ is the misalignment from the ratio model.

$$y_t = \begin{cases} 
\mu_1 + \beta_1 y_{t-1} + u_{1t} & \text{if } y_{t-1} < r \\
\mu_2 + \beta_2 y_{t-1} + u_{2t} & \text{if } y_{t-1} \geq r
\end{cases} \quad (A1)$$

The results are given in Table A1, where the nonlinearity can be seen in each country or area, given $\beta_1$ being always greater than $\beta_2$ in each column. That is, the greater the misalignment is, the faster the misalignment converges (i.e. the shorter half-life). But there is no regular phenomenon in cross-section comparison, as the changes of $\beta_1$ or $\beta_2$ are not regular.

<table>
<thead>
<tr>
<th>Table A1: The coefficients, $\beta_i$ s ($i = 1, 2$), obtained from Eq. (A1)</th>
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<tr>
<td>$\beta_1$</td>
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<td>$\beta_2$</td>
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<td>$r$</td>
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Notes: Nonlinear Least Squares (estimated by Gauss-Newton); Heteroscedasticity-consistent (Eicker-White) standard errors; and *** (**) indicates the significance at 1% (5%) level.

The results from the ordinary AR model, specified as $y_t = \mu + \beta y_{t-1} + u$, also have reference meanings in most countries and areas.

<table>
<thead>
<tr>
<th>Table A2: The coefficients, $\beta$s, obtained from ordinary AR(1) model in different periods</th>
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<tr>
<td>Before 1985</td>
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<td>After 1986</td>
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Notes: OLS estimation; Heteroscedasticity-consistent (Eicker-White) standard errors; and **** (**) indicates the significance at 1% (5%) level.