Cost of Deposit Insurance Under Capital Forbearance: Basel I vs. II

Shih-Cheng Lee

Abstract

The study derives the closed-form solution of the valuation of deposit insurance under forbearance for banks whose capital requirements are either solely based on the 1988 Basel Accord (BA) approach or the VaR-based approach. The study also demonstrates that the deposit insurance liability under BA rises monotonically with portfolio risk, but it is much less risk-sensitive under VaR. It implies that the VaR-based capital regulation is more stable in containing deposit insurance losses and failure probability than the BA capital regulation.

JEL classification numbers: G21, G28, G20.
Keywords: Deposit insurance, Basel Accord, Value-at-Risk, Forbearance, Capital requirement, Option pricing

1 Introduction

Different regulatory methods require different levels of capital adequacy and imply different levels of liability for a deposit insuring agency. The 1988 Basel Capital Accord, which sets down the agreement among the G-10 central banks to apply common minimum capital standards to their banking industries, to be achieved by end-year 1992. The standards are almost entirely addressed to credit risk. The Basel Committee on Banking Supervision’s 1988 capital standards and its subsequent amendments is commonly referred as Basel I. Basel II is an effort by international banking supervisors to update the original international bank capital accord (Basel I). The Basel Committee on Banking Supervision developed proposals that aim to improve the consistency of capital regulations

1Lee: Business School of University of Adelaide and College of Management, Yuan Ze University, Jung-Li 32003, Taiwan
e-mail: sclee@saturn.yzu.edu.tw

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internationally, make regulatory capital more sensitive to market risk, and promote enhanced risk-management practices among large banking organizations.

Value at Risk (VaR), a statistical concept, serves as the conceptual foundation for the supervisory risk-measurement and risk-budgeting the Advanced Internal Ratings based version of Basel II contemplates. VaR was incorporated into the 1996 Market Risk Amendment to Basel I and VaR-like models have been partially incorporated into the Basel II treatment of credit risk. It is commonly believed that there is a regime change in terms of risk measurement and capital requirements from the building-block approach of capital standards as in Basel I to the VaR-based capital requirements in Basel II.

This study intends to examine how the cost of deposit insurance changes when moving from Basel I to Basel II. In order to contrast and highlight the main diferent between Basel I and II, we consider a capital/asset ratio for the capital requirements in Basel I and a pure VaR-based capital to stand for the capital requirements in Basel I and a pure VaR-based capital to stand for the capital requirements in Basel II. Comparing the complete building-block approach of capital standards for Basel I to a combination of building-block approach of capital standards and VaR capital requirements for Basel II would give the same principal results, but would weaken the effects.

The deposit insurance literature has explicitly or implicitly applied the capital/asset ratio approach to reflect the capital requirements in Basel I. For example, Merton (1977), among many others, models deposit insurance as a put option within the Black-Scholes option pricing framework\(^2\). In the traditional Merton-type deposit insurance pricing model, the closure rule plays only a very limited role since depository institutions are assumed to have been liquidated by the end of the period anyway. Obviously, any adjustment made to the deposit insurance payoff in this setting is somewhat artificial and inconsistent with reality. Much of this stream of deposit insurance research has attempted to reflect the policy parameters of capital forbearance\(^3\) in the model. Assuming a fixed capital/asset ratio to reflect the capital requirements in Basel I, Duan, and Yu (1994, 1999) and Cooperstein et al (1995) incorporate the possibility of capital forbearance to study issues related to the deposit insurance, the study herein intends to follow this stream of research and explicitly incorporate capital forbearance to derive the exact relationship of deposit insurance premium under both the capital regulations of Basel I and Basel II. The research also examines the interplay among the cost of deposit insurance, capital standards, and failure probability.

The rest of the paper is organized in three sections. In section 2 we derive a closed-form solution for the valuation of deposit insurance with forbearance in the capital regulations of Basel I and Basel II. We report the deposit insurance premiums from our model and discuss their policy implications in section 3. We summarize the paper in the conclusion section.

\(^2\)Also see in the option framework, for example, McCulloch (1985), Ronn and Verma (1986), Allen and Saunders (1993), Epps et al (1996), etc.

\(^3\)Capital forbearance has long been recognized in the literature as an important determinant of deposit insurance liabilities (see, for example, Kane (1987, 2001) and Nagarajan and Sealey (1995)).


2 The Deposit-Insurance Pricing Model

The typical way to model asset dynamics, such as that presented by Merton (1977) and Ronn and Verma (1986) among others, assumes a lognormal diffusion process for the asset value. The value of a bank’s total assets is governed by the following process:

\[
\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_{A,t}
\]

where \(A_t\) is the value of the bank’s total assets at time \(t\); \(W_{A,t}\) is the Weiner process; \(\mu_A\) is the instantaneous drift; and \(\sigma_A\) is the volatility of the bank’s asset value.

In our deposit insurance pricing model, capital forbearance is incorporated in a manner similar to that of Ronn and Verma (1986), Allen and Saunders (1993) and Duan and Yu (1994), Lee et al (2005) and Chuang et al. (2009). We postulate that a financial institution is subject to periodic audits by regulators. The earned interest is assumed to be plowed back into the deposit base, \(D\). Since the deposits are insured, the interest rate applicable must be the risk-free rate of return, \(r\): The current time is 0. At the time of auditing, \(T\), a depository institution faces an insolvency resolution only when its asset value falls below the forbearance threshold, \(\rho D e^{rT}\), where \(\rho\) is used to model capital forbearance and is taken to be less than or equal to one\(^4\).

Even if its asset value cannot meet the capital standard, as long as it does not fall below the forbearance threshold, the insured bank will not be forced to face an immediate resolution and can extend its operation until \(T + \tau\). A bank in financial distress is able to function “normally” under such circumstances, because the insuring agent guarantees the performance of its deposit liabilities. Although capital forbearance alters the conditions for triggering insolvency resolution, the resolution will, if it takes place, fully restore the asset value to the bank’s outstanding deposit liabilities, \(D e^{rT}\). The amount needed to restore the asset value is the liability of the insuring agent.

2.1 Capital Requirements: Basel I versus Basel II

At auditing time \(T\), the regulator’s goal is to make sure that the bank preserves a safety cushion, \(A_T - D e^{rT}\), which is the difference between asset value and the bank’s outstanding deposit liabilities to meet the capital requirements set in the Basel I or Basel II regulatory mechanism. Under the Basel I regulation, the minimum safety cushion is determined by the capital and the risk weighted assets of the bank. In this study, we simply the capital requirements of Basel I into a fixed capital / asset ratio. This capital requirement can be considered is a fraction \((1 - \frac{1}{q})\) of the bank’s assets, where the parameter \(q\) reflects the capital standard set by the regulatory authority, which is the lower bound of the asset value. The capital standard, based on the Basel I, calls for capital in an amount exceeding 8% of the asset value. This capital standard can be translated into \(q = 4\)

\(^4\)If the government-set capital standard is strictly enforced, the banks will be required to have a capital infusion or face closure before they become insolvent. Failure to close a bank or forcing a capital infusion when the capital standard is violated can be regarded as granting capital forbearance. Based on this interpretation, any closure rule based on zero net worth contains forbearance.
In the case of an audit, the bank will be allowed to continue operations under Basel I only if:

\[ A_T \geq qDe^{rT} \]  

The VaR approach of Basel II is conceptually different from the building-block approach of Basel I, since it includes not only the exposure to risk factors, but also the volatility of the risk factors. As we do not want to model the current regulation of a specific country, but rather wish to compare the effects of different regulatory mechanisms, we assume that capital requirements of Basel II are solely based on VaR. VaR regulation demands that in case of an audit the bank’s safety cushion \( A_T - De^{rT} \) must be at least as high as the \( pr\% \) (say 99\%) VaR for a time horizon of \( H \). Since the asset value of the bank \( (A_t) \) is assumed to follow a geometric Brownian motion, its return between \( T \) and \( T + H \) is normally distributed with mean \( (\mu_A - \frac{\sigma_A^2}{2})H \) and a standard deviation of \( \sigma_A \sqrt{H} \), the \( pr\% \) quantile of the loss distribution is given by

\[
(\Phi^{-1}(pr\%) \sigma_A \sqrt{H} - (\mu_A - \frac{\sigma_A^2}{2})H)AT,
\]

where \( \Phi^{-1}(pr\%) \) is the \( pr\% \) quantile of the standard normal distribution. Dangl and Lehar (2004) model the market with a short period of 10 days, and they ignore \( \mu_A \). In this study, we model bank risk with a longer period of one year, so we cannot ignore \( \mu_A \). That means the bank is allowed to continue its operation under VaR if

\[ A_T \geq \frac{De^{rT}}{1 - \Phi^{-1}(pr\%) \sigma_A \sqrt{H}} \]  

Comparing equations (2) and (3) we can see that the main difference between the two regulatory systems is that VaR regulation explicitly accounts for the risk of the portfolio by adjusting the capital requirements. However, the BA regulation may not be directly related to the asset return and the volatility of the bank’s assets.

### 2.2 Valuation of Deposit Insurance under Capital Forbearance

The payoffs of the deposit insurance contract in both Basel I and II capital regulations at time \( T \), \( (P_T) \); can be characterized as:

\[
P_T = \begin{cases} 
0 & \text{if } A_T \geq QDe^{rT} \\
F_T & \text{if } QDe^{rT} > A_T \geq \rho De^{rT} \\
De^{rT} - A_T & \text{if otherwise,}
\end{cases}
\]  

where

\[
Q = \begin{cases} 
q & \text{if under Basel I} \\
\frac{1}{1 - (\Phi^{-1}(pr\%) \sigma_A \sqrt{H} - (\mu_A - \frac{\sigma_A^2}{2})H)} & \text{if under Basel II}
\end{cases}
\]

and \( FT \) is the value from the extended operation and has a payoff at \( T + \tau \) as follows:

\[
F_{T+\tau} = \begin{cases} 
0 & \text{if } A_{T+\tau} \geq De^{r(T+\tau)} \\
De^{r(T+\tau)} - A_{T+\tau} & \text{if otherwise.}
\end{cases}
\]
The value of $F_T$ at time $T$ can be expressed as the following European option:

$$F_T = D e^{rT} N(-d_r + \sigma_A \sqrt{T}) - A_T N(-d_r),$$

(6)

where

$$d_r = \frac{\ln \frac{A_T}{D e^{rT}} + \frac{\sigma_A^2}{2} T}{\sigma_A \sqrt{T}},$$

and $N(.)$ denotes the cumulative density function of a standard normal variable.

Using the risk-neutral valuation technique, as shown in Appendix A, the value of the deposit insurance under forbearance at current time $0$ can be solved as:

$$P = D N(d_1) - A_0 n(d_2) + D [N(a_1, k_1, m) - N(d_1, k_1, m)] - A_0 [N(a_2, k_2, m) - N(d_2, k_2, m)],$$

(7)

where $N(x, y, c)$ denotes a standard bivariate normal cumulative density function:

$$d_1 = \frac{-\ln \frac{A_0}{\rho D} + \frac{\sigma_A^2}{2} T}{\sigma_A \sqrt{T}}, \quad d_2 = d_1 - \sigma_A \sqrt{T},$$

$$a_1 = \frac{-\ln \frac{A_0}{Q D} + \frac{\sigma_A^2}{2} T}{\sigma_A \sqrt{T}}, \quad a_2 = a_1 - \sigma_A \sqrt{T},$$

$$k_1 = \frac{-\ln \frac{A_0}{D} + \frac{\sigma_A^2}{2} (T + \tau)}{\sigma_A \sqrt{T + \tau}}, \quad k_2 = k_1 - \sigma_A \sqrt{T + \tau},$$

and

$$m = \frac{T}{\sqrt{T + \tau}}.$$

The above closed-form solution is a general deposit insurance pricing formula for both the Basel I and Basel II (VaR) regulations. The only difference is the $Q$ in the definition of $a_1$ which differs for the respective capital regulatory as defined in Equation (4). Since the term $Q$ is not a function of $\rho$ and $\tau$, the impact from changing these key policy variables does not have any distinction between the building-block approach and the VaR approach. That is, lowering the capital standard and prolonging the delay of closure will increase the cost deposit insurance and the failure probability in the same amount for both regulatory mechanisms. We present the derivation in Appendix B. We also note that $N(d_1)$ is the probability that $A_T < \rho D e^{rT}$ which is the failure probability at $T$, and $N(a_1, k_1, m) - N(d_1, k_1, m)$ is the probability that the forbearance banks are resolved at $T + \tau$ and it is the failure probability at that time.

### 3 Cost of Deposit Insurance

Table 1 reports the cost of deposit insurance or the fairly-priced premium rates as derived in equation (7) under alternative levels of portfolio risk and leverage risk. The first panel of Table 1 shows the case whereby banks hold their initial capital exactly according to the amount required by the Basel I and the Basel II. In order to examine the sensitivity of model risk the Table also offers the estimates for VaR capital requirements with the panic
factors of 1 and 5 for comparison. Under BA, the deposit insurance cost rises sharply with the portfolio risk, while under VaR the cost of deposit insurance is less risk-sensitive. It is because the VaR regulation explicitly accounts for the volatility risk of the portfolio and adjusts the capital requirements accordingly, while the BA regulation adopts the building block approach and is less sensitive to the volatility of the bank’s assets. VaR estimates with a low model risk or high panic factor, say VaR(δ = 5), require high capital and therefore have lower estimates of deposit insurance premiums.

Table 1: The Cost of Deposit Insurance Under Capital Forbearance: BA vs. VaR

<table>
<thead>
<tr>
<th>Standard Deviation of Asset Return</th>
<th>Initial Equity/Asset = Required Capital Ratio</th>
<th>Initial Equity/Asset = 15%</th>
<th>Initial Equity/Asset = 10%</th>
<th>Initial Equity/Asset = 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BA</td>
<td>V aR(uA = 0.06)</td>
<td>V aR(uA = 0.09)</td>
<td>V aR(uA = 0.12)</td>
</tr>
<tr>
<td>0.05</td>
<td>25.10</td>
<td>54.14</td>
<td>123.44</td>
<td>227.22</td>
</tr>
<tr>
<td>0.08</td>
<td>106.25</td>
<td>35.98</td>
<td>71.89</td>
<td>128.50</td>
</tr>
<tr>
<td>0.1</td>
<td>174.44</td>
<td>30.19</td>
<td>56.66</td>
<td>97.66</td>
</tr>
<tr>
<td>0.15</td>
<td>362.16</td>
<td>20.42</td>
<td>35.47</td>
<td>57.93</td>
</tr>
<tr>
<td>0.2</td>
<td>559.27</td>
<td>12.40</td>
<td>21.51</td>
<td>35.14</td>
</tr>
</tbody>
</table>

|                                   | BA                                         | V aR(uA = 0.06)           | V aR(uA = 0.09)           | V aR(uA = 0.12)           | Merton’s Put |
| 0.05                              | 0.78                                       | 0.65                      | 0.37                      | 0.12                      | 0.08         |
| 0.08                              | 19.38                                      | 20.71                     | 20.17                     | 18.50                     | 6.76         |
| 0.1                               | 50.10                                      | 54.42                     | 54.20                     | 53.32                     | 23.73        |
| 0.15                              | 173.01                                     | 190.65                    | 190.63                    | 190.56                    | 115.09       |
| 0.2                               | 332.71                                     | 368.51                    | 368.51                    | 368.50                    | 254.27       |

|                                   | BA                                         | V aR(uA = 0.06)           | V aR(uA = 0.09)           | V aR(uA = 0.12)           | Merton’s Put |
| 0.05                              | 11.06                                      | 10.58                     | 8.30                      | 4.47                      | 3.34         |
| 0.08                              | 69.99                                      | 71.89                     | 71.28                     | 68.77                     | 37.01        |
| 0.1                               | 127.69                                     | 132.76                    | 132.59                    | 131.76                    | 79.15        |
| 0.15                              | 299.44                                     | 317.32                    | 317.31                    | 317.27                    | 224.66       |
| 0.2                               | 488.27                                     | 523.16                    | 523.16                    | 523.16                    | 398.79       |

|                                   | BA                                         | V aR(uA = 0.06)           | V aR(uA = 0.09)           | V aR(uA = 0.12)           | Merton’s Put |
| 0.05                              | 69.74                                      | 69.15                     | 8.30                      | 48.37                     | 40.67        |
| 0.08                              | 183.33                                     | 71.89                     | 71.28                     | 182.13                    | 129.35       |
| 0.1                               | 264.28                                     | 132.76                    | 132.59                    | 267.81                    | 198.74       |
| 0.15                              | 469.72                                     | 317.32                    | 317.31                    | 485.30                    | 385.99       |
| 0.2                               | 675.16                                     | 523.16                    | 523.16                    | 706.41                    | 581.00       |

Note: The premium rates for per dollar of deposit liability are in basis points. The auditing period (T) is one year, the delay period (τ) is 0.5 years, VaR confidence level(pr%) is 99%, VaR holding period (H) is one year, Basel Accord capital requirement (q) is 1.087, and the forbearance ratio (ρ) is 0.97.

The other four panels report the cases that allow banks to hold an initial capital position other than the required capital ratio. Limiting our analysis to only the solvent banks whose initial capital position ranges from 15%, 10%, and 5%, we do not consider the case where the initial equity/asset ratio is lower than 0. It is not surprising that the fairly-priced
premium rate, under both the BA and VaR requirements, increases with portfolio risk and leverage risk. The premium estimates for the BA and VaR are similar in various scenarios and the differences between these regulatory mechanisms are much less obvious since the banks have the same initial capital position and differ only in the closure-forbearance condition at the end of the year. The annual premium for Merton’s put has been reported as the benchmark, and it represents the premium rate of the deposit insurance contract with no possibility of forbearance or a ratio of \( \rho \) equal to 1. We also observe that premiums under forbearance are greater than those of Merton’s put and that the difference, forbearance premium, increases with leverage and asset portfolio risk.

4 Conclusion

This study assumes that bank capital requirements are either solely based on BA or on VaR in order to compare and contrast the potential differences of these two regulatory mechanisms. The study derives the closed-form solution of the valuation of deposit insurance under forbearance for these two mechanisms. Our model provides a clear presentation and description of how BA and VaR are different in determining the cost of deposit insurance. It shows that the impact from changing the level of capital forbearance and timing of closure has the same effect on the cost deposit insurance for both regulatory mechanisms. The study also demonstrates that, when banks hold the minimum capital requirements according to BA and VaR, the deposit insurance liability under BA rise monotonically with their portfolio risk, but they are less risk-sensitive under VaR. It indicates that the VaR-based capital regulation is more stable in containing deposit insurance losses than the BA capital regulation. However, when banks hold capital well above or below the BA or VaR standards, the distinction between these regulatory mechanisms is much less obvious in our model. For a more general analysis to achieve more precise results over a longer period of time, a multiperiod model may be a good direction for future research. In addition, VaR regulation may enhance the incentive for solvent banks to reduce their risk since higher asset volatility implies higher capital requirements. In the multiperiod framework, even though an exact formula for the deposit insurance liability probably cannot be found, how moral hazard behavior and risk management will interact under VaR should be interesting and necessary for future research.

References


Appendices

Appendix 1:

Applying the risk-neutral valuation technique, the value of deposit insurance under forbearance can be presented as:

\[ P = e^{-rT} \left[ \int_0^{\phi(T)} (D_e^{rT} - A_T) f(A_T) dA_T + \int_{\phi(T)}^{Q_0} F_T f(A_T) dA_T \right], \]

where \( f(A_T) \) is the probability density function for \( A_T \) conditional on \( A \).

The first term in the above equation is simply a Black-Scholes put option. To derive the second term, we apply the definition of a bivariate normal probability density function:

\[ \int_{-\infty}^{a} f(u)N(\alpha + \beta u) du = N \left( a, \frac{\alpha}{\sqrt{\alpha + \beta^2}}, \frac{-\beta}{\sqrt{\alpha + \beta^2}} \right). \]

Accordingly, we obtain the closed-form solution of the deposit insurance premium under forbearance as presented in Equation (7).

Appendix 2:

This appendix performs comparative statics to understand how the value of deposit insurance under the two regulatory mechanisms varies in response to changes in the critical policy parameters.

1. Capital Forbearance Ratio and Premium Rates

The immediate impact of granting capital forbearance is best demonstrated by the following derivative property:

\[ \frac{\partial P}{\partial \rho} = \frac{A_0 n(-d_1 + \sigma_A \sqrt{T})}{\rho^2 \cdot \sigma_A \sqrt{T}} [1 - \rho N(d_{k2}) - N(d_{k1})] < 0, \]

where \( d_{k1} = -\ln \rho + \frac{\sigma_A \sqrt{T}}{\sigma_A \sqrt{T}} \), \( d_{k2} = d_{k1} - \sigma_A \sqrt{T} \), and \( n(.) \) is a standard normal density function.

The sign of \( \frac{\partial P}{\partial \rho} \) is negative, indicating that a lower capital forbearance ratio will increase the cost of deposit insurance and that capital forbearance cannot be an optimal policy to minimize the cost of deposit insurance. We further show the second-order condition as follows:

\[ \frac{\partial^2 P}{\partial \rho^2} = \frac{A_0}{\rho^2 \cdot \sigma_A \sqrt{T}} [N(d_{k1}) - 1] < 0. \]

This implies that the insurance put is maximized when \( - \) approaches zero. This means that the insured bank operates under the coverage of the insuring agent and has almost no capital of its own.
2. Forbearance Time and Premium Rates
Forbearance provides an extended time period to the insurance contract. The deposit insurance value therefore should be positively related to the forbearance time period. This can be shown as follows:

\[
\frac{\partial P}{\partial t} = D \frac{\sigma_A}{4\sqrt{T+1}} n(k_1) \left[ N\left(\frac{a_1-\mu k_1}{\sqrt{1-m^2}}\right) - N\left(\frac{d_1-\mu k_1}{\sqrt{1-m^2}}\right) + N\left(\frac{a_2-\mu k_2}{\sqrt{1-m^2}}\right) - N\left(\frac{d_2-\mu k_2}{\sqrt{1-m^2}}\right) \right] > 0
\]

The positive sign of the partial differential supports this relationship. The longer the forbearance time is, the higher the value will be of the deposit insurance under forbearance.

3. Cross Effect of Forbearance Ratio and Forbearance Time
The two key policy parameters of forbearance are the threshold forbearance ratio (\(\rho\)) and the length of forbearance time (\(\tau\)). We find that the capital forbearance ratio is negatively related to forbearance time based on the following partial differential:

\[
\frac{\partial^2 P}{\partial \rho \partial \tau} = \frac{-A_0 n(-d_1 + \sigma_A \sqrt{T}) n(d_1)\sigma_A}{2\rho^2 \sigma_A \sqrt{T}} < 0
\]

(9)
The sign of \(\frac{\partial^2 P}{\partial \rho \partial \tau}\) is negative which demonstrates that a lower capital forbearance ratio will increase the marginal impact of forbearance time on the deposit insurance cost (\(\frac{\partial P}{\partial t}\)) and that a longer forbearance time will also increase the marginal impact of the forbearance ratio on the deposit insurance cost (\(\frac{\partial P}{\partial \rho}\)). For the comparative statics presented in this appendix, there is no distinction between the pricing formula of BA and VaR since the term Q is not a function of \(\rho\) and \(\tau\).