Accounting Conservatism, Market Liquidity and Informativeness of Asset Price: Implications on Mark to Market Accounting

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Abstract

This paper theoretically examines the impact of conservatism on the asset price in an asset market allowing for strategic interactions among traders. Due to the trades coming from conservatism traders contain less informational content, the asset price is shown to be less informative in the presence of conservatism traders. In addition, this paper shows that the market liquidity increases as the proportion of conservatism traders increases. With mark to market accounting replacing the conservative accounting practice, the asset price will be more informative and the market liquidity will be reduced. From the perspective of the informativeness of the asset price, the results of this paper support mark to market accounting.

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1 Introduction

Accounting conservatism requires more verifiability for the recognition of gains than for the recognition of losses. An extreme form of accounting conservatism is described in the adage "anticipate no profit, but anticipate all losses" (Bliss (1924)). A large body of empirical research has documented conservatism in accounting and offer explanations for the existence of conservatism. For example, Basu (1997) examined the existence of conservatism empirically and he found that the earnings reflect bad news more quickly than good news. Watts (2003a, 2003b) summarizes the empirical evidence on the existence of conservatism and presents various explanations for conservatism in accounting.

However, few articles examine the impact of such conservative accounting practice on the financial markets. Recently, the working paper by Dierker (2006) provides a theoretical explanation of accounting conservatism used as a means to avoid speculative bubbles and overvaluation. He shows that the conservatism can hurt market efficiency if the financial market is in a frictionless, perfectly efficient state. With a short sale constraint, the market is prone to overvaluation and he shows that in such a market environment, conservatism can actually improve market efficiency. Another working paper in this area is Yu et al (2011) who empirically examine the relationship between conservatism and informational efficiency in Chinese stock markets. They found a positive relationship between conservatism and stock price informativeness. Their empirical evidence supports the view that conservatism makes earnings more informative and price contains more information about future earnings.

This paper extends the framework of Kyle (1985) to examines the impact of conservatism on the asset price and market liquidity in an asset market allowing for the strategic interactions among traders. It proves that conservatism causes the asset price to be less informative although it can also increase the market liquidity.

Specifically, consider a firm that adopts accounting conservatism in its practice. Since conservatism requires more verifiability for the recognition of gains than for the recognition of losses, at any point in time, accounting earnings numbers do not incorporate the expected future profits from the positive net present value projects until these profits are realized while the expected future losses from the negative net present value projects are often recognized in the current accounting earnings. This means that the accounting earnings numbers underestimate the asset’s payoff (economic earnings). Rational traders can correctly figure out the economic earnings and the asset’s pay-
off by taking into account the conservatism practice in the firm’s accounting. However, there is another group of traders who regard the accounting earnings as the asset’s payoff (or economic earnings), hence, they underestimate the asset’s payoff. For the sake of the discussion in this paper, they are called conservatism traders.

In a one-period model with the asset of the accounting conservatism firm as the only asset and one market maker. The payoff of the asset is assumed to be normally distributed. Since rational traders can correctly assess the economic earnings of the conservatism firm, they have correct prior knowledge about the mean and variance of the normal distribution of the asset’s payoff. However, conservatism traders underestimate the mean and variance of the normal distribution of the asset’s payoff. This is because they regard the accounting earnings as the asset’s payoff (or economic earnings) and accounting earnings underestimate the mean and variance of the asset’s payoff.

There are also noise traders in the market who trade for the liquidity reasons. Hence, their demand for the asset is assumed to be random.

Before any trade takes place, rational and conservatism traders receive an informational signal about the asset’s payoff. Both rational and conservatism traders rationally update their conditional mean about the asset’s payoff given their prior knowledge about the asset’s payoff. Rational and conservatism traders submit their market orders for the asset to the market maker. Their market orders are generated from the maximization of their expected profits. The market maker observes the aggregate demand from all traders but not individual demand. The market maker sets the asset price equal to the expected asset’s payoff conditional on the observed aggregate demand for the asset. Here, the market maker provides the liquidity to the market and the cost of doing so is assumed to be zero. The objective of rational and conservatism traders is to maximize their expected profits. Traders are assumed to be risk neutral.

This paper shows that as the proportion of conservatism traders increases the asset price becomes less informative. In addition, this paper shows that the asset price becomes less volatile and the market liquidity increases as the proportion of conservatism traders increases. With mark to market accounting replacing the conservative accounting practice, conservatism traders no longer exist. Hence, the results of this paper suggest that mark to market accounting improves the informativeness of the asset price by eliminating the uninformative trades coming from conservatism traders. In addition, the results of this paper also suggest that the market liquidity will decrease under mark to mar-
ket accounting. This is because the net demand contains more informational content after eliminating conservatism traders. Furthermore, the asset price will become more volatile with mark to market accounting. This is due to the absence of conservatism traders whose prior estimate of the variance of the asset’s payoff being smaller than informed rational traders. From the perspective of the informativeness of the asset price, the results of this paper support mark to market accounting.

The remainder of the paper consists of three sections. The next section describes the framework of the model and presents the equilibrium of the model. Section 3 analyzes the impact of conservatism on the informativeness of the asset price and on the market liquidity. The concluding remarks are presented in Section 4.

2 The Model

Consider a one-period model of an asset market with one asset and one market maker. The market maker supplies the liquidity to the market. The cost of doing so is assumed to be zero for the simplicity. All traders submit their market orders for the asset to the market maker. There are three types of traders: rational traders, conservatism traders and noise traders. Rational traders have correct knowledge that the payoff of the asset as normally distributed with the mean of $\theta$ and variance of $\sigma^2$. Conservatism traders regards the payoff of the asset as normally distributed with the mean of $\theta_c$ and variance of $\sigma^2_c$, where $\theta_c < \theta$ and $\sigma^2_c < \sigma^2$. The underestimation of the mean and variance of the distribution of the asset’s payoff is due to the fact that conservatism traders regard the accounting earnings as the asset’s payoff. Since conservatism in accounting requiring differential verifiability of gains versus losses, under the accounting conservatism practice, at any point in time, accounting earnings numbers do not incorporate the expected future profits from the positive net present value projects until these profits are realized while the expected future losses from the negative net present value projects are recognized in the current accounting earnings. Consequently, the accounting earnings underestimate the mean and variance of the asset’s payoff (economic earnings). In other words, conservatism traders underestimate the mean and variance of the normal distribution of the asset’s payoff.

Noise traders trade based on their liquidity needs. Their demand for the asset is a random variable (denoted as $x$), which is normally distributed with
the mean of zero and variance of $\sigma^2_x$.

No trader knows the payoff of the asset but rational and conservatism traders receive an informational signal about the asset’s payoff before any trade occurs. This informational signal is modeled as $S = \theta + \epsilon$ where $\epsilon$ is normally distributed with the mean of zero and variance of $\sigma^2_\epsilon$. The random variables $\theta$, $\epsilon$ and $x$ are mutually independent.

After receiving the informational signal about the asset’s payoff, rational traders updates their conditional mean about the asset’s payoff according to the following:

$$E(\theta | (S, r)) = \overline{\theta} + \eta_\theta (S - \overline{\theta}),$$

where the parameter $r$ indicates rational traders and $\eta_\theta = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\epsilon}$. The derivation of equation (1) follows from the results of Theorem 1 of Appendix. Similarly, conservatism traders update their conditional mean about the asset’s payoff according to

$$E(\theta | (S, c)) = \overline{\theta}_c + \eta_c (S - \overline{\theta}_c),$$

where the parameter $c$ indicates conservatism traders and $\eta_c = \frac{\sigma^2_c}{\sigma^2_c + \sigma^2_\epsilon}$. Note that $\eta_c < \eta_\theta$ (due to $\sigma^2_c < \sigma^2_\theta$). Equation (2) is derived from the results of Theorem 1 of Appendix. Note that accounting conservatism traders also rationally update their conditional mean about the asset’s payoff given their initial beliefs about the mean of the asset.

Note from equations (1) and (2) that the conditional mean of the asset’s payoff for conservatism traders is smaller than that for rational traders when the informational signal is bigger than the expected asset’s payoff; and it is bigger than that for rational traders when the informational signal is sufficiently smaller than the expected asset’s payoff so that $\overline{\theta} - \overline{\theta}_c + \eta_\theta(S - \overline{\theta}) - \eta_c(S - \overline{\theta}_c) < 0$.

Both rational and accounting conservatism traders are considered as informed traders. It is assumed that there are $N$ informed traders. The proportion of informed traders being conservatism traders is denoted as $f$, where $f \in [0, 1]$.

The market maker behaves competitively. After receiving the aggregate demand of all traders, he sets the asset price equal to the expected asset’s payoff conditional on the observed aggregate demand for the asset. The asset price is denoted as $P$ and the aggregate demand is denoted as $D$. Hence, the asset price is determined by the following equation:

$$P = E(\theta | D).$$
The equilibrium is characterized by the following: (a) Given the asset pricing rule stated in equation (3) and taken into account the impact of his market order on the asset price and on other traders’ market orders, trader \( i \), where \( i \in \{1, 2, \ldots, N\} \), of type \( j, j = r, c \), chooses his market order (denoted as \( X_{ij} \)) to maximize his expected profit

\[
\max_{X_{ij}} [E(\theta \mid (S, j)) - E(P \mid (S, X_{ij}))] X_{ij},
\]

where \( E(\theta \mid (S, j)) = E(\theta \mid (S, r)) \) if \( j = r \); and \( E(\theta \mid (S, j)) = E(\theta \mid (S, c)) \) if \( j = c \). Furthermore, where \( E(P \mid (S, X_{ij})) = E(\theta \mid D) \). (b) Given all the equilibrium market orders coming from all traders, the market maker sets the asset price equal to the expected asset’s payoff according to equation (3).

Note that traders are risk neutral in this framework. The following solves the equilibrium strategies for rational and conservatism traders, and the equilibrium asset price.

Denote the total number of conservatism traders as \( N_c \) and the total number of rational traders as \( N_r \). Hence, \( N = N_r + N_c \). Assume that the equilibrium strategies for rational and conservatism traders are linear functions of the informational signal. That is,

\[
X_{ir} = a_{ir} + b_{ir} S,
\]

for \( i = 1, 2, \ldots, N_r \). And

\[
X_{ic} = a_{ic} + b_{ic} S,
\]

for \( i = 1, 2, \ldots, N_c \). Also assume that the equilibrium asset price follows the linear pricing rule:

\[
P = \mu + \lambda D,
\]

where \( D = \sum_{i=1}^{N_r} X_{ir} + \sum_{i=1}^{N_c} X_{ic} + x \); and all the coefficients \( \mu, \lambda, \ a_{ir}, \ b_{ir}, \ a_{ic}, \ b_{ic} \) (for \( i = 1, 2, \ldots, N_c \)) are to be determined later.

Substituting equations (1), (2), (5), (6) and (7) into the optimization problem (4), it follows that the first order condition for the optimization problem (4) is as follows:

\[
\bar{\theta} + \eta \theta (S - \bar{\theta}) - \mu - \lambda (2X_{ir} + \sum_{n=1}^{N_r} (a_{nr} + b_{nr} S) + \sum_{n=1}^{N_c} (a_{nc} + b_{nc} S)) = 0,
\]

and

\[
\bar{\theta}_c + \eta_c (S - \bar{\theta}_c) - \mu - \lambda (2X_{kc} + \sum_{n=1}^{N_c} (a_{nc} + b_{nc} S) + \sum_{n=1}^{N_r} (a_{nr} + b_{nr} S)) = 0.
\]
Again, substituting equation (5) and (6) into equation (8) and (9) respectively, it follows that
\[
a_{ij} = \frac{\bar{\theta}_j - \mu - \eta_j \bar{\theta}_j - A}{\lambda},
\]
and
\[
b_{ij} = \frac{\eta_j}{\lambda} - B,
\]
where \( A = \sum_{n=1}^{N_r} a_{nr} + \sum_{n=1}^{N_c} a_{nc} \), \( B = \sum_{n=1}^{N_r} b_{nr} + \sum_{n=1}^{N_c} b_{nc} \); and where \( \bar{\theta}_j = \bar{\theta} \), and \( \eta_j = \eta_\theta \) when \( j = r \); and \( \theta_j = \theta_c \), \( \eta_j = \eta_c \) when \( j = c \).

Notice from equations (10) and (11), that for \( i' \neq i \), \( a_{ij} = a_{i'j} \) and \( b_{ij} = b_{i'j} \) for the same \( j \in \{ r, c \} \) (the same type of traders). Hence, let \( a_{ir} = a_r, b_{ir} = b_r \) when \( j = r \); and \( a_{ic} = a_c, b_{ic} = b_c \) when \( j = c \). Equations (10) and (11) imply the following four equations are true:

\[
a_r = \frac{\bar{\theta} - \mu - \eta_\theta \bar{\theta} - N f (\bar{\theta}_c - \bar{\theta} + \eta_\theta \bar{\theta} - \eta_c \bar{\theta}_c)}{\lambda(N + 1)},
\]
\[
a_c = \frac{\bar{\theta}_c - \mu - \eta_c \bar{\theta}_c - N(1 - f) (\bar{\theta} - \bar{\theta}_c - \eta_\theta \bar{\theta} + \eta_c \bar{\theta}_c)}{\lambda(N + 1)},
\]
\[
b_r = \frac{\eta_c + (1 + N f)(\eta_\theta - \eta_c)}{\lambda(N + 1)},
\]
and
\[
b_c = \frac{\eta_c - N(1 - f)(\eta_\theta - \eta_c)}{\lambda(N + 1)}.
\]

Using equation (3),
\[
P = E(\theta | A + BS + x = D)
\]
\[
= \bar{\theta} + \frac{B^2 \sigma_\theta^4}{B^2 \sigma_S^2 + \sigma_x^2} (D - A - B\bar{\theta}).
\]

Using equations (7) and (16) along with the definitions of \( A \) and \( B \), one can show that
\[
\mu = \bar{\theta} + N f (1 - \eta_c)(\bar{\theta} - \bar{\theta}_c) + N^2 f \bar{\theta}_c(1 - f)(\eta_\theta - \eta_c),
\]
and
\[
\sigma_x^2 \lambda^3 + \lambda \left( \frac{N}{N + 1} \right)^2 \sigma_S^2 (\eta_\theta - f \eta_\theta + f \eta_c)^2 - \left( \frac{N}{N + 1} \right)^2 \sigma_\theta^4 (\eta_\theta - f \eta_\theta + f \eta_c)^2 = 0,
\]
\[
\eta_c - N(1 - f)(\eta_\theta - \eta_c) = 0.
\]
where $\sigma_{S}^{2} = \sigma_{\theta}^{2} + \sigma_{\epsilon}^{2}$.

Note that $\lambda$ is determined by equation (18). Since $\eta_{c} < \eta_{\theta}$, it is true that there exists at least one positive root from equation (18). Hence, the positive root from equation (18) is used to ensure that the second order condition of the optimization problem (4) holds and ensure that the equilibrium price is increasing in the total demand for the asset.

Since all traders of the same type have the same equilibrium strategy, the equilibrium market order for rational and conservatism traders are denoted as $X_{r}$ and $X_{c}$, respectively. Using equations (5) through (7), (12) through (15), (17) and (18), the equilibrium strategies for rational and conservatism traders, and the equilibrium asset price are computed as the following:

$$X_{r} = \frac{N^{2}f\overline{\theta}_{c}(\eta_{\theta} - \eta_{c})(f - 1) + (Nf(\eta_{\theta} - \eta_{c}) + \eta_{\theta})(S - \overline{\theta})}{\lambda(N + 1)},$$

$$X_{c} = \frac{N(f\eta_{\theta} - f\eta_{c} - \eta_{\theta})(S - \overline{\theta}) + N^{2}f\overline{\theta}_{c}(\eta_{\theta} - \eta_{c})(f - 1)}{\lambda(N + 1)} + \frac{\eta_{c}(S - \overline{\theta}_{c}) + \overline{\theta}_{c} - \overline{\theta}}{\lambda},$$

$$P = \overline{\theta} + \lambda x + \frac{N(\eta_{\theta} - \eta_{c}f + \eta_{c}f)(S - \overline{\theta})}{N + 1},$$

where $\lambda$ is described in equation (18).

Note from equation (21) that the expected asset price equals to the expected asset’s payoff.

Furthermore, from equations (19) and (20) that the following is true:

$$X_{r} - X_{c} = \frac{1}{\lambda}(\overline{\theta} - \overline{\theta}_{c} + \eta_{\theta}(S - \overline{\theta}) - \eta_{c}(S - \overline{\theta}_{c})).$$

Hence, the demand for the asset coming from conservatism traders is smaller than that coming from rational traders when the informational signal is bigger than the expected asset’s payoff; however, it is bigger than the demand coming from rational traders when the informational signal is sufficiently smaller than the expected asset’s payoff so that the right hand side of equation (22) is negative. The reason for this is because the conditional mean of the asset’s payoff for conservatism traders is smaller than that for rational traders when the informational signal is bigger than the expected asset’s payoff; and it is bigger than that for rational traders when the informational signal is sufficiently smaller than the expected asset’s payoff so that

$$\overline{\theta} - \overline{\theta}_{c} + \eta_{\theta}(S - \overline{\theta}) - \eta_{c}(S - \overline{\theta}_{c}) < 0.$$
The next section analyzes the impact of conservatism traders on market liquidity and the informativeness of the asset price.

3 The Results

This section shows that as the proportion of conservatism traders increases, the asset price becomes less informative. It also shows that the asset price becomes less volatile and the market liquidity increases as the proportion of conservatism traders increases. The following begins with the discussion on the market liquidity.

The market liquidity is defined as the extent to which one unit of excess net demand for the asset move the asset price. The excess net demand for the asset is the difference between the net demand for the asset and the mean of the net demand (i.e. $D - A - B\theta$). In this framework, it is measured by $\frac{1}{\lambda}$, which is the amount of excess net demand needed to move the asset price by one dollar.

Taking the derivative of equation (18) results in the following equation:

$$\frac{d\lambda}{df} = \frac{2 \left(\frac{N}{N+1}\right)^2 (\eta f - f \eta + f \eta c) (\eta f - \eta c) (\lambda \sigma_S^2 - \sigma_D^2)}{(\frac{N}{N+1})^2 (\eta f - f \eta + f \eta c)^2 \sigma_S^2 + 3 \sigma^2 \lambda^2}.$$

(23)

Note from the result of Lemma 1 in the appendix that $\lambda \sigma_S^2 - \sigma_D^2 < 0$. This, together with equation (23), implies that $\frac{d\lambda}{df} < 0$. This means that the market liquidity increases as the proportion of conservatism traders increases. This result is stated in the following proposition.

**Proposition 1.** The market liquidity increases as the proportion of conservatism traders increases. That is, $\frac{d\lambda}{df} < 0$.

The intuition behind this result is as follows. In the standard model of Kyle (1985), the informed traders have incentive to make the market less liquid so that they can capture more benefit of their received informational signal. In other words, the asset price is more sensitive to more informative net demand (i.e., higher $\lambda$). This suggests that the market liquidity is lower when the net demand is more informative. Here, due to the conservatism traders’ conservative prior beliefs about the asset’s payoff, as the proportion of conservatism traders increases, the excess net demand contains less informational content.
and consequently, the market liquidity increases. This is consistent with the above intuition in the model of Kyle (1985).

Furthermore, since conservatism traders’ prior belief about the variance of the asset’s payoff is smaller than rational traders’ prior belief, in responding to the informational signal, the presence of conservatism traders causes the asset price to fluctuate less than rational traders do. Hence, the variance of the asset price decreases as the proportion of conservatism traders increases. This can be seen from the following computation. Using equation (21), the variance of the asset price is computed as the following:

\[ \text{Var}(P) = \lambda^2 \sigma_x^2 + \frac{N^2 (\eta_\theta - \eta_\theta f + \eta_c f)^2}{(N + 1)^2} \left( \sigma_\theta^2 + \sigma_c^2 \right). \]  

(24)

Taking the derivative of equation (24) yields the following: for \( K = \eta_\theta - \eta_\theta f + \eta_c f, \)

\[ \frac{d(\text{Var}(P))}{df} = 2 \left( \frac{N}{N + 1} \right)^2 (\eta_c - \eta_\theta) K \left( \sigma_\theta^2 + \frac{2\lambda \sigma_x^2 (\sigma_\theta^2 - \lambda \sigma_\theta^2)}{3 \sigma_\theta^2 \lambda^2 + \left( \frac{N}{N + 1} \right)^2 \sigma_\theta^2 K^2} \right). \]  

(25)

which is negative due to \( \eta_c < \eta_\theta \) or \( \sigma_\theta^2 > \sigma_c^2 \). This result is stated in Proposition 2 below.

**Proposition 2.** *The variance of the asset price decreases as the proportion of conservatism traders increases. That is, \( \frac{d(\text{Var}(P))}{df} < 0 \).*

To examine the impact of conservatism traders on the informativeness of the asset price, the difference between the posterior variance of the asset’s payoff conditional on the asset price and the posterior variance of the asset payoff conditional on the informational signal is computed. This difference is shown to increase as the proportion of conservatism traders increases. Specifically, the derivative (with respect to \( f \)) of \( \text{Var}(\theta | P) - \text{Var}(\theta | S) \) is computed from equation (21) as, (see the appendix for the detailed derivation of the following equation)

\[ \frac{d(\text{Var}(\theta | P) - \text{Var}(\theta | S))}{df} = \frac{2 \left( \frac{N}{N + 1} \right)^2 (\eta_\theta - \eta_c) \lambda K \sigma_x^2 \sigma_\theta \left[ K \left( \frac{N}{N + 1} \right)^2 (3 \sigma_\theta^2 \lambda - 2 \sigma_\theta^2) + 3 \sigma_\theta^2 \lambda^3 \right]}{\left( 3 \sigma_x^2 \lambda^2 + \left( \frac{N}{N + 1} \right)^2 \sigma_\theta^2 K^2 \right) \left( \lambda^2 \sigma_x^2 + \left( \frac{N}{N + 1} \right)^2 \sigma_\theta^2 K^2 \right)^2}. \]  

(26)
Note from the result of Lemma 1 in the appendix that \(3\sigma_\lambda^2 \lambda - 2\sigma_\theta^4 > 0\). This, together with the fact that \(\eta_c < \eta_\theta\), implies that

\[
\frac{d (\text{Var}(\theta | P) - \text{Var}(\theta | S))}{df} > 0
\]

(see equation (26)). This means that as the proportion of conservatism traders increases, the asset price becomes less informative. The reason for this is because the market orders coming from conservatism traders have less informational content. The following proposition states this result.

**Proposition 3.** The asset price is less informative as the proportion of conservatism traders increases. That is, \(\frac{d (\text{Var}(\theta | P) - \text{Var}(\theta | S))}{df} > 0\).

The results of Proposition 1, 2 and 3 suggest that the asset price will be more informative, the asset price will be more volatile and the market liquidity will decrease if the conservative accounting practice is replaced with mark to market accounting. This is because with mark to market accounting, conservatism traders will no longer exist and the uninformative trades coming from conservatism traders are eliminated. As a result, the net demand contains more informational content. From the aspect of the informativeness of the asset price, the results of this paper support mark to market accounting.

### 4 Concluding Remarks

This paper theoretically examines the impact of conservatism on the asset price’s informativeness and market liquidity in an asset market allowing for strategic interactions among traders. Due to conservatism traders having a smaller prior belief about the variance of the asset’s payoff than rational traders, the asset price is less volatile as the proportion of conservatism traders increases. Furthermore, the asset price is shown to be less informative in the presence of conservatism traders. This is because the trades coming from conservatism traders contain less information content. In addition, this paper shows that the market liquidity increases as the proportion of conservatism traders increases.

With mark to market accounting, conservatism traders no longer exist. Mark to Market accounting eliminates the uninformative trades coming from conservatism traders and consequently improves the informativeness of the asset.
price. Furthermore, with mark to market accounting, the market liquidity will decrease. This is because the net demand contains more informational content with mark to market accounting. Furthermore, with the absence of conservatism traders whose prior estimate of the variance of the asset price being smaller than informed rational traders, the asset price becomes more volatile. The results of this paper support mark to market accounting from the perspective of the informativeness of the asset price.

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Appendix

Theorem 1: If the random variables $X^*$ and $Y^*$ are jointly normally distributed, then

$$E(X^* | Y^* = Y) = EX^* + \frac{Cov(X^*, Y^*)}{Var(Y^*)}(Y - EY^*)$$

and

$$Var(X^* | Y^* = Y) = Var(X^*) - \frac{[Cov(X^*, Y^*)]^2}{Var(Y^*)}$$

(See Hoel, p.200).

Lemma 1: The following two inequalities are true:

$$\lambda \sigma_S^2 - \sigma^4_\theta < 0, \quad (27)$$

and

$$3\sigma_S^2\lambda - 2\sigma^4_\theta > 0. \quad (28)$$

Proof. If $\lambda = \frac{\sigma_\theta^2}{\sigma_S^2}$, then the left hand side of equation (18) becomes $\sigma_S^2\left(\frac{\sigma_\theta^2}{\sigma_S^2}\right)^3$, which is bigger than zero. For the parameter $\lambda$ to solve equation (18), it must be the case that $\lambda < \frac{\sigma_\theta^2}{\sigma_S^2}$. Hence, the inequality (27) must be true. Furthermore, if $\lambda = \frac{2\sigma_\theta^4}{3\sigma_S^2}$, then the left hand side of equation (18) is negative. Hence, the inequality (28) must be true. \qed
Derivation of equation (26): Using Theorem 1, the following two equations are obtained:

\[ \text{Var}(\theta | S) = \sigma^2_{\theta} - \frac{\sigma^4_{\theta}}{\sigma^2_{\theta} + \sigma^2_{\epsilon}}, \quad (29) \]

and

\[ \text{Var}(\theta | P) = \sigma^2_{\theta} - \frac{\sigma^4_{\theta} \left( \frac{N}{N+1} \right)^2 (\eta_{\theta} + f_{\eta_{\epsilon}} - f_{\eta_{\theta}})^2}{\lambda^2 \sigma^2_{\epsilon} + \left( \frac{N}{N+1} \right)^2 (\eta_{\theta} - \eta_{\theta} f + \eta_{\epsilon} f)^2 \sigma^2_{\epsilon}}. \quad (30) \]

Equation (26) follows from equations (29) and (30). □

References


