An empirical analysis of trading volume and return volatility relationship on Istanbul stock exchange national -100 Index[*]

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Abstract

It is a well-known fact that most of the asset returns tend to be skewed and heavy-tailed. Heavy tailed distributions such as the Student's t distribution and Stable distribution are commonly used in finance to model asset returns that areheavy tailed. Additionally, Stable distribution allows not only for leptokurtosis but alsoskewness. Researchers that investigate the relationship between stock return volatility and trading volume have found a positive correlation between the volatility of returns and the volume traded. This paper focuses on this relationship by assuming the Student's t and the Stable distributions for innovations. In this paper, GARCH and Threshold GARCH (TGARCH) models are applied on the Istanbul Stock Exchange National-100 Index with the purpose of analyzing the relationships between the volatility of stock returns and the trading volume.

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   the views of the Central Bank of the Republic of Turkey.

1 Introduction

Financial decisions are exclusively based on expected returns and risk of alternative investment opportunities. The distributional assumptions for financial processes have important theoretical implications. Solutions to such problems like portfolio selection, option pricing, and risk management depend crucially on distributional specifications. Modern finance relies heavily on the assumption that the random variables under investigation follow a normal distribution (Rachev and Mittnik, 2000). Yet, finance data often depart from the Gaussian model, in that their marginal distributions are fat-tailed. The works of Mandelbrot (1963) and Fama (1965) introduced the use of stable distributions to finance. The peaked and heavy-tailed nature of the return distribution led the authors to reject the standard hypothesis of normally distributed returns in favor of the stable distribution. In general, the upper and lower tails of stable distributions decrease like a power function. In literature, this is often characterized as heavy or long tails. Since then, the stable distribution has been used to model both the unconditional and conditional return distributions. It is now commonly accepted that financial asset returns are indeed heavy-tailed (Marinelliet.al., 2001).

While different heavy-tailed distributions such as Student's t or GED can be used for modeling financial variables, stable distributions are preferred due to the generalized Central Limit Theorem. This theorem states that regardless of the existence of the variance, the limiting distribution of a sum of independent and identically distributed random variables is stable. (Borak, Härdle and Weron, 2005).

The stable Paretian distribution including the normal distribution as a special case allows heavy tailed distributions for GARCH innovations. In addition to this feature, it allows for skewness compared with the Student's t. Over and above, it also has the attractive property that it is the only suitable distribution that occurs as a limiting distribution of sums of independently, identically distributed (iid) random variables. This property is extremely desirable since error terms in econometric models are usually interpreted as random variables that represent the sum of the external effects not being described by the model.

The relationship between stock return volatility and trading volume has been investigated by many researchers, and researchers have documented that equity prices and trading volume indicate positive and contemporaneous correlation. The aim of this paper is to examine the relationship between the volatility of ISE National-100 returns and the trading volume by using the GARCH and Threshold GARCH (TGARCH) models with heavy tailed innovations. TGARCH will take into account the leverage effects, e.g., negative shocks or news have larger impact on the volatility than positive shocks.

2 Literature review

Stable distributions are a rich class of probability distributions that allow skewness and heavy tails. Therefore, they are widely used in modeling heavy tailed data. Stable distributions, also called "Levy-Pareto distributions" are used to describe complex systems in physics, biology, sociology and economics (Zolotarev, 1986). Another important property of the stable distributions with respect to its use as a model is the Generalized Central Limit Theorem which states that regardless of the existence of the variance, the limiting distribution of a sum of independent and identically distributed random variables is stable (Marinelli et.al., 2001).

There have been many studies that examine the suitability of the stable Paretian distribution for modeling the unconditional distribution of asset returns (Fama 1963, 1965 and Mandelbrot 1963, Mittnik and Rachev, 1993; McCulloch, 1997; Rachev and Mittnik, 2000). Mittnik et al. (2000) broaden the stable-GARCH models to include more general GARCH structures with stable innovations and use these models in conjunction with tests on estimated model residuals to test Stable Paretian hypothesis. The stable Paretian distributions have been examined by Liu and Brorsen (1995) (Curto et al., 2009). Liu and Brorsen (1995) developed a new method that allows attaining maximum likelihood estimates of the parameters of a GARCH process when the conditional distribution of the residuals is stable. They used data on five spot foreign exchange rates and constructed GARCH stable model. McCulloch (1985) estimated a GARCH-stable model with the stable distribution taken as symmetric. Liu and Brorsen (1995) estimated the same model but they focused on asymmetric stable distributions.

Frain (2008) found that the stable GARCH(1,1) model for losses gives the best measure of Value at Risk (VaR) compared to Normal distribution. Its performance is about the same to the t-distribution but is excellent at the conventional levels.

Lamoureux and Lastrapes (1990), who are the first to use GARCH models, find that the trading volume as a proxy for information arrival time has positive effects on the volatility of stock returns. Fong (2003), Darrat et al. (2003), Chuang et al. (2009), and others show that they are dynamically collated or indicating causal relationships. Girard and Biswas (2007) also find similar results using 22 developed and 27 emerging markets with asymmetric GARCH models. Suominen (2001) develops a theoretical model showing the GARCH like conditional variance as a function of trading volume, and the trading volume and the price volatility can be positively or negatively correlated.

This paper focuses on trading volume and return volatility relationship by using the Student's t and Stable distributions that fit with characteristics of observed financial data. In this paper, GARCH and Threshold GARCH (TGARCH) models are applied on the Istanbul Stock Exchange National-100

Index with the purpose of analyzing the relationships between the volatility of stock returns and the trading volume.

3 The model

Let a Stable Paretian distribution, $S_{\alpha\beta}(\delta, c)$, be expressed as its characteristic function:[2]

$$\phi_X(t) = E(e^{itX}) = \exp\left(i\delta t - |ct|^{\alpha}\left[1 + i\beta\,\mathrm{sgn}(t)\,w(t,\alpha)\right]\right) \qquad (1)$$

where

$$w(t,\alpha) = \begin{cases} -\tan\left(\dfrac{\pi\alpha}{2}\right), & \alpha \neq 1 \\[2mm] (2/\pi)\ln|t|, & \alpha = 1 \end{cases}$$

Assume that $-\infty < t < \infty$, $0 < \alpha \leq 2$, $|\beta| \leq \min(\alpha, 1-\alpha)$, $c > 0$, $-\infty < \delta < \infty$. '$c$' is the scale parameter. The parameter α is called characteristic exponent and interpreted as a shape parameter. When $\alpha = 2$, a stable distribution becomes a Gaussian distribution, which is the only stable distribution that the second and higher absolute moments exist. The tail thickness increases as α decreases, and the distribution becomes more leptokurtic. When β (skewness parameter) is positive (negative), the distribution is skewed to the right (left). If β is zero, the distribution becomes symmetric about δ (location parameter). As α approaches to 2, the distribution approaches to a Gaussian distribution regardless of β (Fama and Roll, 1968).

Let the time series y_t be given as ARMA(p,q) process (Curto et al. (2009)):

$$y_t = \mu + \sum_{i=1}^{p}\gamma_i y_{t-i} + \sum_{j=1}^{q}\xi_j u_{t-j} + u_t \qquad (2)$$

The conditional standard equation of a Stable Threshold GARCH ($S_{\alpha,\beta}$ TGARCH(r;s))is expressed as:[3]

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[2]Refer to Rachev and Mittnik (2000) for detailed description of Stable distributions.

[3]A Gaussian (distribution) TGARCH model is given:

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{r}\theta_i u_{t-i}^2 + \sum_{i=1}^{r}\eta_i S_{t-i} u_{t-i}^2 + \sum_{j=1}^{s}\phi_j \sigma_{t-j}^2.$$

$$\sigma_t = \theta_0 + \sum_{i=1}^{r} \theta_i \left| u_{t-i} \right| + \sum_{i=1}^{r} \eta_i S_{t-i} \left| u_{t-i} \right| + \sum_{j=1}^{s} \phi_j \sigma_{t-j}, \qquad (3)$$

where

$$S_{t-i} = \begin{cases} 1, & u_{t-i} < 0 \\ 0, & u_{t-i} \geq 0 \end{cases}, \ u_t = \sigma_t \varepsilon_t, \text{ and } \ \varepsilon_t \overset{iid}{\sim} S_{\alpha,\beta}(0,1). \ S_{\alpha,\beta}(0, 1) \text{ denotes the standard}$$

asymmetric Stable Paretian distribution with location parameter ($\delta = 0$), and unit scale parameter ($c=1$).[4] The probability density and the likelihood functions of the Stable distribution GARCH are nontrivial, and a Fast Fourier Transforms (FFT) procedure is employed for the maximum likelihood estimation (MLE) algorithm. We follow the MLE procedure of conditional heteroskedasticity models with Stable distributions presented by Mittnik et al. (1999).[5]

To examine the stock returns and volume relationships, assume the mean equation follows:

$$y_t = \mu + u_t, \qquad (4)$$

where y_t is the returns, and $u_t = \sigma_t \varepsilon_t$, $\varepsilon_t \overset{iid}{\sim} S_{\alpha,\beta}(0,1)$. The trading volume ($V$) is added to the TGARCH(1,1) equation to analyze how the volume affects the volatility:[6]

$$\sigma_t = \theta_0 + \theta_1 \left| u_{t-1} \right| + \eta S_{t-1} \left| u_{t-1} \right| + \phi_1 \sigma_{t-1} + \gamma \sqrt{V} \qquad (5)$$

Certain stability conditions are imposed to estimate the Stable GARCH and TGARCH models(Rachev and Mittnik, 2000).

4 Empirical results

Table 1 presents the summary statistics of the data. The sample starts from January 3, 2005 and ends March20, 2012. There are 1817 total observations (1816 for returns) obtained from Bloomberg. Returns are the log differences of ISE National-100, and Volume is the trading volume at the end of trading day. V* is the square root of the absolute value of the logarithmic difference of Volume. Returns show negative skewness, but Volume shows positive skewness. Returns

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[4]These assumptions simplify the estimation, but will not alter the properties of the stable distribution. See, for example, Curto et al. (2009) and Tavares et al. (2008).

[5]I thank Professor Curto for providing Matlab codes to estimate the model.

[6]A Gaussian TGARCH(1,1) is given as: $\sigma_t^2 = \theta_0 + \theta_1 u_{t-1}^2 + \eta S_{t-1} u_{t-1}^2 + \phi_1 \sigma_{t-1}^2 + \gamma V$

indicate high leptokurtic patterns. According to the Jarque-Bera statistics, all variables indicate non-normal distributions.

Table 1: Summary Statistics

Statistics	ISE-100	Returns	Volume	V*
Mean	45093.36	0.000492	1466799	0.420372
Median	44385.71	0.001145	1324665	0.407111
Maximum	71543.26	0.121272	4428298	1.239977
Minimum	21228.27	-0.090137	139597.8	0.014917
Std. Dev.	12669.78	0.01865	678856.3	0.193991
Skewness	0.065815	-0.193392	0.882682	0.460197
Kurtosis	2.025815	5.804774	3.517904	3.167666
Jarque-Bera	73.12142	606.5711	256.1113	66.2263

Table 2 presents Gaussian, Student's t and Stable GARCH as benchmark models, where V is the absolute value of the logarithmic difference of Volume. The results show that the parameters are significant after adding the volume in the GARCH processes. The estimates of volume are also significant and they are positive implying that the trading volume contributes to the volatility of returns. The Stable GARCH models indicate that the shape parameters are significant and both are less than 2 based on t-tests indicating leptokurtic pattern. The Stable GARCH model shows higher log likelihood values compared to the Gaussian GARCH model and yields much smaller values of AIC and SBC. Student's t GARCH model indicates the same result compared to the Gaussian model. Both of them are preferable models based on the goodness of fits. These results are no surprise since the Stable and Student's t distributions take into account the non-normal distribution of the time series estimated. When Stable and Student's t GARCH models are compared, a minor difference is observed. According to AIC values, Student's t GARCH model slightly outperforms Stable GARCH model whereas SIC value makes us choose the Stable GARCH model over Student's tGARCH model.

Table 3 presents the estimation results of Gaussian, Student's t and Stable TGARCH models. The main findings are similar to those found by benchmark GARCH models. Stable and Student's t TGARCH models are compared and it is observed that AIC values are nearly the same. In addition, Stable TGARCH model outperforms Student's t TGARCH model according to SIC values.

The asymmetric parameters are positive and significant for all specifications. The results support strong leverage effects, where negative shocks have larger effect on volatility of ISE National-100 returns. TGARCH models indicate higher log likelihood values than those of the GARCH models.

Table 2: GARCH Models with Volume

	Gaussian GARCH	Student's t GARCH	Stable GARCH
	V	V	V*
Intercept	0.001291 (3.99E-04) [3.23]	0.001301 3.81E-04 [3.42]	0.0011065 (4.15E-04) [2.67]
θ_0	0.0000123 (5.55E-06) [2.22]	0.0000113 6.77E-06 [1.68]	0.0002617 (2.03E-04) [1.29]
θ_1	0.147589 (1.78E-02) [8.27]	0.129708 2.19E-02 [5.92]	0.072865 (1.11E-02) [6.58]
ϕ_1	0.767981 (2.72E-02) [28.19]	0.797765 3.27E-02 [24.38]	0.86111 (2.20E-02) [39.09]
γ	0.0000785 (2.35E-05) [3.34]	0.0000627 2.95E-05 [2.12]	0.0008978 (4.46E-04) [2.01]
α	-	-	1.8967 (3.18E-02) [59.60]
β	-	-	-0.38579 (2.22E-01) [-1.74]
Log Likelihood	4795	4812	4801
AIC	-9580	-9613	-9600
SBC	-9552	-9580	-9602

Standard errors are presented in parentheses, and t-statistics are in square brackets.

Table 3: TGARCH models with Volume

	Gaussian TGARCH	Student's t TGARCH	Stable TGARCH
	V	V	V*
Intercept	0.000597 (4.03E-04) [1.48]	0.000802 3.83E-04 [2.09]	0.0004584 (4.20E-04) [1.09]
θ_0	0.0000124 (6.57E-06) [1.88]	0.000013 0.00000783 [1.66]	0.0003576 (2.49E-04) [1.43]
θ_1	0.038937 (1.79E-02) [2.18]	0.038955 2.22E-02 [1.76]	0.02486 (1.33E-02) [1.87]
ϕ_1	0.717369 (2.97E-02) [24.12]	0.733816 3.58E-02 [20.5]	0.79702 (2.69E-02) [29.63]
η	0.227715 (3.34E-02) [6.82]	0.212705 4.04E-02 [5.26]	0.11022 (1.63E-02) [6.77]
γ	0.000142 (2.75E-05) [5.19]	0.000123 3.39E-05 [3.62]	0.0022208 (5.76E-04) [3.86]
α	-	-	1.9073 (3.36E-02) [56.82]
β	-	-	-0.38972 (2.27E-01) [-1.72]
Log Likelihood	4818	4829	4822
AIC	-9625	-9644	-9642
SBC	-9592	-9606	-9644

Standard errors are presented in parentheses, and t-statistics are in square brackets.

5 Conclusion

This paper examines volatility of stock returns and trading volume by employing the Student's t and Stable TGARCH models. The results suggest that these kinds of distributions clearly outperform the Gaussian case. However, when Stable and Student's t distributions are compared to each other, a wise conclusion should be avoided since SIC favors the Stable distribution while AIC points out the Student's t distribution. The results also indicate that the trading volume significantly contributes to the volatility, and indicate the strong leverage effects on volatility. The goodness of fit supports the use of TGARCH over GARCH specifications.

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