A parametric measure of productivity change from hyperbolic distance function: Application to the Vietnamese banking industry

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Abstract

This paper introduces a new parametric measure of productivity change using the hyperbolic distance function. More specifically, the paper first estimates a stochastic translog hyperbolic distance function, which satisfies regularity conditions, using the Bayesian approach. Then it derives an expression to measure scale efficiency from this estimated function. Finally, it shows measurements of three components of productivity change. The first component, technical efficiency change, is computed by a ratio of hyperbolic distances between two consecutive periods. The second component, technological change, is measured as a ratio of distances along a chosen input-output vector. The third component, scale efficiency change, is determined by a ratio of scale efficiency estimates between two consecutive periods. This new measure is then applied to investigate productivity change of the Vietnamese banking sector during the 2000–2006 period. The empirical results show that the Vietnamese banking industry experiences modest productivity growth. The thrust of this growth is technological progress, and to some degree technical change, whereas scale efficiency change contributes negatively to productivity growth. Moreover, foreign banks are found to have the highest productivity growth.

JEL classification numbers: C11, D24, G21

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Article Info: Received: June 22, 2012. Revised: August 4, 2012. Published online: October 15, 2012
Keywords: Bayesian approach, Hyperbolic distance function, Productivity change, Vietnamese banking sector

1 Introduction

Productivity is an important measure of bank performance, because an increase in productivity is expected to induce lower prices, improved service quality, and an enhancement of resource allocation and productivity of the whole economy. Moreover, productivity progress also improves safety and soundness of the banking system if this productivity improvement is directed towards strengthening capital buffers that absorb risk (Casu et al. 2004). Productivity varies due to differences in production technology, in the efficiency of the production process, and in the scale of operations. Thus, a change in productivity can be decomposed into three components: technical efficiency change, technological change, and scale efficiency change. In particular, technical efficiency change measures the capacity of a bank to improve its production position relative to the production frontier from one period to another. Scale efficiency change measures changes in the scale of operations of that bank relative to the most productive scale size over time. In the meantime, technological change captures the shift in the production frontier from period to period and reflects improvement or deterioration in the performance of best-practice banks.

Productivity change can be measured by various approaches, such as the Hicks-Moorsteen approach, the profitability ratio approach, and the Malmquist Productivity Index (MPI) approach. Among these, the MPI approach is widely used. The main feature of MPI is that it is based on distance functions, and thus does not require price information or behavioral assumptions; however, it requires calculation or estimation of a representation of production technology (Kumbhakar and Lovell 2000).

Two commonly used distance functions for the measurement of MPI are output and input distance functions. An input distance function seeks the maximum contraction in inputs to produce a given output bundle, whereas an output distance function seeks the maximum expansion in outputs that can be produced from provided inputs. These distance functions entail the restriction of keeping the opposite orientation unchanged: outputs unchanged in an input distance function, and inputs unchanged in an output distance function. Cuesta and Zofio (2005) argued that these restrictions could be unacceptable if a measure of productive and economic performance that takes into account the adjustability of both inputs and outputs is desired. In that sense, a more flexible distance function, which allows simultaneous change in inputs and outputs, should be considered. This paper utilizes the hyperbolic distance function, a function allows inputs and outputs to be adjusted at the same time along a hyperbolic path, to measure the magnitude of productivity change and its various components.

In particular, this paper specifies a stochastic hyperbolic distance frontier to
estimate pure technical efficiency level. To ensure that the estimated translog hyperbolic distance function is well-behaved, this paper imposes monotonicity conditions in the estimation process using the Bayesian approach. The paper then shows how, based on the estimated stochastic hyperbolic distance function, scale efficiency can be measured. Finally, it describes how productivity change and its components are computed.

This measure of productivity change is applied to investigate the productivity evolutions of banks in Vietnam during 2000–2006 and the main drivers of these changes. Like banking sectors in other developing countries, the banking sector in Vietnam has experienced substantial changes over recent decades, commencing with the transformation from a mono- to a two-tier banking system, followed by reform and restructuring of the national banking system, the removal of many restrictions on foreign banks, the enlargement of bank branch networks, increases in equity capital, and the integration of Vietnam into the global economy. The signing of the BTA with the US in 2000, and Vietnam becoming a member of the WTO in late 2006, marked the global integration of Vietnam. The studied period, 2000–2006 also witnessed substantial changes in banking technology, including the application of banking software to computerize transactions, the expansion of automatic teller machine (ATM) networks, the issuing of debit and credit cards, and the development of internet and electronic banking services. Therefore, it is crucial to examine productivity change of banks over this period. Furthermore, the use of a hyperbolic distance function to analyze performance of banks in Vietnam will enhance the appropriateness of the estimation. As discussed by Cuesta and Zofio (2005) if profit maximization is an appropriate behavioral assumption, the hyperbolic distance function is a natural choice since it is dual to a measure of profitability which is represented by ‘return to the dollar’.3 The main objective of banks in Vietnam practically is to maximize profits, even though some state-owned banks have some directed duties by the government. Thus the choice of hyperbolic distance function is the best suit.

In this paper, the following questions of concern will be clarified: whether the Vietnamese banking industry has experienced productivity progress; what is the major driver of productivity change; whether productivity change has responded positively to banking reforms; whether banks with foreign ownership have outperformed domestic banks; and finally what policy implications can be drawn to improve bank productivity.

This paper makes contributions to the literature in both theoretical and empirical dimensions. By introducing a new measure of productivity change and its various components based on the hyperbolic distance function, it diversifies

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3 In this context, profit efficiency is measured based on a profitability function represented by the revenue-to-cost ratio, and it can be decomposed into technical efficiency represented by the hyperbolic distance function and allocative efficiency as the remaining.
methodological choices for practical researchers. Also, by examining productivity features of banks in Vietnam, it fills in a gap of lacking productivity research in the Vietnamese banking sector, thus extending the archive of empirical studies for future reference. The remainder of the paper is organized as follows. The next section provides a review of bank productivity. Section 3 describes the methodology, and Section 4 defines bank inputs and outputs, and outlines practical implementation. Section 5 discusses the findings, and Section 6 concludes the paper.

2 Literature review

The first paper estimating productivity growth in banking using an econometric approach was that of Kim and Weiss (1989). They specified a parametric cost function with a time-trend variable to estimate productivity growth for Israeli banks during 1979–1982. The results showed that productivity grew at an annual average rate of 7.8%. Since then, a large proportion of productivity studies have concentrated on the US and European banking markets. Bauer et al. (1993) estimated productivity change in the US banking industry during the 1977–1988 period, using a cost frontier as a reference technology. They found a low level of productivity growth. This was due to higher costs of funding driven by high market rates, the removal of deposit rate ceilings, and increased competition from non-bank financial institutions.

Using the same data set as used by Bauer et al. (1993), Humphrey (1993) also found a negative cost productivity growth in US banks, with costs increasing at the average of 0.8% to 1.4% per year for alternative models. Again, this finding was attributed to deregulation of deposit rates, which benefited customers/depositors. The third study using the same data set as used by Bauer et al. (1993) was that of Humphrey and Pulley (1997). In contrast to the two earlier studies, they chose an alternative profit function as a benchmark to decompose productivity change into various components. They found that larger banks enjoyed a shift in the profit function and changes in business conditions, particularly following deposit deregulation. Tirtiroğlu et al. (1998) examined whether regulation had influenced productivity growth of commercial banks in the US during 1946–1995. They found that the overall impact of regulation on productivity was largely negative and the relationship between productivity growth and technological investment was very weak.

For European banking, Orea (2002) assessed productivity change in the Spanish banking sector during the period 1985–1998. His results suggested an increase in productivity for both merged and non-merged banks. The main factor for this growth was strong technological progress. Returns to scale also had a positive effect on productivity, implying that the scale effect should be considered when analyzing productivity growth of banks.

Rezitis (2008) examined the impact of mergers and acquisitions on total
factor productivity of the Greek banking industry over the 1993–2004 period. They specified a stochastic translog distance function. The findings demonstrated that banks participating in merging activities experienced a decrease in productivity, from about –3.7% for years before merger, to about –0.63% afterwards. Meanwhile, non-merging banks experienced an annual average productivity growth of 2.08%. The decrease in total factor productivity of merging banks can be interpreted as the result of a decline in technical efficiency and the disappearance of scale economies, while technological change remained constant.

Koutsomanoli-Filippaki (2009) investigated the productivity growth of the banking industry in 10 Central and Eastern European countries over the period 1998–2003, using a stochastic directional technology distance function approach. They parameterized the directional distance function via a quadratic functional form that permitted the imposition of the translation property (the additive analog of the homogeneity property). They reported that productivity for the whole region initially declined, but improved more recently with further progress of institutional and structural reforms. This productivity growth was driven by technological change rather than efficiency change. Furthermore, they found that foreign banks outperformed both domestic private and state-owned banks in terms of productivity gains.

Other studies addressing the issue of bank productivity using parametric approaches including those of Kumbhakar et al. (2001), Berger and Mester (2003), Casu et al. (2004), Kumbhakar and Lozano-Vivas (2005), Molyneux and Williams (2005), Tirtiroğlu et al. (2005), Kondeas et al. (2008), Olgu and Weyman-Jones (2008), Fiorentino et al. (2009), and Tanna (2009).

A few studies have been devoted to investigating productivity change in Asian banking. Among these, Gilbert and Wilson (1998) investigated the effects of privatization and deregulation on the productivity of Korean banks during 1980–1990. They found that Korean banks responded to privatization and deregulation by substantially altering their mix of inputs and outputs, yielding large changes in productivity. Park and Weber (2006) employed the directional distance function to evaluate Korean banks’ productivity change expressed in terms of the Luenberger productivity indicator. Observing that technological progress actually occurred in the Korean banking sector during the period studied, 1992–2002, they constructed an accumulative technology. They found that over the sample period, the Korean banking industry experienced productivity growth mainly due to technological progress, which offset a decline in efficiency.

Analyzing productivity change of Malaysian commercial banks, Abdul-Majid et al. (2009) showed that during 1996–2002, on average, Malaysian banks experienced moderate scale economies and annual productivity change of 2.68%. The primary driver of productivity growth was technological change, which declined over time. Merged banks were found to have higher input usage and lower productivity change, suggesting that bank mergers did not contribute positively to bank performance.

Sufian (2009) attempted to identify the sources of productivity change in
Chinese banks over the period 2000–2005. Their findings suggested that the Chinese banking sector exhibited productivity progress, attributed to increases in efficiency. Moreover, the productivity growth of state-owned banks was lower than that of joint stock banks, 0.2% compared to 1.3%.

Examining the Indonesian banking sector, Hadad et al. (2008) reported that the main driver of productivity growth for Indonesian banks over the period January 2006 to July 2007 was technological progress. They also suggested that a strategy based on the gradual adoption of advanced technology seemed to have the highest potential for boosting the productivity of Indonesian banks. Huang et al. (2008) employed the output distance DEA approach to investigate the productivity change of Taiwanese commercial banks between 2001 and 2004. They found that, on average, banks experienced an increase in productivity growth, which was mainly caused by technological progress.

In Vietnam, the only study of bank productivity is that of Hung (2007), who utilized an input-oriented DEA to measure the Malmquist Productivity Index of a sample of 13 Vietnamese banks over the period 2001–2003. His findings showed that total factor productivity increased by 5.7% in 2003 compared to 2001. This improvement was primarily due to increases in technical efficiency and, to some extent, technological advancement. Hung studied a small sample of banks over a short period (just 3 years). This limitation makes the present study worthwhile, since it employs a much larger sample (56 banks, including all types of ownership), and a longer period (7 years), in addition to the fact that it employs a methodology that appears to be superior.

Summing up, some noteworthy points can be drawn from the reviewed studies: (i) It seems that many studies have reported the improvement in the performance of best-practice banks as the main contributor to productivity growth. Meanwhile, findings as to the impact of deregulation on productivity growth were mixed; (ii) No study employed the hyperbolic distance function as the reference technology to measure productivity change and its decompositions; and (iii) There is real paucity in productivity research on the Vietnamese banking sector. This paper addresses some of the deficiencies pointed out above.

3 Methodology

3.1 Hyperbolic distance function

Consider an N-input and M-output production technology. Let \( x \in R^N \) denote a vector of inputs and \( y \in R^M \) a vector of outputs. The production technology is represented by the production set as

\[
T = \{(x,y) : x \text{ can produce } y\},
\]

with the assumption that \( T \) satisfies the traditional regularity conditions (i.e., \( T \) is
closed and convex with freely disposable inputs and outputs and no free lunch).

For a geometric illustration, consider a bank operating at point A using a single input \((x)\) to produce a single output \((y)\), as shown in Figure 1. By construction, the bank is technically inefficient as it lies below the frontier. If the bank seeks only the maximum increase in outputs with inputs unchanged, or the maximum decrease in inputs with outputs unchanged, it will move to point B or C respectively. Alternatively, the bank can seek the maximum input contraction and maximum output expansion simultaneously by moving along the hyperbolic path AD.

Algebraically, the hyperbolic distance function \(D_H\) is defined as

\[
D_H(x, y) = \min \left\{ \lambda : \left( \lambda x, \lambda^{-1} y \right) \in T \right\}.
\]

In words, provided \((x, y) \in T\), the smallest nonnegative scalar \(\lambda\), such that \((\lambda x, \lambda^{-1} y) \in T\) will be located on the frontier, is the hyperbolic distance. Under the assumption of weak free disposability, \(\lambda < 1\), and it measures the maximum equiproportionate contraction of all inputs and expansion of all outputs that remains technically feasible. Put another way, inputs will be scaled down by \(\lambda\) and outputs will be scaled up by \(1/\lambda\). If \(\lambda = 1\), the bank is technically efficient.

![Figure 1: Hyperbolic distance function](image)

The hyperbolic distance function is almost homogeneous of degree –1 and +1 in inputs and outputs respectively, and then +1 in itself (Färe et al. 1985), i.e.
Parametric measure of productivity

\[ D_H(\mu x, \mu^{-1} y) = \mu^{-1} D_H(x, y), \mu > 0. \]

In other words, if a vector of inputs and a vector of outputs are deflated and inflated respectively by the same proportion, then the hyperbolic function’s value will be inflated by the same proportion. Because of this property, the measurement of hyperbolic distances in the non-parametric context requires non-linear optimization techniques which are quite problematic. Thus, it is much easier to estimate hyperbolic distance in the parametric context. Furthermore, to optimize the behavior of the hyperbolic distance function, this paper assumes that \( D_H(x, y) \) satisfies the monotonicity constraints, i.e.

\[ D_H(\mu x, y) \leq D_H(x, y), \mu \geq 1, \text{ indicating that } D_H(x, y) \text{ is decreasing in inputs,} \]

and

\[ D_H(x, \mu y) \leq D_H(x, y), \mu \in [0,1], \text{ indicating that } D_H \text{ is increasing in outputs.} \]

Because

\[ D_H(x, y) \leq 1 \Leftrightarrow (x, y) \in T, \]

the hyperbolic distance can be considered as a measure of technical efficiency.

That means if a bank is technically efficient, or lies on the production frontier, \( D_H(x, y) = 1 \). Otherwise, \( D_H(x, y) < 1 \), indicating that the bank could improve its production performance by contracting its inputs by \( (1-D_H) \times 100 \) percent and at the same time expanding its outputs by \( (1/D_H-1) \times 100 \) percent.

3.2 Stochastic hyperbolic distance function and its Bayesian estimation

3.2.1 Stochastic Hyperbolic distance function

As first advanced by Cuesta and Zofio (2005), the translog hyperbolic distance function can be written for the case of M outputs and N inputs as

\[ F\text{äre et al. (1985) mentioned that the hyperbolic distance function is non-increasing in inputs and non-decreasing in outputs. That means that at some points the function can be flat or non-monotonic. Thus, to meet monotonicity conditions, this study imposes the strong assumption that the hyperbolic distance function is decreasing in inputs and increasing in outputs.} \]
\[
\ln D_{H}(x, y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_m + \frac{1}{2} \sum_{m=1}^{M} \sum_{k=1}^{M} \alpha_{mk} \ln y_m \ln y_k + \sum_{n=1}^{N} \beta_n \ln x_n \\
+ \frac{1}{2} \sum_{m=1}^{M} \sum_{j=1}^{N} \beta_{mj} \ln x_m \ln x_j + \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} \ln y_m \ln x_n 
\]

(1)

For simplicity, the observation subscript and the time subscript are dropped from (1). The frontier of the feasible input-output set corresponds to

\[\ln D_{H}(x, y) = 0, \quad \text{or} \quad D_{H}(x, y) = 1,\]

and the interior points correspond to

\[-\infty < \ln D_{H}(x, y) < 0, \quad \text{or} \quad 0 < D_{H}(x, y) < 1.\]

By symmetry:

\[\alpha_{mk} = \alpha_{km}, \quad \text{and} \quad \beta_{ml} = \beta_{lm}, \quad \text{for all} \quad m, k = 1, 2, ..., M \quad \text{and} \quad n, l = 1, 2, ..., N.\]

The conditions for ensuring that the hyperbolic distance function defined by (1) is almost homogeneous in inputs and outputs of order –1 and +1 respectively are:

\[\sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n = 1; \quad \sum_{k=1}^{M} \alpha_{mk} - \sum_{n=1}^{N} \delta_{mn} = 0, \quad \text{with} \quad m = 1, 2, ..., M;\]

\[\sum_{m=1}^{M} \delta_{mn} - \sum_{l=1}^{N} \beta_{nl} = 0 \quad \text{with} \quad n = 1, 2, ..., N.\]

To meet these homogeneity conditions, the translog hyperbolic distance function defined by (1) is transformed into a normalized function. This paper arbitrarily chooses the last output ($M^{th}$) as a normalizing variable; thus when the almost homogeneity condition is imposed, the translog hyperbolic distance function can be written as:

\[
\ln(D_{H}/y_M) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y^*_m + \frac{1}{2} \sum_{m=1}^{M} \sum_{k=1}^{M-1} \alpha_{mk} \ln y^*_m \ln y^*_k + \sum_{n=1}^{N} \beta_n \ln x^*_n \\
+ \frac{1}{2} \sum_{n=1}^{N} \sum_{l=1}^{N} \beta_{nl} \ln x^*_n \ln x^*_l + \sum_{m=1}^{M-1} \sum_{n=1}^{N} \delta_{mn} \ln y^*_m \ln x^*_n 
\]

(2)

where $y^*_m = y_m/y_M$ with $m = 1, 2, ..., M - 1$; $x^*_n = x_n y_M$ with $n = 1, 2, ..., N$. The summations related to $y^*_m$ are over $M - 1$ because when $y_m = y_M$, $y_m/y_M = 1$, then $\ln(y_m/y_M) = 0$.

Denote the right hand side of (2) as $TL(x^*, y^*; \theta)$, with $\theta$ as a vector of unknown parameters. Then (2) is equivalent to

\[-\ln y_M = TL(x^*, y^*; \theta) - \ln D_{H},\]

(3)
A stochastic frontier model is formed by adding a random noise term, \( v \), to (3) and set the inefficiency term \( u = -\ln D_{ij} \). Hence, for a panel data set of \( I \) banks \((i = 1, 2, \ldots, I)\) and \( T \) periods \((t = 1, 2, \ldots, T)\), the stochastic translog hyperbolic distance function is given by

\[
\ln y_{M_{it}} = TL(x^*_{it}, y^*_{it}; \Theta) + v_{it} + u_{it} = TL(x^*_{it}, y^*_{it}; \Theta) + \varepsilon_{it}
\]  

(4)

where \( \varepsilon_{it} = v_{it} + u_{it} \) is the composite error term; the random noise component \( v_{it} \) is assumed to be identically independently distributed as normal with mean zero and constant variance, \( v_{it} \sim i.i.d N(0, \sigma_{v}^2) \). For the ease of many technical derivations, the precision rather than the variance is used. Thus, \( v_{it} \sim i.i.d N(0, h^{-1}) \), where \( h = 1/\sigma_{v}^2 \). The inefficiency term follows a non-negative distribution (e.g., a half-normal, exponential, truncated, or gamma distribution). In this paper, \( u_{it} \) is assumed to be exponentially distributed with mean \( 1/\phi \) (where \( \phi \) is a parameter) and variance \( 1/\phi^2 \), \( u_{it} \sim i.i.d \ Exp(\phi) \). The \( v_{it} \)s and \( u_{it} \)s are further assumed to be distributed independently of each other.

A concerning issue when estimating the stochastic frontier model as specified in (4) is the endogeneity problem. As discussed by Cuesta and Zofio (2005), the condition of degree one homogeneity in inputs and outputs implies that the error term equally affects all the inputs and outputs, thus the ratios and products regressors can be considered as exogenous. In addition, provided the assumption of profit maximization behavior, outputs themselves can be considered as endogenous.

The stochastic frontier model (4) can be estimated by the maximum likelihood estimation which relies only on the observed data and parameters, or by the Bayesian estimation. The former does not take into account prior information such as prior belief on parameters and economic constraints for functions, whereas the latter does. Thus, the Bayesian estimation is deemed to produce more appropriate estimates for parameters and efficiency scores.

### 3.2.2 Bayesian estimation and monotonicity constraints

The Bayesian estimation can combine information from two sources: prior information, which describes the researcher’s beliefs about the parameters of interest, and the observed information contained in the data. The information from these two sources is combined, using the Bayes theorem, to produce posterior distributions of the parameters on which the statistical inferences of interest are formed. The advantages of the Bayesian approach are thus exact finite sample inference as to efficiencies, easy incorporation of prior ideas and restrictions such as regularity conditions, and formal treatment of parameter and model uncertainty (Griffin and Steel 2007).

For Bayesian estimation, beside the distributional assumptions for the
random noise term and the inefficiency term as presented above, prior distributions for the unknown parameters are also required. Especially, specification of the prior distribution (usually the prior mean and variance) is important in Bayesian inference as it affects the posterior inference. Prior distributions are classified into two main groups: non-informative and informative. A non-informative prior conveys ignorance, and normally assumes that the parameters are all independently distributed with equal probability. In the case of no prior information, it is necessary to specify a prior distribution that will not influence the posterior distribution and will “let the data speak for themselves” (Ntzoufras 2009). In contrast, an informative prior, as its name suggests, contains information about the parameters.

In the present study, the parameters introduced in (4) are assigned uninformative prior distributions with large variance to display ignorance, i.e., a multivariate normal $\theta \sim N(0, \Sigma)$, where 0 is a vector of zero means, and $\Sigma$ is a positive definite variance-covariance matrix with large elements. The prior distribution for $\phi$ is assumed to be exponential distribution with mean $(-1/\ln r^*)$, where $r^*$ is a prior median efficiency. The value of $r^*$ varies from study to study depending on the efficiency expectation. Following the common literature, this paper sets $r^* = 0.85$. The variance parameter, $h$, is granted a gamma distribution, with small values for scale and shape parameters, $h \sim \text{Gamma}(0.001, 0.001)$.

Furthermore, to allow time-varying efficiency, this paper utilizes the specification proposed by Battese and Coelli (1992),

$$u_i = \exp\left\{-\eta(t-T)\right\}u_i,$$

where $\eta$ is an unknown scalar parameter, $t$ represents a present period, and $T$ is the total number of periods (in this study $T=7$). A positive or negative $\eta$ indicates an increase (with a decreasing rate) or a decrease (with an increasing rate) in technical efficiency over time. Following Griffin and Steel (2007), $\eta$ is assumed to be normally distributed with zero mean and variance 0.25, which represents the prior indifference between increasing and decreasing efficiency.

In addition, as mentioned earlier, this paper assumes that the hyperbolic distance function is increasing in outputs and decreasing in inputs. That means

$$\frac{\partial D_H(x, y)}{\partial x_n} = \frac{\partial \ln D_H(x, y)}{\partial \ln x_n} \times \frac{D_H(x, y)}{x_n} = e^{\gamma_x} \times \frac{D_H(x, y)}{x_n} < 0 \iff e^{\gamma_x} < 0,$$

$$\frac{\partial D_H(x, y)}{\partial y_m} = \frac{\partial \ln D_H(x, y)}{\partial \ln y_m} \times \frac{D_H(x, y)}{y_m} = e^{\gamma_y} \times \frac{D_H(x, y)}{y_m} > 0 \iff e^{\gamma_y} > 0$$

5 The prior mean provides a prior point estimate for the parameter of interest while the variance expresses our uncertainty concerning this estimate (Ntzoufras, 2009).
where \( e^{i_n} = \frac{\partial \ln D_{n}(x, y)}{\partial \ln x_n} \) and \( e^{o_m} = \frac{\partial \ln D_{m}(x, y)}{\partial \ln y_m} \) are the elasticities of the hyperbolic distance function with respect to the \( n^{th} \) input and the \( m^{th} \) output respectively.

The two constraints (5) and (6) are incorporated into the estimation of the hyperbolic distance function through an index variable \( I(\theta) \), specified such that \( I(\theta) \) equals 1 if the estimated frontier meets the monotonicity conditions, and zero otherwise.

For the computational implementation, this paper employs the Markov Chain Monte Carlo (MCMC) with a Gibbs sampler algorithm, and runs the simulation in WinBUGS. To overcome high correlation among the draws, to avoid sensitivity of the initial values, and to increase convergence, this paper simulates 500,000 iterations, drops the first 100,000 as a burn-in, and uses the remainder to produce averages of the posterior means, standard deviations, and 95% coverage regions.

3.2.3 A measure of scale efficiency from the estimated hyperbolic distance function

Ray (1998) derived expressions for output- and input-oriented measures of scale efficiency which can be computed from a fitted translog frontier for the case of single output. Then Ray (2003) extended his previous work for the case of multiple inputs and multiple outputs. Based on Ray’s work, this paper derives an expression for calculating scale efficiency from the estimated hyperbolic distance function.

As a simple illustration, consider a production function using a single input \( (x) \) and a single output \( (y) \) such that \( y = f(x) \). Suppose that this production function is concave and twice differentiable continuous. Then, the scale elasticity at any input level \( x \) is

\[
\xi(x, y) = \frac{d \ln f(x)}{d \ln x} = \frac{d f(x)}{d x} \times \frac{x}{f(x)}.
\]

The average productivity (\( AP \)) is determined as the ratio of output quantity and input quantity

\[
AP = \frac{y}{x} = \frac{f(x)}{x}.
\]

The average productivity is maximized at a most productive scale size (MPSS) point. Denote \( (x^{e}, y^{e}) \) a MPSS point. Then, a necessary condition for a maximizing average productivity is that the first derivative of \( AP \) with respect to \( x^{e} \) equals zero:
\[ \frac{dAP}{dx} \bigg|_{x=x'} = 0. \]  

(7)

Condition (7) is equivalent to

\[ \frac{df(x^c)}{dx^c} \times \frac{x^c}{f(x^c)} = \frac{d \ln f(x^c)}{d \ln x^c} = 1, \quad \text{or} \quad \xi(x^c, y^c) = 1 \]

where \( \xi(x^c, y^c) = \frac{d \ln f(x^c)}{d \ln x^c} \) is the scale elasticity at MPSS input level \( x^c \).

In other words, at a MPSS point, the scale elasticity is equal to 1, and the average productivity is maximum (given the second-order condition for maximization is satisfied).

Then, scale efficiency (SE) can be defined as the ratio of the actual average productivity to the maximum average productivity obtained at the MPSS (Ray, 1998),

\[ SE = \frac{AP(x)}{AP(x')} \]

To extend the work for the case of multiple inputs and multiple outputs, a method for aggregating inputs and outputs is needed. Following Ray (1998), this paper expresses inputs and outputs by means of a composite input and a composite output respectively. So, let \( k_1x \) represent \( k_1 \) units of the composite input, and \( k_2y \) represent \( k_2 \) units of the composite output.

In Figure 2, \( T_{VRS} \) and \( T_{CRS} \) represent the production frontiers under the variable returns to scale (VRS) and constant returns to scale (CRS) technology respectively. If a bank at point \( A(x, y) \) (where \( x \) and \( y \) are a composite input and output respectively) is technically inefficient, by construction, then \( D_n(x, y) < 1 \) as it lies underneath the frontier. After a simultaneous contraction of inputs and an expansion of outputs by the same proportion, this bank moves to point B, a technically efficient projection, on the VRS frontier. At B, the combination of composite input and output is \( (x^T, y^T) \), in which \( x^T = \mu x \), \( y^T = y / \mu \), and \( D_n(x^T, y^T) = 1 \). Point B is technically efficient but may not be a MPSS. In Figure 2, the MPSS occurs at point C where the CRS frontier is tangent to the VRS frontier. To find inputs and outputs at the MPSS, this study assumes that there are optimum constant scalars \( k_1^* \) and \( k_2^* \) to make a feasible movement along the VRS frontier from point B to point C such that:

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6 It is easy to show that the sufficient condition for maximization \( \frac{d^2 AP}{dx^2} \bigg|_{x=x'} < 0 \) is also satisfied.
(i) \((x^C, y^C) = (k_1^* x^T, k_2^* y^T) = (k_1^* \mu x, k_2^* y / \mu)\),

(ii) \(D_{H}(x^C, y^C; VRS) = D_{H}(x^C, y^C; CRS) = 1\), and

(iii) the average productivity at C is maximum.

Then, a measure of scale efficiency is defined as the ratio of the average productivity at point B and the maximum average productivity at C,

\[
SE = \frac{y^T / x^T}{y^C / x^C} = \frac{y^T / x^T}{k_2^* y^T / k_1^* x^T} = \frac{k_1^*}{k_2^*}.
\]  

(8)

Figure 2: Scale efficiency

Next, the paper shows how to solve for \(k_1^*\) and \(k_2^*\) in terms of the elasticities of the hyperbolic distance function with respect to inputs and outputs. At the MPSS point

\((x^C, y^C) = (k_1^* \mu x, k_2^* y / \mu)\),

the translog hyperbolic distance function can be written as
\[ \ln D_{ij}(x^C, y^C) = \alpha_o + \sum_{m=1}^{M} \alpha_m (\ln y_m + \ln k_2^* - \ln \mu) + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} (\ln y_m + \ln k_2^* - \ln \mu)(\ln y_n + \ln k_2^* - \ln \mu) + \sum_{n=1}^{N} \beta_n (\ln x_n + \ln k_1^* + \ln \mu) + \frac{1}{2} \sum_{n=1}^{N} \sum_{j=1}^{N} \beta_{nj} (\ln x_n + \ln k_1^* + \ln \mu)(\ln x_j + \ln k_1^* + \ln \mu) + \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} (\ln y_m + \ln k_2^* - \ln \mu)(\ln x_n + \ln k_1^* + \ln \mu) \]

Equation (9) can be reduced to

\[ 0 = \ln k_2^* \varepsilon^y(x, y) + \ln k_1^* \varepsilon^x(x, y) + \frac{1}{2} \Delta \left[ \ln k_1^* + \ln k_2^* \right]^2. \]  

where

\[ \varepsilon^y(x, y) = \sum_{m=1}^{M} \varepsilon^y_{x_m}; \quad \varepsilon^x(x, y) = \sum_{n=1}^{N} \varepsilon^x_{y_n}; \quad \text{and} \quad \Delta = \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} = \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} = \sum_{n=1}^{N} \sum_{j=1}^{N} \beta_{nj}. \]

Another equation beside (10) is needed to solve for \( k_1^* \) and \( k_2^* \). For this purpose, it is worth noting that in the case of multiple inputs and multiple outputs, the scale elasticity at a given point is determined as the negative of the sum of input elasticities over the sum to output elasticities (see Färe and Primont 1995), i.e.

\[ \xi(x, y) = -\frac{\varepsilon^x(x, y)}{\varepsilon^y(x, y)}. \]

Therefore, the scale elasticity at point C \((x^C, y^C)\) can be computed as

\[ \xi(x^C, y^C) = -\frac{\varepsilon^x(x^C, y^C)}{\varepsilon^y(x^C, y^C)} = 1 \quad \text{(as point C is also a MPSS point)} \]

where

\[ \varepsilon^x(x^C, y^C) = \varepsilon^x(x, y) + \Delta (\ln k_1^* + \ln k_2^*), \quad \text{and} \quad \varepsilon^y(x^C, y^C) = \varepsilon^y(x, y) + \Delta (\ln k_1^* + \ln k_2^*). \]

Thus,

\[ \xi(x^C, y^C) = -\frac{\varepsilon^x(x^C, y^C)}{\varepsilon^y(x^C, y^C)} = -\frac{\varepsilon^x(x, y) + \Delta (\ln k_1^* + \ln k_2^*)}{\varepsilon^y(x, y) + \Delta (\ln k_1^* + \ln k_2^*)} = 1. \]

Or,

\[ \ln k_2^* + \ln k_1^* = -\frac{\varepsilon^x(x, y) + \varepsilon^y(x, y)}{2\Delta}. \]  

(11)
Combine (10) and (11) to provide
\[
\ln k_2^* = \frac{\varepsilon^x(x, y) + \varepsilon^y(x, y) + 3\varepsilon^x(x, y) - \varepsilon^x(x, y)}{2\Delta} \times \frac{3\varepsilon^y(x, y) - \varepsilon^x(x, y)}{4},
\]
and
\[
\ln k_i^* = -\frac{\varepsilon^x(x, y) + \varepsilon^y(x, y) + 3\varepsilon^y(x, y) - \varepsilon^x(x, y)}{2\Delta} \times \frac{3\varepsilon^y(x, y) - \varepsilon^x(x, y)}{4}.
\]
So,
\[
k_1^* = \exp \left[ -\frac{\{\varepsilon^y(x, y) + \varepsilon^x(x, y)\}^2}{4\Delta} \right].
\]

In summary, a measure of scale efficiency from an estimated translog hyperbolic distance function can take the form
\[
SE = \frac{k_1^*}{k_2^*} = \exp \left[ -\frac{\{\varepsilon^y(x, y) + \varepsilon^x(x, y)\}^2}{4\Delta} \right]. \tag{12}
\]

As demonstrated earlier, if \((x, y)\) is an MPSS,
\[
\varepsilon^y(x, y) + \varepsilon^x(x, y) = 0,
\]
scale efficiency is 1. Otherwise, scale efficiency lies between 0 and 1 as long as \(\Delta > 0\). Practically, to guarantee that scale efficiency is not greater than 1, this study imposes the restriction that \(\Delta > 0\) into the Bayesian estimation.

### 3.3 Productivity change and its components

The translog hyperbolic distance function specified in (1) can be used to calculate technical efficiency change and scale efficiency change. However, to capture technological change, the function needs to include a time trend variable. Thus, (1) is extended to
\[
\ln D_H(x, y, z, t) = \alpha_n + \sum_{m=1}^{M} \alpha_m \ln y_m + \sum_{k=1}^{M} \sum_{m=1}^{M} \alpha_{mk} \ln y_m \ln y_k + \sum_{n=1}^{N} \beta_n \ln x_n + \frac{1}{2} \sum_{n=1}^{N} \sum_{j=1}^{N} \beta_{nj} \ln x_n \ln x_j + \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} \ln y_m \ln x_n + \phi t + \frac{1}{2} \phi t^2
\]
\[
+ \sum_{n=1}^{N} \gamma_{2+n} \ln x_n + \sum_{m=1}^{M} \phi_{2+n} \ln y_m + \gamma_1 \ln z + \frac{1}{2} \gamma_2 (\ln z)^2
\]
\[
+ \sum_{n=1}^{N} \gamma_{2+n} \ln z \ln x_n + \sum_{m=1}^{M} \gamma_{2+n} \ln y_m \ln y_m \tag{13}
\]
where $z$ is the level of equity capital and treated as a fixed input to capture risk preference; $t$ is a time trend representing technological change and appears in three different forms: (i) standalone in a first and second order; (ii) cross products with inputs; and (iii) cross products with outputs. Technological change over time will depend on all these terms. $t$ in (13) represents period-$t$ technology $T'_t$, and it will be replaced by $s$ to represent period-$s$ technology $T'_s$.

### 3.3.1 Technical efficiency change

Technical efficiency change (TEFFCH) indicates the capacity of individual banks to improve technical efficiency from period $s$ to period $t$. If technical efficiencies in periods $s$ and $t$ are defined as

$$TE_s = D_H(x_s, y_s, T'_s) = \exp[\ln D_H(x_s, y_s, T'_s)] = \exp(-u_s),$$

and

$$TE_t = D_H(x_t, y_t, T'_t) = \exp[\ln D_H(x_t, y_t, T'_t)] = \exp(-u_t),$$

then TEFFCH is computed by

$$TEFFCH = \frac{TE_t}{TE_s} = \frac{D_H(x_t, y_t, T'_t)}{D_H(x_s, y_s, T'_s)} = \exp(-u_t + u_s). \quad (14)$$

### 3.3.2 Technological change index

Technological change captures the shift in technology between two periods with regard to the actual best practice frontiers. Technological change index (TECHCH) between period $t$ and period $s$ can be redefined by means of hyperbolic distances as

$$\text{TECHCH} = \left[ \frac{D_H(x_t, y_t, T'_t)}{D_H(x_s, y_s, T'_s)} \times \frac{D_H(x_s, y_s, T'_s)}{D_H(x_t, y_t, T'_t)} \right]^{1/2}$$

where the first ratio is technological change index with the period-$t$ data, and the second with the period-$s$ data.

Based on (13), technological change with the period-$s$ data is given by

$$\text{TECHCH}(x_s, y_s) = D_H(x_s, y_s, T'_s)/D_H(x_t, y_t, T'_t)$$

$$= \exp[\ln D_H(x_s, y_s, T'_s) - \ln D_H(x_t, y_t, T'_t)]$$

$$= \exp \left[ \phi_1(s-t) + \frac{1}{2} \phi_2(s^2-t^2) + \sum_{n=1}^{N} \phi_{2+n}(s-t) \ln x_{n,s} + \sum_{m=1}^{M} \phi_{2+m+n}(s-t) \ln y_{m,s} \right] \quad (15)$$

Assume $s$ and $t$ are two consecutive years, i.e., $t - s = 1$, then (15) reduces to
\[ \text{TECHCH}(x_s, y_s) = \exp \left[ - \left( \phi_1 + \frac{1}{2} \phi_2 (s + t) + \sum_{n=1}^{N} \phi_{2,n} \ln x_{n,s} + \sum_{m=1}^{M} \phi_{2,n+m} \ln y_{m,s} \right) \right]. \quad (16) \]

In a similar way, the technological change index with the period-\(t\) data is computed by

\[ \text{TECHCH}(x_t, y_t) = \exp \left[ - \left( \phi_1 + \frac{1}{2} \phi_2 (s + t) + \sum_{n=1}^{N} \phi_{2,n} (\ln x_{n,t} + \ln x_{n,s}) + \sum_{m=1}^{M} \phi_{2,n+m} (\ln y_{m,t} + \ln y_{m,s}) \right) \right]. \quad (17) \]

From (16) and (17), the technological change index between period \(s\) and period \(t\) is defined by

\[ \text{TECHCH} = \left[ \text{TECHCH}(x_s, y_s) \times \text{TECHCH}(x_t, y_t) \right]^{1/2} \]

\[ = \exp \left[ -0.5 \times \left\{ 2\phi_1 + \phi_2 (s + t) + \sum_{n=1}^{N} \phi_{2,n} (\ln x_{n,t} + \ln x_{n,s}) + \sum_{m=1}^{M} \phi_{2,n+m} (\ln y_{m,t} + \ln y_{m,s}) \right\} \right] \]

\[ = \exp \left[ -0.5 \times \left\{ \frac{\partial \ln D_H(x_s, y_s, T^s)}{\partial s} + \frac{\partial \ln D_H(x_t, y_t, T^t)}{\partial t} \right\} \right]. \quad (18) \]

3.3.3 Scale efficiency change

As derived earlier, the scale efficiency of individual banks in a certain period is measured by

\[ SE = \frac{k_s^*}{k_t^*} = \exp \left[ - \frac{\{ e^x(x, y) + e^y(x, y) \}^2}{4\Delta} \right]. \]

Like technical efficiency change, scale efficiency change (SEFFCH) is the ratio of two consecutive scale efficiencies corresponding to period \(s\) and period \(t\),

\[ \text{SEFFCH} = \frac{SE_t}{SE_s}. \quad (19) \]

Finally, the productivity change index (PROD) is obtained as the product of the three components shown in (14), (18) and (19):

\[ \text{PROD} = \text{TEFFCH} \times \text{TECHCH} \times \text{SEFFCH}. \]

4 Data selection and empirical implementation

4.1 Data selection

As the banking industry is a service industry, there are a number of ways to define inputs and outputs of banks. The \textit{intermediation approach} assumes that
banks collect deposits to transform them, using labor and capital, into loans and other assets. Hence, banks are considered financial intermediaries connecting savers and investors. The production approach views banks as producers, using labor and capital to produce deposits and loans in terms of the number of accounts. The value added approach which states that all liabilities and assets of banks have some output characteristics, rather than categorizing them as either inputs or outputs only. In empirical research, the intermediation approach seems to be preferred as it better represents the role of banking in providing financial services (Altunbas et al. 2001; Berger and Mester 1997; Resti 1997; Maudos et al. 2002; Färe et al. 2004; Koetter 2006).

According to Vietnam’s Law on Credit Institutions, commercial banks are a type of credit institution, which are established to conduct monetary business, provide payment services, and provide banking services in the form of receiving deposits and using those deposits to extend credits. Therefore, in accordance with this law and following the approach of the majority of studies in the literature, this study employs the intermediation approach to define outputs and inputs of banks in Vietnam.

Regarding outputs, as commonly realized, two traditional outputs are customer loans ($y_1$) and other earning assets ($y_2$), including balances with credit institutions, securities and equity investments. It is important to point out that non-performing loans (NPLs) are extracted from customer loans to reflect the exact amount of outstanding loans. The study further considers off-balance sheet (OBS) items as a third output ($y_3$) because recent literature (Altunbas and Chakravarty 2001; Berger and Mester 1997; Rogers 1998) recognized that not considering OBS activities would create a biased assessment of banks’ performance. There are some common proxies for OBS activities, including credit equivalents, asset equivalents, as well as measures of non-interest income. Each proxy is measured by different methods and contains different limitations, but they have a common feature in that they do not reflect the actual value of OBS activities. In this study, instead of using a proxy, we directly choose the actual volume of OBS activities as a third output. Three measures of banking inputs include the number of full-time equivalent employees ($x_1$), physical capital ($x_2$) measured by the amount of fixed assets, and total borrowed funds ($x_3$), which includes customer deposits and balances with other credit institutions. Furthermore, to capture the effect of risk propensity on efficiency, adopting the common approach in the literature, equity capital (denoted as $z$) is treated throughout this thesis as a fixed input. The definitions of all the variables mentioned above are presented in Table 1.

The sample studied comprises 56 banks operating in Vietnam over the period 2000–2006. These banks can be divided into three types of ownership according to the State Bank of Vietnam’s classification:

i. State-owned commercial banks (SOCBs), which are entirely owned by the government or state sector;
ii. Joint stock commercial banks (JSCBs), which are jointly owned by both the public and private sectors. They are organized as joint-stock companies, which have partnerships limited by shares;

iii. Foreign banks (FBs), including joint venture banks which are 50% owned by foreign banks and 50% owned by a state-owned bank, and foreign bank branches, which belong to overseas-headquartered foreign banks.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outputs</td>
<td></td>
</tr>
<tr>
<td>( y_1 )</td>
<td>Customer loans (excluding nonperforming loans)</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>Other earning assets (balances with credit institutions, securities, and equity investments)</td>
</tr>
<tr>
<td>( y_3 )</td>
<td>Actual value of off-balance sheet items</td>
</tr>
<tr>
<td>Inputs</td>
<td></td>
</tr>
<tr>
<td>( x_1 )</td>
<td>Number of full-time equivalent employees</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>Fixed assets</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>Total borrowed funds including customer deposits, and balances with credit institutions</td>
</tr>
<tr>
<td>( z )</td>
<td>Equity capital</td>
</tr>
</tbody>
</table>

Accordingly, the observed sample consists of 5 SOCBs, 30 JSCBs, and 21 FBs. Among the SOCBs, four are very large and are involved in all aspects of banking with national branch networks. JSCBs concentrate on providing universal banking services in their particular regions, although some maintain networks of branches that allow them to operate on a multi-regional or national basis. The domestic asset mix of JSCBs is broadly similar to that of SOCBs, but much lower in terms of magnitude. Foreign banks’ business is mainly geared to wholesale activities with a limited customer base and transaction points. Altogether, these 56 banks account for more than 95% of the Vietnamese banking industry’s assets. Table 2 displays the means and standard deviations of all variables for each group of banks and for all banks. The means and standard deviations of all outputs and inputs indicate a significant difference across banks and across bank categories. More specifically, output and input portfolios of SOCBs are much larger than those of JSCBs and FBs, but the differences in output and input composition of JSCBs and FBs are not substantial. These figures support the fact SOCBs dominate the banking industry in Vietnam.
Table 2: Means and standard deviations of variables by bank category

<table>
<thead>
<tr>
<th>Variable</th>
<th>SOCBs</th>
<th>JSCBs</th>
<th>FBs</th>
<th>All banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std</td>
<td>Mean</td>
<td>Std</td>
</tr>
<tr>
<td>( y_1 )</td>
<td>46,700.0</td>
<td>35,300.0</td>
<td>1,337.2</td>
<td>1,914.1</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>16,200.0</td>
<td>16,000.0</td>
<td>726.9</td>
<td>1,548.6</td>
</tr>
<tr>
<td>( y_3 )</td>
<td>5,087.2</td>
<td>4,615.9</td>
<td>341.4</td>
<td>636.3</td>
</tr>
<tr>
<td>( x_1 ) *</td>
<td>10,841</td>
<td>9,248</td>
<td>362</td>
<td>500</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>980.8</td>
<td>738.4</td>
<td>47.3</td>
<td>71.7</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>67,800.0</td>
<td>42,500.0</td>
<td>2,204.3</td>
<td>3,552.9</td>
</tr>
<tr>
<td>( z )</td>
<td>3,695.0</td>
<td>2,190.7</td>
<td>215.4</td>
<td>281.1</td>
</tr>
</tbody>
</table>

Note: All variables are measured in billion dong (VND), except * measured in the number of staff

4.2 Empirical implementation

A normalized form (by the last output) of the translog hyperbolic distance function for the case of three inputs, three outputs, a fixed input, and a time variable can be written as:

\[-\ln y_{3it} = \alpha_0 + \sum_{m=1}^{2} \alpha_m \ln y^*_{mit} + \frac{1}{2} \sum_{m=1}^{2} \sum_{k=1}^{2} \alpha_{mk} \ln y^*_{mit} \ln y^*_{kit} + \sum_{n=1}^{3} \beta_n \ln x^*_{nit} + \frac{1}{2} \sum_{n=1}^{3} \sum_{l=1}^{3} \beta_{nl} \ln x^*_{nit} \ln x^*_{nit} + \frac{1}{2} \sum_{m=1}^{3} \sum_{n=1}^{3} \delta_{mn} \ln y^*_{mit} \ln x^*_{nit} + \phi t + \frac{1}{2} \phi^2 t^2 + \sum_{n=1}^{3} \phi_{2+n} t \ln x^*_{nit} + \sum_{n=1}^{3} \phi_{5+n} t \ln y^*_{mit} + \chi_1 \ln z_{it} + \frac{1}{2} \gamma_2 (\ln z_{it})^2 + \sum_{n=1}^{3} \gamma_{2+n} \ln z_{it} \ln x^*_{nit} + \sum_{n=1}^{3} \gamma_{5+n} \ln z_{it} \ln y^*_{mit} + v_{it} + u_{it}\]

where

\( y^*_{mit} = y_{mit} / y_{3it} \) with \( m = 1,2 \); \( x^*_{nit} = x_{nit} / y_{3it} \) with \( n = 1,2,3 \);

\( u_{it} = -\ln D_{ita} \) and \( u_{it} = \exp \{-\eta (t-T)\} u_{it}, t = 1, ..., T, \ (T = 7) \) to allow time-varying efficiency.

It might be noteworthy that, as discussed earlier about the equal effect of the error term on the outputs and inputs, the choice of an output as the numeraire would not result in any significant difference in the estimation of efficiency.

This model is estimated by the Bayesian approach with the following prior distributional assumptions and monotonicity constraints:

\( v_{it} \sim \text{iidN}(0, h^{-1}) \); \( u_{it} \sim \text{i.dExp}(\phi) \); \( \phi \sim \text{Exp}(-1 / \ln r^*) \), where \( r^* = 0.85 \);
\( \theta \sim N(0, \Sigma) ; \quad h \sim Gamma(0.001, 0.001); \quad \eta \sim N(0, 0.25). \)

\( \varepsilon'_{\theta} < 0 ; \quad \varepsilon'_{\eta} > 0. \)

Regard to technological change, technological regress is not an unusual phenomenon, but its presence may cause some concerns and questions in some periods of the evolution of some industries, particularly service industries. The banking sector is a service and high-technology-applied industry. For customers’ convenience and bank efficiency, banks must upgrade banking technology and offer more electronic services. Moreover, empirical research on banking industries in general, and in Asia particularly, shows that technological progress has actually occurred. For example, Abdul-Majid et al. (2009) found that the Malaysian banking industry experienced significant technological progress over the period 1996–2002. Hadad et al. (2008) suggested that the main driver of productivity growth in Indonesian banks is technological progress. Huang et al. (2008) found that Taiwanese commercial banks experienced technical progress over the period of 2001–2004.

The level of technology in the banking industry in Vietnam has been increasing in recent years. For example, the number of banks offering internet banking increased from three in 2004 to 17 in 2007. There were 3,820 ATMs in 2007 compared to 800 in 2004. The numbers of electronic cards in 2004 and 2007 were 1.9 million and 6.2 million respectively. This paper therefore assumes that over the studied period the Vietnamese banking experienced technological progress. To satisfy this assumption, the following condition is added to the estimation process:

\[ \varepsilon' \leq 0 , \]

where \( \varepsilon' \) is the elasticity of the hyperbolic distance function with respect to time \( t \).

5 Emprical findings
5.1 Monotonicity conditions for the hyperbolic distance function

To illustrate the effects of imposing monotonicity conditions in the stochastic hyperbolic frontier, the study estimates an unconstrained model (no monotonicity conditions imposed) and checks whether its input and output elasticities have correct signs at the sample mean and at each sample data point. As shown in Table 3, under the unconstrained model, most of the input and output elasticities have correct signs at the sample mean and at each sample data point. As shown in Table 3, under the unconstrained model, most of the input and output elasticities have correct signs at the sample mean and at each sample data point. At the sample data points there are no violations for the third input (deposits) and the first output (loans). However, monotonicity conditions are not satisfied at many data points for other factors. For example, 309 (out of 392) bank observations have an incorrect sign for elasticity with respect to fixed assets, and 94 observations have an incorrect sign for elasticity with respect
to OBS items. This finding suggests that if the hyperbolic distance function does not meet the monotonicity conditions, its elasticities with respect to inputs and outputs will have incorrect signs at many sample data points, leading to incorrect estimates of returns to scale, scale efficiency, and productivity growth.

Now the model including monotonicity conditions (constrained model) is estimated. The elasticities with respect to all factors achieved from the constrained model (as displayed in Table 3) have correct signs at the sample mean and each sample data points (by construction). The results also show that most of the estimated coefficients from the constrained model are statistically significant and MC errors are quite small, suggesting that convergence has been achieved.

### Table 3: Elasticities from unconstrained and constrained models

<table>
<thead>
<tr>
<th></th>
<th>Unconstrained Model</th>
<th>Constrained Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std</td>
</tr>
<tr>
<td><strong>Input elasticities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td>-0.0369</td>
<td>0.0382</td>
</tr>
<tr>
<td>Fixed assets</td>
<td>0.0259</td>
<td>0.0315</td>
</tr>
<tr>
<td>Deposits</td>
<td>-0.4659</td>
<td>0.0570</td>
</tr>
<tr>
<td><strong>Output elasticities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loans</td>
<td>0.3584</td>
<td>0.1120</td>
</tr>
<tr>
<td>Other earning assets</td>
<td>0.1547</td>
<td>0.1084</td>
</tr>
<tr>
<td>OBS items</td>
<td>0.0100</td>
<td>0.0131</td>
</tr>
</tbody>
</table>

### 5.2 Economies of scale

Returns to scale (RTS) indicate what happens to a bank’s outputs if it increases all inputs by a small proportion. Returns to scale are measured by the elasticity of outputs with respect to inputs—the elasticity of scale. In the context of the hyperbolic distance function and the framework of multiple inputs and multiple outputs, the elasticity of scale is determined by a negative ratio of the sum of input elasticities and the sum of output elasticities. If $RTS < 1$, the technology exhibits decreasing returns to scale (DRS) and the bank suffers diseconomies of scale; if $RTS = 1$, the bank achieves the optimal scale size or the MPSS; and if $RTS > 1$, the technology exhibits increasing returns to scale (IRS) and the bank enjoys economies of scale.

Table 4 provides descriptive statistics of returns to scale for all banks and each category of bank. The overall mean RTS is 0.9951, indicating that a 1% increase in all inputs leads to a 0.995% increase in all outputs on average. Thus, the average bank experiences slightly decreasing returns to scale. At the bank category level, foreign banks have the highest returns to scale on average, closely followed by JSCBs, and SOCBs have the lowest returns to scale. More than
two-thirds of FBs and around one-half of JSCBs enjoy economies of scale, but none of the SOCBs benefit from economies of scale. This result suggests that SOCBs have been operating beyond the optimal scale size. In contrast, FBs in Vietnam are small in scale and limited in scope, indicating the potential to expand production as well as provide branch networks.

The finding of diseconomies of scale in SOCBs can be due to the excess use of labor, too much borrowing, and a nationwide network with many ineffective branches. In fact, the average number of full-time equivalent staff at SOCBs is nearly 30 and 80 times greater than the average number of full-time equivalent staff at JSCBs and FBs respectively (as shown in Table 2). Moreover, borrowings are expensive resources to finance lending, and most of the lending of SOCBs is based on borrowing funds. Thus, SOCBs face high interest expenses, accounting for nearly 80% of total costs. Finally, the cost of maintaining a wide and ineffective network also contributes to the diseconomies of scale faced by all SOCBs. For example, the Vietnam Bank for Agriculture and Rural Development (VBARD)—the largest SOCB in terms of total assets—for both political and socio-economic reasons, provides a broad presence not only in populated areas but also in remote areas. Many branches of this bank in remote areas deliver financial services on a non-profitable basis.

Looking further at the evolution, Figure 3 reveals a decreasing trend in returns to scale over time for all banks and all bank categories. This implies that the growth rate of outputs reduces over time given the same proportional increase in inputs. In particular, the average bank in Vietnam achieves economies of scale for the first three years of the studied period, but then copes with diseconomies of scale for the rest of the period. This decreasing trend can be explained by increased competition and regulations. As commonly accepted in the banking literature, competition is one of the main drivers of an increase in bank size, which then leads to a decrease in returns to scale. To compete with large SOCBs, most of the JSCBs enlarged their branch networks rapidly and diversified their product range (e.g., offering more types of loans). This explains why JSCBs began to experience diseconomies of scale in 2005. To compete with domestic banks, FBs offered more cyber products, which are associated with less cost than products offered through physical branches. Thus, FBs suffered a smaller decreasing rate in

| Table 4: Descriptive statistics of returns to scale |
|---------------------------------|------|-----|-----|-----|-----------------|
| All banks                      | 392  | 0.9951 | 0.0262 | 0.9106 | 1.0529 | 203(51.8%) |
| SOCBs                          | 35   | 0.9397 | 0.0174 | 0.9106 | 0.9769 | 0(0%) |
| JSCBs                          | 210  | 0.9943 | 0.0218 | 0.9398 | 1.0529 | 88(41.9%) |
| FBs                            | 147  | 1.0096 | 0.0117 | 0.9825 | 1.0462 | 115(78.2%) |

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returns to scale than JSCBs. The second factor is government regulation. Since the early 2000s, the State Bank of Vietnam has required independent banking institutions (except foreign bank branches) to meet the minimum capital requirement of 8%. This ratio is calculated as the ratio of capital over risk-adjusted assets. Also, the State Bank of Vietnam has increased the level of legal capital to enhance capital capacity. Thus, banks have had to increase their capital to meet these requirements. Increases in capital give rise to total costs, and then fall to returns to scale (because capital acts as a fixed input in the cost function).

![Figure 3: Returns to scale during 2000–2006](image)

5.3 Scale efficiency

As presented earlier, this paper develops a new measure of scale efficiency based on the estimated hyperbolic distance function. Table 5 reports the yearly and period mean scale efficiency for all banks and each category of bank. The overall mean scale efficiency of the average bank is reasonably high (over 93%), suggesting that the average bank operates quite close to the most productive scale size. Over time, scale efficiency decreases gradually from 0.9664 in 2000 to 0.8944 in 2006, indicating that the average bank moves further away from the MPSS.

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>All banks</td>
<td>0.9664</td>
<td>0.9702</td>
<td>0.9552</td>
<td>0.9336</td>
<td>0.9210</td>
<td>0.9036</td>
<td>0.8944</td>
<td>0.9345</td>
</tr>
<tr>
<td>SOCBs</td>
<td>0.8831</td>
<td>0.8713</td>
<td>0.8517</td>
<td>0.8306</td>
<td>0.8160</td>
<td>0.7876</td>
<td>0.7797</td>
<td>0.8306</td>
</tr>
<tr>
<td>JSCBs</td>
<td>0.9596</td>
<td>0.9689</td>
<td>0.9470</td>
<td>0.9134</td>
<td>0.8933</td>
<td>0.8717</td>
<td>0.8623</td>
<td>0.9157</td>
</tr>
<tr>
<td>FBs</td>
<td>0.9974</td>
<td>0.9972</td>
<td>0.9939</td>
<td>0.9903</td>
<td>0.9900</td>
<td>0.9829</td>
<td>0.9734</td>
<td>0.9893</td>
</tr>
</tbody>
</table>
At the bank category level, FBs are the most scale efficient, followed by JSCBs, and SOCBs are the least. Scale inefficiency in FBs is because these banks operate below the optimal scale size, whereas scale inefficiency in domestic banks (especially SOCBs) is because they perform beyond the optimal scale size. Moreover, FBs experience a moderate decrease in scale efficiency over the 7 years under study (just 2%), whereas SOCBs and JSCBs suffer quite significant deterioration in scale efficiency, around 10%. The main reason for this deterioration is the number of domestic banks operating in the region of decreasing returns to scale and size increases over time. Thus their actual average productivity is more distant from the maximum average productivity.

It should be noted that the feature of scale efficiency is well matched with the feature of returns to scale and size as assessed earlier. Because of these features, it can be anticipated that scale efficiency would contribute negatively to productivity growth.

5.4 Productivity change and its components

Table 6 presents the overall mean estimates of productivity change (PROD) and its components of technical efficiency change (TEFFCH), scale efficiency change (SEFFCH), and technological change (TECHCH), for all banks and each bank category. The period mean estimate is the geometric mean of all annual means (which are the geometric means of individual bank estimates). It is worth recalling that this study assumes out technological regress by constructing the accumulative frontier technology, and thus all TECHCH indexes are greater than one.

<table>
<thead>
<tr>
<th></th>
<th>TEFFCH</th>
<th>SEFFCH</th>
<th>TECHCH</th>
<th>PROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>All banks</td>
<td>1.0127</td>
<td>0.9849</td>
<td>1.0205</td>
<td>1.0178</td>
</tr>
<tr>
<td>SOCBs</td>
<td>1.0072</td>
<td>0.9753</td>
<td>1.0245</td>
<td>1.0064</td>
</tr>
<tr>
<td>JSCBs</td>
<td>1.0088</td>
<td>0.9790</td>
<td>1.0202</td>
<td>1.0076</td>
</tr>
<tr>
<td>FBs</td>
<td>1.0196</td>
<td>0.9958</td>
<td>1.0198</td>
<td>1.0354</td>
</tr>
</tbody>
</table>

Overall, the industry achieves an average annual increase of 1.27% in technical efficiency and 2.05% in technological progress. However, scale efficiency falls at an average rate of 1.51% per year. These changes lead to a mild productivity growth of 1.78% for the average bank. Put differently, the combination of the positive effects of technical efficiency change and technological change is adequate to offset the negative effect of scale efficiency.

At the category level, throughout the sample period SOCBs undergo an
average increase of 0.72% in technical efficiency and an average improvement of 2.45% in technological progress. Nonetheless, they suffer a deflation of 2.47% in scale efficiency every year. That explains why SOCBs operate at a poor productivity growth of 0.64% on average.

JSCBs experience something quite similar to what happens to their domestic counterpart SOCBs. In particular, the positive contributions of technical efficiency change (0.88%) and technological progress (2.02%) to productivity growth are together large enough to offset the negative contribution of scale efficiency change (−2.1%). Thus, moderate productivity growth of 0.76% per year is recorded to the average JSCBs.

Finally, the findings show that foreign banks achieve the greatest rate of productivity growth, with an annual rate of 3.5%. The major sources of this outcome are positive changes in technical efficiency (1.96%) and technological progress (1.98%), while the negative change in scale efficiency is modest (−0.42%). The plausible explanation for the high growth rate of productivity in foreign banks lies in their business feature. Unlike domestic banks, they focus on corporate and wholesale banking rather than retail banking. Thus, they can adjust their production plans and operation scale better and more quickly than others when facing changes in the banking environment. The other likely explanation is that most foreign banks have more advanced technology platforms than SOCBs and especially JSCBs. Clearly, they have been the leaders in the introduction of credit cards, debit cards, ATMs, factoring, forfeiting, and other modern financial services. With these advantages, the penetration of foreign banks into the Vietnamese market has promoted productivity gains of the whole banking system.

Figure 4 shows the evolution of productivity change and its components over 2000–2006. The first observation is that there seems to be some common patterns. TEFFCH and TECHCH for all banks and all groups of banks decrease smoothly at a gentle rate throughout the examined period, while SEFFCH and PROD follow a downward trend until 2003 and then recover slightly in 2004 before experiencing a downward trend again toward the end of the period. In particular, the whole banking system experiences a decreasing rate of productivity growth, from around 4.6% in 2001 (compared to 2000) to nearly 0.2% in 2006 (compared to 2005). This decrease is consistent with diminishing rates of technical efficiency gains and technological progress, and increasing rates of scale efficiency loss. These findings suggest that the productivity of the Vietnamese banking system has not reacted positively to deregulation and the bank reforms occurring during the period analyzed.

Likewise, the rate of productivity growth for SOCBs reduces from 2% in 2001 to −1% in 2006. This contraction is mainly generated by an increasing rate of scale efficiency deterioration from around −1.5% to −3.5%. Similarly, the traceable source for the decreasing productivity gains in JSCBs (from 4.6% to −0.8%) is a significant drop in the scale efficiency index from a growth rate of nearly 1% in 2001 to a contraction rate of −3.1% in 2006. This result could be explained by the fact that during the sample period JSCBs began to enlarge their
operations by opening more new branches. Correspondingly, their inputs, such as the number of staff and fixed assets, increased dramatically. Meanwhile, no equivalent increase in outputs occurs, at least in the short term. This causes a decrease in actual average productivity, and then a lower rate of productivity growth.

Like the domestic banks, foreign banks experience a weakening rate of productivity growth, from above 5% in 2001 to below 2% in 2006. However, unlike domestic banks, foreign banks enjoy productivity gains throughout the 7 years under analysis. This is the result of their superior management of production plans and scale operations as well as their advanced technology.

Figure 4: Productivity change over time
It is further worth pointing out that, at both the industry and the category level, technical efficiency change and technological change follow a monotonic trend. That is because this study models the inefficient term as an exponential function of time and an unknown parameter $\eta$ to capture inefficiency change over time. A positive sign of $\eta$ then signals an increasing trend of technical efficiency at a decreasing rate. Moreover, productivity change seems to follow the same shape as scale efficiency evolution, but at a different magnitude, indicating that scale efficiency change plays a pivotal role in shaping productivity change. This finding supports the argument that ignoring scale efficiency change as a component of total productivity change would cause erroneous estimation of productivity growth. For instance, in the present study, ignoring scale efficiency change would cause an overestimation of productivity change.

To sum up, the findings exhibit a generally low level of productivity growth over the sample period, at 1.78% per year. This growth is first engendered by technological progress, then by technical efficiency improvements, while scale efficiency deteriorates. Moreover, foreign banks are found to attain the highest productivity growth, followed by JSCBs and then SOCBs.

Based on the analyzed productivity features of the Vietnamese banking industry, some solutions can be outlined to promote productivity growth. First, domestic banks need to manage their operation scale effectively. This issue can be addressed through several ways, such as:

i. Reducing the number of physical branches and developing new channels for delivering customer services, such as internet banking, phone centers, and automated teller machines, because these channels may exhibit greater economies of scale than traditional branching networks;

ii. Diversifying output portfolios to provide more non-traditional outputs (which require less physical resources but higher skill management);

iii. Reallocating resources by focusing on qualified and skilled staff rather than the number of staff; and

iv. Reducing borrowings which are quite costly by developing non-traditional products (e.g., asset-backed security, collateralized debt obligations) to free up capital and grant more lending with less burden of borrowing costs.

Second, domestic banks need to develop a wide range of technologies including information technology, telecommunications, and financial product technologies. Technological innovations can help to increase economies of scale in variety of bank products and services such as payment processing, cash management, and bank office operations. Technological advances can lead to the development of new products and services that have greater economy of scale than traditional banking products. Therefore there is the potential for an increase in the productive efficiency of banks.

Third, low productivity growth in SOCBs might be improved if these banks are privatized. The literature provides evidence that banks that were privatized experienced improvements in their productivity (see Fiorentino et al. 2009; Bonin
et al. 2005; Nakane and Weintraub 2005). In fact, a road map exists to equitize four out of the five SOCBs. So far, one of four SOCBs has completed its equitization and in 2008 started operating as a joint stock bank. The other three are in the process of change. Thus, to enhance the productivity growth of these banks, the equitization process should be accelerated.

Finally, foreign banks can attain higher productivity growth by increasing their scale of operation. In fact, they had been restricted in opening offices and transaction points (such as location of ATMs). However, under Vietnam’s commitments to the WTO, foreign banks may now increase their presence in terms of locating ATMs in public areas, opening new branches in the same city, or establishing a full-legal status foreign bank. Thus, it can be expected that the productivity of foreign banks will grow significantly, increasing the overall productivity growth level of the industry.

6 Conclusion

This paper has introduced a new parametric measure of productivity change as a product of three components, namely technical efficiency change, technological change, and scale efficiency change, based on the hyperbolic distance function. The estimation of the hyperbolic distance function was implemented to meet the monotonicity conditions. Furthermore, it also derived a new expression for the measurement of scale efficiency.

In the investigation of the productivity feature of the Vietnamese banking industry, the paper found that the Vietnamese banking industry experienced modest productivity growth, 1.78% per year on average. The thrust of this growth was technological progress, and then technical efficiency improvements, whereas scale efficiency change contributed adversely to productivity growth. Moreover, FBs experienced the highest productivity growth, followed by JSCBs and SOCBs. Possible explanations for the high productivity growth of FBs are that they possessed modern technology infrastructure (leading to a high level of technological progress), and they operated at an appropriate scale (thus they did not face the same strong deterioration in scale efficiency as domestic banks). In contrast, many domestic banks operated well beyond the optimal scale size, and consequently coped with diseconomies of scale, and then deterioration in scale efficiency. This scale efficiency regress almost offsets technological advancement, leading to poor productivity growth for SOCBs and JSCBs.

The paper revealed no evidence of improvement in productivity growth over the period studies as a positive response to banking reforms and deregulation. This finding suggests that positive effects of banking reforms and deregulation on bank performance might take some time to materialize. To promote productivity growth, domestic banks need to manage their scale of operation effectively and further upgrade their information technology platform. Meanwhile, foreign banks need enlarge their scale size through opening new offices and transaction points.
Furthermore, the equitization process for SOCBs should be accelerated to boost productivity growth in SOCBs.

References


