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Default Forecasting Considering Correlation Between Business and Credit Cycles

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Abstract

Bank's major approach in her internal rating system is credit scoring valuation which focused on corporates' idiosyncratic risks and based on their financial indexes. Hence, an influence on corporates' credit risks by business variation is not considered in her system. We model the effect on corporates' credits by macroeconomic variables and analyze it. Firstly we model a corporate's credit variable by the credit cycle index and her idiosyncratic risk factor, and consider the correlation between the business and credit cycles. And we decompose the business cycle into a trend and a cycle using Hodrick-Prescott filter and show we can build the more explanatory model than one based on macroeconomic variables themselves. Secondly we optimize the weights of credit cycle index by some distance measures for Japanese corporates and quantify the severity on historical Japanese corporates' credits by macro economy.

JEL classification numbers: C35, G21, G32, G33
Keywords: Business cycle, Credit cycle index, Hodrick-Prescott filter, Probit model, Distance measure

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1 Introduction

Recently banks adopting the “internal rating” to evaluate corporate’s credit risk are steadily increasing as ones practicing advanced credit risk management. Nowadays, the internal rating has been a material indicator for the valuation of corporate’s credit risk.

On the other hand, there is a incredibility in corporate finance that when economy worsens, the number of defaulting corporates increases. Quantifying the relation between business cycle and corporates’ credit cycle is not necessarily mature. The endowment of internal rating with obligor’s corporate is mainly followed by the credit scoring utilizing micro data such as the obligor’s financial ratios.

Here we assume the probability of default (PD) in a rating pool equals the average of obligor’s unique PDs which are assigned in the pool bucket. Then there exist two types of internal ratings related to pool’s PD. One is “PIT (Point-in-time) rating”, which a rating entity uses to evaluate the corporate’s credit in the light of economy condition at the time. Major entity is said to be a bank, but in her internal risk management, internal ratings are decided from the data for over latest five years for the consistency with Basel 2. The other is TTC (Through-the-cycle) rating, which reflects the average of long economy variation that is stable against business variation and evaluates corporates’ credit risk. The rating agency is a typical example of this type of rating entity.

In addition, we distinguish between “unstressed PD” and “stressed PD” in the light of whether background macro economy is stressed or not. The former is a non-biased estimate for defaults over the next year given all currently-available information such as static or dynamic obligor’s attribute and portfolio’s aggregated data. And the former drops when macro economy recovers and reversely increases when it worsens. The latter uses all available obligor’s information, but measures the probability that obligors will default over the next year based on the assumption of stressed scenarios that economy condition will worsen. And as the latter uses dynamic obligor’s attributes, obligor’s each attribute changes depending on macroeconomic change, but it will tend not to be highly correlated with business cycle.

Then the relation between obligor’s idiosyncratic PD and bank’s rating approach is shown on Table 1. The obligor’s unstressed PD assigned to a PIT risk bucket is stable against business cycle, to the contrary, the unstressed PD assigned to a TTC risk bucket is negatively correlated with business cycle. On the other hand, the obligor’s stressed PD assigned to the PIT risk bucket is positively correlated with business cycle, to the contrary, the stressed PD

Table 1: Correlation between pooled PD and business cycle

Attribute for obligor's PD	Rating philosophy	
	PIT	TTC
Unstressed	Stable	Negative
Stressed	Positive	Stable

assigned to the TTC risk bucket is stable against business cycle.

Now we review the existing researches treating with credit cycle and/or business cycle.

First of all, we can mention the models of Belkin et al. (1998) and Kim (1999). These models adopt "one factor approach" to incorporate business variation into a rating migration probability matrix. This one factor is a systematic factor to represent the credit condition in whole financial market and called "credit cycle index" in Kim (1999). As you know the number of corporates to deteriorate their credits or default will increase when business worsens, this index indicates macroeconomic condition. Kim (1999) recognized that a high rating bond holds much lower PD. He used PDs for speculative rating bonds (bonds with the rating below BB) and selected by a probit model only one macroeconomic variable which affected PDs. Following Belkin et al. (1998) and Wilson (1997), Trück (2008) used PDs for speculative rating bonds in US market and selected macroeconomic variables by multiple regression analysis based on a probit model. Tsai and Chang (2010) predicted the probability of financial distress of listed Taiwanese corporates based on the discrete-time hazard models of Schumway (2001) and set the cut-off point to distinguish distressed corporates from non-distressed ones by credit cycle index of Kim (1999).

Next, as the research treated with the correlation between business and credit cycles, we refer to Koopman et al. (2005). They used Hodrick-Prescott filter and decomposed PDs for US corporates into the trend component and the cycle component, and empirically analyzed the effect of cycle component. Here, Hodrick-Prescott filter is the filter presented on Hodrick-Prescott (1997), a major econometric approach which decompose a business cycle into the trend component and the cycle component.

Section 2 sets up the credit risk valuation model to consider the business cycle by the credit cycle index and the idiosyncratic risk factor. Section 3 decomposes the time-series data into the trend component and the cycle component by Hodrick-Prescott filter, selects macroeconomic variables which affect corporate credits and optimizes the weight for the credit cycle index using some distance measures. Finally Section 4 concludes.

2 Default Risk Model

We consider an asset return $Y(t)$ as the random variable which represents a corporate's credit. $Y(t)$ is decomposed into two factors, one of which is the systematic risk factor $Z(t)$ that represents the economy condition and the other of which is the corporate's idiosyncratic risk factor $u(t)$. By setting $\mathbb{R}_+ = (0, \infty)$, $Y(t)$ is formulated as the linear combination of two factors as follows.

$$Y(t) = w(t)Z(t) + \sqrt{1 - w(t)^2}u(t), \quad t \in \mathbb{R}_+ \tag{1}$$

where $Z(t)$ and $u(t)$ are mutually independent standard normally distributed random variables, hence so is $Y(T)$. A parameter $w(t) \in [0, 1]$ $(t \in \mathbb{R}_+)$ is the factor loading which represents the degree that corporate's asset return is affected by credit cycle index. And $w(t)^2$ is a time-dependent correlation between the credit cycle index $Z(t)$ and the asset return $Y(t)$, so is also recognized as "asset correlation". The nearer $w(t)$ approaches to one, the more severely by macroeconomic risk than by corporate's idiosyncratic risk the asset return will be affected.

We set one year stressed PD in PIT rating approach at time t as $P^S_{PIT}(t)$. This probability is a time-deterministic function which is calculated from the probability of default in historical one year $(t-1, t]$. In existing research, historical probability of default for speculative grade is usually used. According to Belkin et al. (1998) and Wilson (1997), the speculative grade bond is more severely affected by the macro economy than the investment grade bond. Hence we also set this assumption. Here stressed PD indicates the probability of default which is much strongly stressed by systematic risk than by obligor's idiosyncratic one.

One method to specify stressed PD in PIT rating approach is to directly assign macroeconomic variable to $Z(t)$. One of existing researches related to this method is Nickell et al. (2000). Though the PD for any rating pool which reflects its obligor's stressed PD tends to move to the same direction as macro economy, it peaks around the top of business cycle and falls around the trough.

When a speculative grade's bond defaults, we assume that it depends on the credit cycle index $Z(t)$, but not on the corporate's idiosyncratic risk factor $u(t)$. If we assume that a default rate $P^S_{PIT}(t)$ is the probability on which the asset return $Y(t)$ is below the default threshold $y^S_{PIT} = \bar{z}$, then we can get the following equation.

$$P^S_{PIT}(t) = \Phi(\bar{z}) \tag{2}$$

hence, $\bar{z}(t) = \Phi^{-1}(P^S_{PIT}(t))$ comes out of the inverse $\Phi^{-1}(\cdot)$ of standard normally distributed function $\Phi(\cdot)$. We set the average of actual data for $\bar{z}(t) = \Phi^{-1}(P^S_{PIT}(t))$ as $\mu_{\Phi^{-1}(P(t))}$ and the variance as $\sigma^2_{\Phi^{-1}(P(t))}$. The standardization of $\bar{z}(t)$ is followed by the credit cycle index $Z(t)$.

$$Z(t) = \frac{\Phi^{-1}(P^S_{PIT}(t)) - \mu_{\Phi^{-1}(P(t))}}{\sigma_{\Phi^{-1}(P(t))}} \tag{3}$$

The credit cycle index $Z(t)$ is an index reflects the macroeconomic condition from which all corporates are affected by t. In case of the good economy, $Z(t)$ is minus, reversely in case of the bad economy it is plus. Hence default threshold $\bar{z}^S_{PIT}(t) = \Phi^{-1}(P^S_{PIT}(t))$ for stressed PD by PIT rating approach is expressed by multiple regression model as follows.

$$\bar{z}^S_{PIT}(t) = \Phi^{-1}(P^S_{PIT}(t)) = \beta_0 + \sum_{j=1}^{m} \beta_j X_j(t - lag) + \sum_{j=m+1}^{n} \beta_j X_j(t) + e(t) \tag{4}$$

where $X_j(t - lag),\ j = 1, 2, \ldots, m$ are macroeconomic variables lag months before time t, for which Real GDP growth rate, CPI Composite Index, Overall unemployment rate, etc. are applicable. The lag is the duration fixed in accordance with each renewal cycle and variable's attribute, the duration is three months for real GDP growth rate and one month is for overall unemployment rate. The figures at the same time as one for the credit cycle index are fixed for $X_j(t),\ j = m+1, m+2, \ldots, n$. And $e(t)$ is a standard normally distributed random error.

Furthermore we decompose original macroeconomic variables into a trend component and a cycle component by Hodrick and Prescott filter (hereafter "HP filter", Hodrick and Prescott (1997) for the reference.) and assign some decomposed components selected by the regression model to $X_j(t - lag),\ j = 1, 2, \ldots, m$ and $X_j(t),\ j = m + 1, m + 2, \ldots, n$.

On the other hand, we assume that when a corporate bond's rating migrates from state i to state j, the credit cycle index $Y(t)$ take a value on $(y_i^j, y_i^{j+1}]$. Then, if the credit cycle index $Z(t)$ takes a realized value $z(t)$, the rating migration probability matrix $p(t; i, j|z(t))$ conditional on $Z(t) = z(t)$ is expressed as follows.

$$p(t; i, j|z(t)) = \Phi\left(\frac{y_i^{j+1}(t) - w(t)z(t)}{\sqrt{1 - w(t)^2}}\right) - \Phi\left(\frac{y_i^j(t) - w(t)z(t)}{\sqrt{1 - w(t)^2}}\right) \tag{5}$$

where rating threshold $y_i^j(t)$ and $y_i^{j+1}(t)$ is the given values calculated from rating migration probability matrix. Especially in case of migration to default state K, the probability of default conditional on $Z(t) = z(t)$, namely stressed PD $P_{PIT}^S(t) := p(t; i, K|z(t))$ in PIT rating approach is expressed by setting $y_i^K := y_{PIT}^U$ as follows.

$$P_{PIT}^S(t) = \Phi\left(\frac{y_{PIT}^U(t) - w(t)z(t)}{\sqrt{1 - w(t)^2}}\right) \tag{6}$$

where y_{PIT}^U is the inverse of cumulative normal distribution function with regard to unstressed PD $P_{PIT}^U(t)$ by PIT rating approach, hence,

$$y_{PIT}^U = \Phi^{-1}(P_{PIT}^U(t)) \tag{7}$$

where unstressed PD is the unbiased estimate for obligor's default over next year by currently-available macroeconomic information, and when economy enters into boom, unstressed PD goes downward, reversely when economy enters into recession, unstressed PD goes upward.

We model macroeconomic variables such as real GDP growth rate from among $X_j(\cdot),\ j = 1, \ldots, m$ in eq.(4) by time-series model before completing the multiple regression analysis. Time series data can be decomposed into level term, cycle term and error term. So macroeconomic time-series has the long trend and the cyclical variation around the trend. It has indeed features to hold the trend with different growth rate to different duration and time-varying cycle. With regard to seasonal variation, we can avoid to face with the treatment by using time-series data from Cabinet Office of Japanese Government. As this data is seasonally-adjusted time-series data coordinated by seasonal adjustment method X12-ARIMA, seasonal variation is exempted from this modeling. Then time-series model is defined as follows.

$$v(t) = \mu(t) + \psi(t) + \sigma_\epsilon \epsilon(t), \quad t = 1, 2, \ldots, n \tag{8}$$

where $v(t)$ indicates actual seasonally-adjusted time-series, $\mu(t)$, $\psi(t)$ and $\epsilon(t)$ are respectively trend term, cycle term and error term. The error term $\epsilon(t)$ is standard normally distributed random variable, mutually independent from all other random variables and its value at different time.

We conduct the modeling of HP filter type to specify trend component $\mu(t)$. Then,

$$\mu(t+1) = \mu(t) + \nu(t) \tag{9}$$

$$\nu(t+1) = \nu(t) + \sigma_\xi \xi(t) \tag{10}$$

where $\xi(t)$ is a standard normally distributed random variable. The residual component $\bar{\psi}(t) := \psi(t)+\sigma_\epsilon \epsilon(t)$ of time-series $v(t)$ from which trend component is exempted is the cycle component (cycle term and error term).

If $\sigma_\xi = 0$, trend term is constant. And cycle component $\bar{\psi}(t)$ is formulated by triangular function in accordance with the method of Harvey et al. (1985) as follows.

$$\begin{pmatrix} \bar{\psi}(t) \\ \bar{\psi}^*(t) \end{pmatrix} = \phi \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} \bar{\psi}(t-1) \\ \bar{\psi}^*(t-1) \end{pmatrix} + \sqrt{1-\phi^2} \begin{pmatrix} \kappa(t) \\ \kappa^*(t) \end{pmatrix} \tag{11}$$

where $\lambda \in [0, \pi]$ is cycle length (radian), $\phi \in [0, 1]$ is decay factor of amplitude, and $\kappa(t)$ and $\kappa^*(t)$ are mutually independent and standard normally distributed random variables. This cycle component express the stationary cycle process with average cycle length $2\pi/\lambda$, but $\epsilon(t)$, $\xi(t)$, $\kappa(t)$, and $\kappa^*(t)$ are all normally stochastic processes. Besides, as to modeling of cycle, refer to Harvey (1991).

Solving eq. (11) using lag operator, then,

$$\bar{\psi}(t) = \frac{(1-\phi\cos\lambda \cdot L)\sqrt{1-\phi^2}\kappa(t) + (\phi\sin\lambda \cdot L)\sqrt{1-\phi^2}\kappa^*(t)}{1-2\phi\cos\lambda \cdot L + \phi^2 \cdot L^2} \tag{12}$$

$\bar{\psi}(t)$ is expressed by ARMA(2,1) model. A general expression for ARMA(2,1) model is expressed in setting $\eta(t)$ as a standard normally distributed random variable as follows.

$$\bar{\psi}(t) = \alpha_0 + \alpha_1\bar{\psi}(t-1) + \alpha_2\bar{\psi}(t-2) + \theta_1\eta(t-1) + \eta(t) \tag{13}$$

Hence parameters ϕ and λ in eq.(12) are calculated as given of α_1 and α_2 in eq. (13) as follows.

$$\phi = \sqrt{-\alpha_2}, \quad \lambda = \cos^{-1}\frac{\alpha_1}{2\sqrt{-\alpha_2}} \tag{14}$$

We can derive the following equation using eq. (4) and eq. (6).

$$\bar{z}^S_{PIT}(t) = \sqrt{1 + b(t)^2}\mu^U_{PIT}(t) + b(t)\bar{\psi}(t) \tag{15}$$

where given $\bar{z}^S_{PIT}(t) = \Phi^{-1}(P^S_{PIT}(t))$, namely $P^S_{PIT}(t)$, we set the asset correlation $w(t)^2$, the default threshold $y^U_{PIT}(t)$ calculated from unstressed PD by PIT rating approach, and the credit cycle index $z(t)$ at default as follows.

$$w(t)^2 = \frac{b(t)^2}{1 + b(t)^2} \tag{16}$$

$$y^U_{PIT}(t) = \mu^U_{PIT}(t) \tag{17}$$

$$z(t) = -\bar{\psi}(t) \tag{18}$$

where as $w(t)^2 \in [0, 1)$ is derived from $b(t)^2 \in \mathbb{R}_+$, $w(t) = 1$ is exempted in eq.(1). And $\mu^U_{PIT}(t)$ is a long-term factor which is the inverse of standard normally distributed function on the time-series $P^U_{PIT}(t)$ of unstressed actual default rate.

3 Data Analysis

3.1 Trend and cycle decomposition of time-series data

The default threshold $\bar{z}(t)^S_{PIT}$ in eq.(15) is time-varying, and is affected by cycle component $\bar{\psi}(t)$ in business cycle, the sensitivity of which is b. Figure 1 indicates the result of decomposition by HP filter (smoothing coefficient: 100 for annual data) of annually actual default rates for Japanese corporates of which R&I's ratings[2] for fiscal year 1980 to 2011 belong to speculative grade (below BB grade). Looking at the figure, the trend component of actual default rates are consistently increasing. Because for a period of Japanese financial crisis around 1997 to 1998 and a period of sub-prime loan problem and global financial crisis around 2007 summer to 2009, the number of defaults had increased.

On the other hand, Figure 2 indicates the result of decomposition by HP filter (smoothing coefficient: 1600 for quarterly data) of actual GDP growth rate (seasonally-adjusted). Looking at the figure, both trend component and cycle component of business cycle are not monotonically increasing, and the

[2] R&I is a Japanese rating agency and give most ratings toward Japanese corporates.

influence that macro economy exerts the individual corporate's default is restrictive. Rather we can mention the following cause; there increased the corporates with worse performance than previous one among corporates of which rating pool with the ratings below R&I BB grade are composed, there increased the heterogeneity with regard to corporates in the rating pool, and the volume of actual default data is not enough.

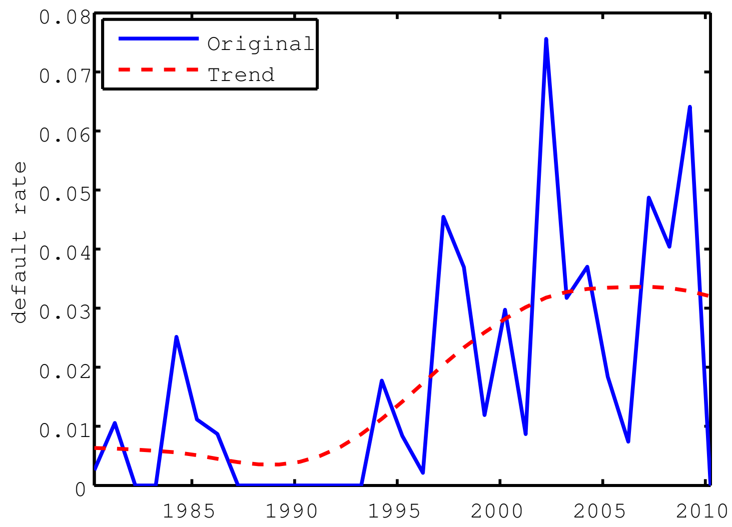

Figure 1: Decomposition by HP filter of PD for Japanese corporates (smoothing coefficient: 100 for annual data)

Using real GDP growth rate data (seasonally-adjusted, compared with the previous quarter, annualized rate) for 30 years from 1981Q2 to 2011Q1, we estimate the parameters $(\alpha_0, \alpha_1, \alpha_2, \theta_1)$ in eq.(13). Then, as we derive the estimates of Table 2, the parameters for the cycle component in eq.(14) are as follows.

$$\phi = 0.7413, \quad \lambda = 0.6201 \tag{19}$$

Hence we derive $2\pi/\lambda = 10.13$ years as the average cycle length.

Figure 4 indicates the result for HP filter decomposition into the coordination part $\sqrt{1+b(t)^2}\mu^U_{PIT}(t)$ with the trend component of the actual default rate time-series $P^U_{PIT}(t)$ and the coordination part $b(t)\bar{\psi}(t)$ with cycle component of macroeconomic variables of default threshold $\bar{z}^S_{PIT}(t)$ in eq.(15) through the use of the annually actual default rates PDs for Japanese corporates ranked the

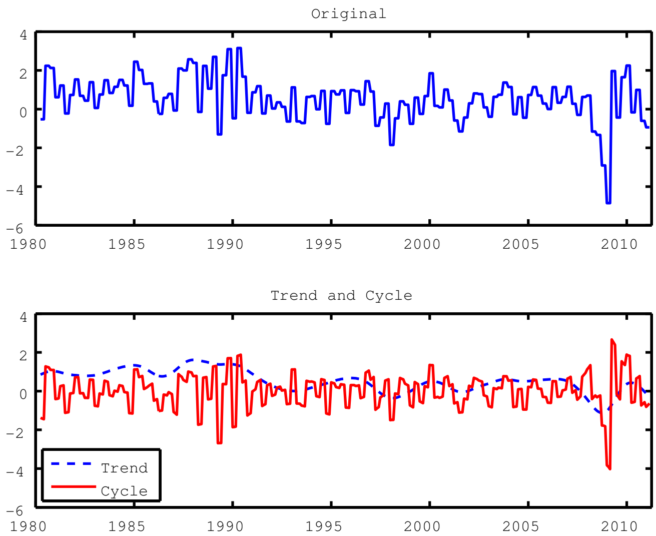

Figure 2: Decomposition by HP filter of actual GDP growth rate (upper: original, lower: trend and cycle, smoothing coefficient: 1600 for quarterly data)

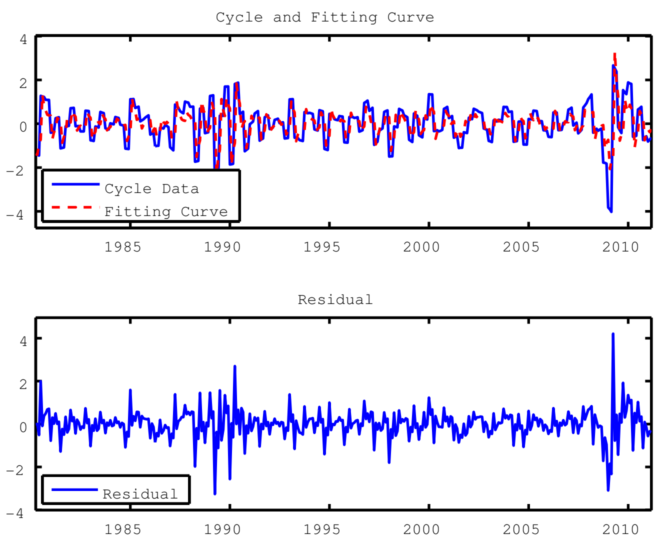

Figure 3: Estimation by ARMA(2,1) model of cycle component of actual GDP growth rate (upper: cycle term and fitting curve, lower: residuals)

Table 2: Parameter estimates for cycle component of real GDP growth rate by ARMA(2,1) model (Least squares method)

Parameters				F-statistic
α_0	α_1	α_2	θ_1	
0.0012	1.2066	-0.5496	-0.5104	81.8462*
(0.2087)	(0.1087)*	(0.0627)*	(0.1242)*	

Note: Standard Errors in parentheses; *significant at 1% and 5%.

speculative grade (below BB grade) by R&I. From the upper figure of Figure 4, we see that as the collapse of Japanese bubble economy grew increasingly apparent nationwide from the early 1990s, Japanese corporates' speculative grade default rate signaled a upward trend. And as we can see much zero values for actual default rates, cycle component is so roughly varying on the lower figure of Figure 4.

Figure 5 and Figure 6 indicate obligors' average of unstressed PD assigned to Moody's and S&P rating by EDF(Expected Default Frequency) by Moody's Analytics, and is a monthly time-series process from 2004/1 to 2012/1 by single logarithmic scale. As PIT rating system is designed to adjust obligors' rating grade when unstressed PD changes, average PD by rating grade is said to be usually stable as to time. Looking at two figures, till 2008/8 before Lehman shock, it is indeed stable, but owing to Lehman shock, all levels of PDs by rating grade once have increased, hereafter they have returned to previous levels.

3.2 Model Selection

By the multiple regression analysis based on probit model in eq.(4), we select the parameters for macroeconomic variables from which the credit cycle index is affected. Here parameters' candidates are shown on Table 3. Real GDP growth rate, CPI (excluding fresh food), Overall Unemployment rate are all seasonally-adjusted variables excluding seasonal variation terms. Time lags are set in accordance with renewal cycle (monthly, quarterly or annually) and variable's attribute. We provide two types of datasets, one of which is original dataset "Set 1", and the other of which is the decomposed dataset "Set 2",

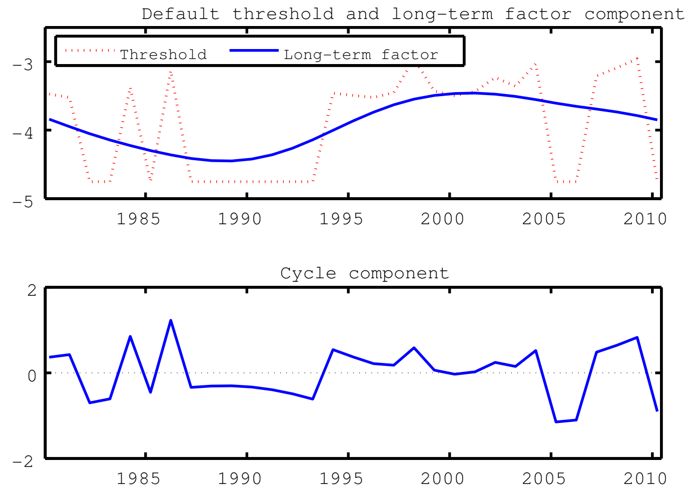

Figure 4: Upper figure: Default threshold $y^U_{PIT}(t) = \Phi^{-1}(P^U_{PIT}(t))$ and the long-term factor component $\sqrt{1+b(t)^2}\mu^U_{PIT}(t)$ of the default threshold. Lower figure: Cycle component $b(t)\bar{\psi}(t)$ of default threshold.

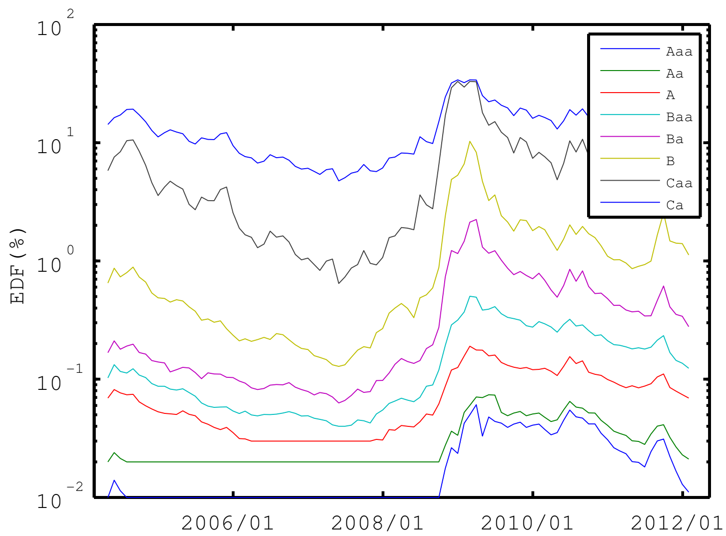

Figure 5: EDF time-series process by Moody's rating grades

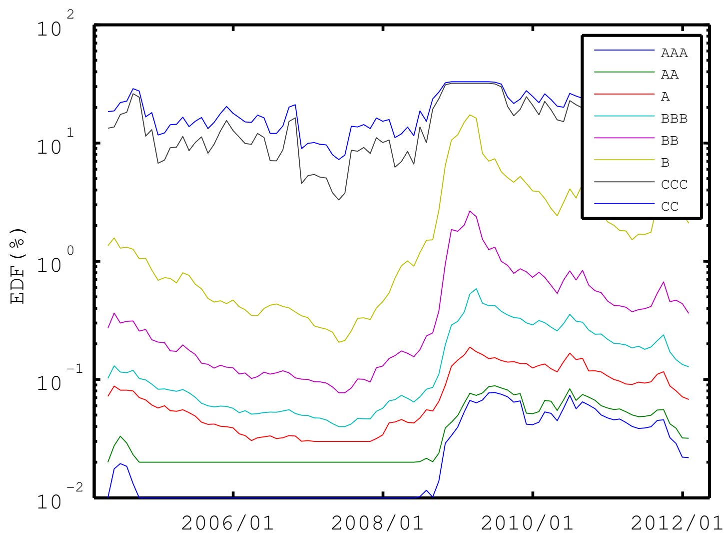

Figure 6: EDF time-series process by S&P rating grades

which includes the trend components and the cycle components derived from HP filter decomposition of real GDP growth rate, CPI(excluding fresh food), and overall unemployment rate, and Composite Index (Leading series and Coincident series).

As for CPI (excluding fresh food), as consumer price is strongly affected from the variation of fresh vegetable price moving up and down owing to climate variation, the index excluding fresh food is used as the explanation for consumer price variation. The seasonality's exemption is based on a seasonally-adjustment method "X12-ARIMA".

As for Confusion Index, there are three types of leading series, coincident series, and lagging series. The lagging series is composed of six indexes, but Completely Unemployment rate out of which is solely selected. Hence, the lagging series is off the candidates. The leading series is the indexes that react before business variation, twelve indexes such as Consumer attitude index and Tokyo Stock Exchange index are utilized. The leading indexes calculated from the leading series are said to forecast a few months forward business variation. Hence we set the time lag as −3 months for leading series. And the coincident series are indexes that react with business variation, eleven items such as industrial output index and job-offers-to-seekers ratio are utilized. The coincident indexes calculated from the coincident series are said to show current business condition, so we set the time lag as zero month for coincident series. Completely Unemployment rate is a major index that composes lagging series and lagging indexes calculated from lagging series are said to react behind a half year to one year. Nonetheless, to incorporate the index as a forecasting variable, we set the time lag as -1 month.

As for interest rate variables, we use 10 years JGB yield (month end, latest value) as a long-term interest rate, and 10 years JGB yield minus overnight call rate (month end) as a spread between long- and short-term interest rates. And as for stock variables, we use TOPIX (latest value, month-to-month basis). As the cyclicality is not generally recognized for both variables, we do not use the variables after HP decomposition.

Here we set some time-series datasets. Looking at Figure 1, it is after year 1994 that default events occurred in rating grades below R&I BB grade. Hence we split dataset from 1994/4 to 2011/3 into two parts, set 10 years' dataset from 1994/4 to 2004/3 as "Sample 1", and 7 years' dataset from 2004/4 to 2011/3 as "Sample 2". And as Sample 2 was affected by sub-prime loan problem and global financial crisis, to investigate the influence, we also set dataset from 2008/9 to 2011/3 after Lehman shock. Totally we set three types of datasets for original time-series. Furthermore, we decompose three types of original dataset excluding interest rates and stock variables into a trend component and a cycle component respectively, and set three types of decomposed dataset as HP filter decomposition datasets. As a result, total number of datasets is $3 \times 2 = 6$.

We select the step-down procedure as the variable selection method for multiple regression analysis by probit model. Taking care of the sign of variables, we exclude the variable which sign is reversed. In addition, to avoid the multicollinearity, we exclude the variable candidate with VIF beyond 10, and we repeat the procedure till all variable candidates' VIFs are below 10.

We give the selected explanations about variables' signs below. Firstly, we explain about original dataset. Real GDP growth rate is a major index which expresses business variation, and if this index goes up, reversely the ratio of defaulting corporates will reduce and PD by rating grade will also become small. Hence as the credit cycle index's value becomes small, the sign is $-$.

As for TOPIX (month-to-month basis, latest value), through stock price's increase is said to be the leading index for business uptrend, this idea is not necessary applicable historically. As this index is the value of month-to-month basis and one month's short term increase is not connected with business uptrend, the sign is either $+$ or $-$.

As for CI leading series (month-to-month basis), the index's upward or downward movements and economic expansion or recession are respectively coincident. As we can consider the magnitude for the index's change indicates the speed for boom or recession, the sign for original dataset's index is $-$ as is the case with real GDP growth rate. In contrast, as the trend or cycle components after HP decomposition are respectively not necessarily consistent with the original index's up or down trends, their signs are either $+$ or $-$.

Real private final consumption expenditure is one of GDP consumption items, and is also called "Personal Consumption". As the expense for necessity such as food is included in its item, its variation is not necessarily volatile compared to other necessities such as Private Housing Investment. Referred to Figure 7, though real GDP growth rate dropped by 0.6 times compared with the previous quarter in 2010 Q4 after Lehman shock, real private final consumption expenditure increased 0.1 times compared with the previous quarter. Hence, even if real private final consumption expenditure is plus, it does't necessarily lead to business boom and the sign is either $+$ or $-$.

Next as for trend component and cycle component after HP decomposition, as their signs are not coincident with ones for up and down of original variables, all their signs are either $+$ or $-$.

Table 5 shows modified R-squared of multiple regression analysis. As a remarkable feature common among six datasets is that R-squared goes up with the order of Sample 1, Sample 2 and after Lehman shock. Lehman shock is an event that most severely affected to Japanese corporate's default for this period. The corporate's default for the dataset after Lehman shock can be most accountable by only macroeconomic variables. Next, on Sample 2 dataset, an influence from global financial crisis is smaller than the dataset after Lehman shock, finally Sample 1 from 1994/4 to 2004/3 comes.

We decompose only six variables such as real GDP growth rate, and compare three types of datasets with variables of decomposed trend and cycle components with respective original datasets. Then, it is proved that the utilization of macroeconomic data much improves the explanatory power on all pairs. Especially on dataset after Lehman shock, macroeconomic variables can account for about ninety percent of Japanese corporate's default, and we can conclude that the cause for corporate default for this period is business worsening.

Next, looking at selected variables on Table 6, as for original data, the data besides CI leading series and overnight call rate as short term interest rate are selected evenly. On the other hand, as for dataset after HP decomposition, trend component or cycle component are selected in most of datasets. As for CPI Composite Index (excluding fresh food), on some cases both components are selected exceptionally. Depending each variable's attribute, either trend component or cycle component has a meaning. Hence, to draw a valid conclusion, we must discuss the analysis in more detail.

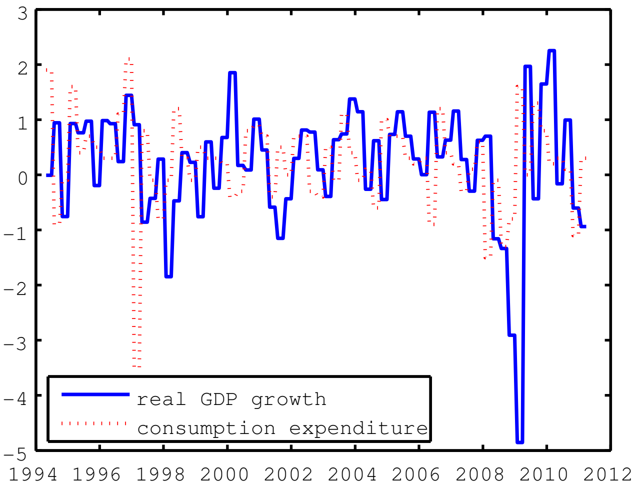

Figure 7: Comparison of real private final consumption expenditure's change with real GDP growth rate's change (Both curves are respectively based on the comparison with the previous quarter.)

3.3 Decision of Weight for Credit Cycle Index

In this section, we decide the weight for credit cycle index by two methods. One method is based on EDF data, the other method is the optimization of weight based on distance measure.

3.3.1 Derivation of Weight Based on EDF Data

We calculate the weight of the credit cycle index included in eq.(1) from Sample 2. We can calculate from the coefficient $b(t)$ in trend component of default threshold of unstressed PD by PIT rating approach in eq.(15), and substitute Moody's Analytics's EDF for unstressed PD. But EDF data is expected default probability corresponding to ratings without notch. It is pointed out that there

Table 3: Candidates of explanatory variables on multiple regression model of credit cycle index as object variable (Set 1: original dataset, Set 2: Trend components and cycle components decomposed by HP filter)

NO	Explanatory variables candidates	Set	Renewal cycle	Time lag (mo)
1	Real GDP growth rate (s.a., p.q., annualized %)	1	quarterly	−3
2	Real GDP growth rate (s.a., p.q., annualized %) Trend	2	quarterly	−3
3	Real GDP growth rate (s.a., p.q., annualized %) Cycle	2	quarterly	−3
4	CPI Composite Index (s.a., year-to-year basis) (%)	1	monthly	−1
5	CPI Composite Index (s.a., year-to-year basis) Trend (%)	2	monthly	−1
6	CPI Composite Index (s.a., year-to-year basis) Cycle (%)	2	monthly	−1
7	Overall Unemployment rate(s.a., year-to-year basis) (%)	1	monthly	−1
8	Overall Unemployment rate(s.a., year-to-year basis) Trend (%)	2	monthly	−1
9	Overall Unemployment rate(s.a., year-to-year basis) Cycle term (%)	2	monthly	−1
10	CI leading series (month-to-month basis)	1	monthly	−3
11	CI leading series (month-to-month basis) Trend	2	monthly	−3
12	CI leading series (month-to-month basis) Cycle	2	monthly	−3
13	CI coincident series (month-to-month basis)	1	monthly	zw0
14	CI coincident series (month-to-month basis) Trend	2	monthly	zw0
15	CI coincident series (month-to-month basis) Cycle	2	monthly	zw0
16	Real private final consumption expenditure (s.a., p.q.)	1	quarterly	−3
17	Real private final consumption expenditure (s.a., p.q.) Trend	2	quarterly	−3
18	Real private final consumption expenditure (s.a., p.q.) Cycle	2	quarterly	−3
19	Overnight call rate (month end) latest value	1 & 2	daily	−1
20	10 years JGB yield (month end) latest value	1 & 2	monthly	−1
21	10 years JGB yield (month end) − overnight call rate (month end)	1 & 2	monthly	−1
22	TOPIX (first section of Tokyo Stock Exchange, month-to-month basis) latest value	1 & 2	monthly	−1

Note: s.a.: seasonally-adjusted, p.q.: compared with the previous quarter

Table 4: Description statistics for explanatory variables and object variable

	zwVariable candidates	zwSign	Sample 1			Sample 2			After Lehman shock		
			Ave. (runs test)	stderr	N	ave. (runs test)	stderr	N	Ave. (runs test)	stderr	N
Obj. func.	unstandardized threshold below R&I BB		-3.5571	0.2573	120	-4.3713	2.3186	84	-5.8739	3.3337	31
zwOriginal zwvariables	Real GDP growth rate	−	1.1760	3.1250	120	0.3363	5.5580	84	-0.4028	7.7943	31
	CPI CI	+	0.0265	0.8210	120	-0.1533	0.9383	84	-0.8415	0.9154	31
	Overall unemployment rate (%)	+	6.9470	8.8507	120	-0.6043	15.7722	84	7.8269	22.8019	31
	CI leading series (m-to-m)	−	0.0833	1.2010	120	0.0179	1.7150	84	0.7258	2.1259	31
	CI coincident series (m-to-m)	−	0.1183	0.8562	120	0.0774	1.2392	84	0.1194	1.7526	31
	Real private final consumption expenditure (s.a., p.q.)	+/−	0.2800	0.8923	120	0.1357	0.7577	84	0.2194	0.8388	31
	Overnight call rate (month end) latest value	+/−	0.4314	0.6248	120	0.1980	0.2185	84	0.1134	0.0655	31
	10 years JGB yield (month end) latest value	+	2.0114	1.0203	120	1.4498	0.2289	84	1.2542	0.1473	31
	10 years JGB yield − overnight call rate (month end)	+/−	1.5800	0.5634	120	1.2518	0.2589	84	1.1408	0.1381	31
	TOPIX (m-to-m) latest value	+/−	-0.0015	0.0444	120	42.8603	193.0377	84	116.1368	307.0865	31
zwHP decomp. zwvariables	GDP real trend	+/−	1.1478	1.2167	120	0.3640	2.1787	84	-0.3477	1.8389	31
	GDP real cycle	+/−	0.0282	2.6404	120	-0.0276	4.5349	84	-0.0551	6.7336	31
	CPI CI (s.a., y-to-y) Trend (%)	+/−	0.0379	0.5606	120	-0.1201	0.4065	84	-0.5417	0.3223	31
	CPI CI (s.a., y-to-y) Cycle (%)	+/−	-0.0115	0.5155	120	-0.0331	0.7365	84	-0.2998	0.8177	31
	Overall unemployment rate(s.a., y-to-y) Trend (%)	+/−	7.2355	5.0374	120	-1.9394	9.5794	84	-0.3288	13.7146	31
	Overall unemployment rate(s.a., y-to-y) Cycle (%)	+/−	-0.2885	6.7375	120	1.6660	9.6635	84	9.0523	11.3903	31
	CI leading series (m-to-m) Trend	+/−	0.0763	0.1785	120	0.0606	0.4440	84	0.5045	0.3247	31
	CI leading series (m-to-m) Cycle	+/−	0.0071	1.1415	120	-0.0427	1.6232	84	0.2213	2.1209	31
	CI coincident series (m-to-m) Trend	+/−	0.0884	0.1416	120	0.1029	0.2923	84	0.2140	0.3167	31
	CI coincident series (m-to-m) Cycle	+/−	0.0299	0.8136	120	-0.0255	1.1710	84	-0.0946	1.6558	31
	Real private final consumption expenditure Trend	+/−	0.2646	0.1689	120	0.1655	0.2180	84	0.2302	0.1474	31
	Real private final consumption expenditure Cycle	+/−	0.0154	0.8561	120	-0.0298	0.6820	84	-0.0108	0.7735	31

Note: Obj. func.: Object function, s.a.: seasonally-adjusted, p.q.: compared with the previous quarter, m-to-m: month-to-month basis, y-to-y: year-to-year basis

Table 5: Modified R-squared for multiple regression analysis based on probit model

zwDataset	Data period		
	Sample 1	Sample 2	After Lehman shock
Original	0.394	0.605	0.798@
HP decomp.	0.588	0.836	0.895

Note: Sample 1F1994/4–2004/3CSample 2F2004/4–2011/3C
After Lehman shockF2008/9–2011/3

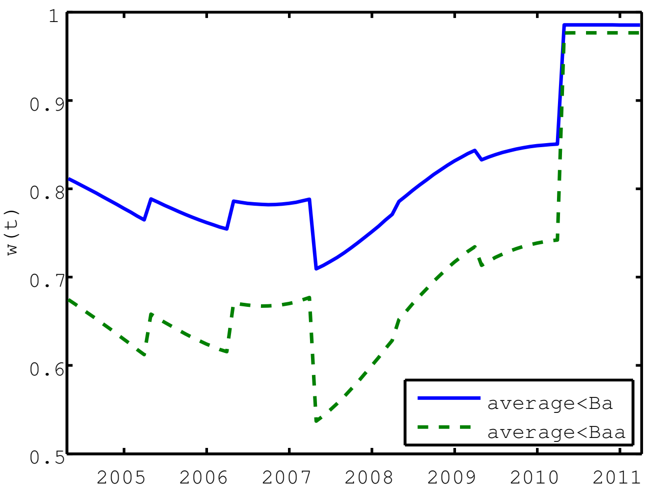

Figure 8: Change of weight $w(t)$ for credit cycle index by using EDF data

Note: Legend's "average<Ba" and "average<Baa" respectively indicate the averages of EDFs by rating grade without notch below Moody's Ba or below Baa in calculation of $P^U_{PIT}(t)$.

Table 6: Parameter estimates for multiple regression analysis based on probit model

zwData type		zwSelected explanatory variables	Unstd. coeff. β_j	stderr	Std. coeff. beta	t-value	Signif. prob.	Multico. VIF
zwSample 1	zwOriginal	(const.)	−3.085	0.056		−55.095	0.000	
		CPI CI (%)	0.049	0.023	0.155	2.110	0.037	1.064
		10 years JGB yield − overnight call rate (month end)	−0.300	0.034	−0.656	−8.917	0.000	1.064
	zwHP decomp.	(const.)	−3.289	0.052		−63.255	0.000	
		Real GDP growth rate (s.a., p.q., annualized) Cycle	−0.015	0.007	−0.151	−2.080	0.040	1.517
		CPI CI (s.a., y-to-y basis) Cycle (%)	0.197	0.032	0.395	6.098	0.000	1.214
		Overall unemployment rate(s.a., y-to-y basis) Trend	0.010	0.004	0.199	2.787	0.006	1.467
		CI leading series (m-to-m basis) Trend	0.670	0.103	0.465	6.503	0.000	1.476
		10 years JGB yield − overnight call rate (month end)	−0.248	0.029	−0.542	−8.521	0.000	1.169
		Real private final consumption expenditure Cycle	0.060	0.022	0.201	2.788	0.006	1.499
zw Sample 2	zwOriginal	(const.)	−11.074	1.139		−9.721	0.000	
		10 years JGB yield (month end) latest value	4.754	0.767	0.469	6.199	0.000	1.205
		TOPIX (m-to-m basis) latest value	−0.003	0.001	−0.269	−3.374	0.001	1.338
		Overall unemployment rate(s.a., y-to-y basis) (%)	0.050	0.012	0.343	4.200	0.000	1.401
		CPI CI (s.a., y-to-y basis) (%)	0.692	0.210	0.280	3.293	0.001	1.520
		Real private final consumption expenditure	0.624	0.233	0.204	2.672	0.009	1.223
	zwHP decomp.	(const.)	−5.845	0.676		−8.640	0.000	
		CPI CI (s.a., y-to-y basis) Trend (%)	4.492	0.432	0.788	10.395	0.000	2.899
		CPI CI (s.a., y-to-y basis) Cycle (%)	0.436	0.201	0.139	2.167	0.033	2.066
		Overall unemployment rate(s.a., y-to-y basis) Trend (%)	0.163	0.016	0.675	10.271	0.000	2.180
		CI coincident series (m-to-m basis) Cycle	0.182	0.097	0.092	1.870	0.065	1.222
		10 years JGB yield − overnight call rate (month end)	1.011	0.598	0.113	1.691	0.095	2.248
		TOPIX (m-to-m basis) latest value	0.002	0.001	0.133	2.097	0.039	2.031
		Real private final consumption expenditure Trend	6.137	1.090	0.577	5.631	0.000	5.304
zw zwAfter Lehman shock	zwOriginal	(const.)	−22.048	2.657		−8.299	0.000	
		Real GDP growth rate (s.a., p.q., annualized %)	−0.182	0.062	−0.426	−2.932	0.007	3.138
		Overall unemployment rate(s.a., y-to-y basis) (%)	0.037	0.019	0.253	1.977	0.059	2.427
		10 years JGB yield (month end) latest value	12.588	2.203	0.556	5.715	0.000	1.410
		TOPIX (m-to-m basis) latest value	−0.003	0.001	−0.287	−2.648	0.014	1.750
		Real private final consumption expenditure	1.757	0.589	0.442	2.982	0.006	3.268
	zwHP decomp.	(const.)	−4.025	0.498		−8.087	0.000	
		Real GDP growth rate (s.a., p.q., annualized) Cycle (%)	−0.114	0.040	−0.230	−2.862	0.008	1.835
		CPI CI (s.a., y-to-y basis) Trend (%)	14.866	1.074	1.437	13.843	0.000	3.068
		CPI CI (s.a., y-to-y basis) Cycle (%)	1.445	0.468	0.354	3.086	0.005	3.753
		Real private final consumption expenditure Trend	28.814	3.844	1.274	7.496	0.000	8.218

is a little difference between EDF's ratings and R&I rating, Moody's rating is nearly two notches lower than R&I's one (refer to Kanno (2011)Cpp.147–149.). Hence, we calculate the time-series of weight $w(t)$ for EDFs below Moody's Ba and ones below Baa and drawn Figure 8. Looking at Figure 8, though the weight $w(t)$ once decreased in 2007/4, hereafter it has increased constantly for the period of Sub-prime loan problem and Global financial crisis. The weight average is 0.66363 for rating grade below Baa and 0.79293 for one below Ba.

3.3.2 Optimization of Weight Based on Distance Measure

We estimate the weight $w(t)$ of the credit cycle index $Z(t)$ by some distance measures for the rating migration probability matrix. Here we use two types of variables synthesized from selected explanatory variables on Table 6 which are selected in previous section as credit cycle index $Z(t)$. Then we estimate weight $w(t)$ by solving the following nonlinear optimization problem.

$$\min_{w(t)\in[0,1]} \sum_j \sum_i F(i,j,p(t;i,j),\hat{p}(t,w(t);i,j|z(t))) \tag{20}$$

where $p(t;i,j)$ indicates an actual value for (i,j) entry on the rating migration probability matrix, and $\hat{p}(t;i,j|z(t))$ indicates an estimate for (i,j) entry on the rating migration probability matrix conditional on $z(t)$ in eq.(5). The weight of each entry on both matrices is allocated by function F. To get better estimates, we set some risk sensitive distance measures which optimize the distance between actual rating migration probability matrix and forecasting rating migration probability matrix. First of all, we set two distance measures proposed in Trück and Rachev (2005).

$$D_1(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n-1} d(i,j) + \sum_{i=1}^{n} n \cdot d(i,n) \tag{21}$$

$$D_2(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n-1} d(i,j) + \sum_{i=1}^{n} n^2 \cdot d(i,n) \tag{22}$$

where distance $d(i,j)$ for (i,j) entry is as follows.

$$d(i,j) = (i-j)\cdot(p_{ij} - q_{ij}) \tag{23}$$

where p_{ij} is the actual value $p(t,i,j)$ of (i,j) entry on the rating migration probability matrix P in eq.(20), and q_{ij} is the estimated value $\hat{p}(t,w(t);i,j|z(t))$ of (i,j) entry on the conditional rating migration probability matrix Q in eq.(20).

As $d(i,n)$ indicates a distance between default migration probability from rating grade i to default grade, two measures D_1 and D_2 respectively impose a coefficient of n times and n^2 times larger on migration to default than on one to other entry.

In Trück and Rachev (2009), some distance measures are proposed, and we present the following standardized absolute difference measure.

$$D_{NSD}(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{|p_{ij} - q_{ij}|}{p_{ij}} \tag{24}$$

where we mention $D_{NSD}(Q,P)$ as another absolute difference measure different from $D_{NSD}(P,Q)$. As $D_{NSD}(P,Q) \neq D_{NSD}(Q,P)$ apparently, we must take care that symmetric condition is not applicable.

And we mention weighted absolute difference WAD as another measure as follows.

$$D_{WAD}(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}|p_{ij} - q_{ij}| \tag{25}$$

where we can mention $D_{WAD}(Q,P)$ as a absolute difference measure different from $D_{WAD}(P,Q)$ as well as NSD, but as $D_{WAD}(P,Q) \neq D_{WAD}(Q,P)$ apparently, we must also take care that symmetric condition is not applicable.

Finally as ordinary measures, we mention the measures for first order norm L_1 and second order norm L_2. The two measures are expressed as follows.

$$D_{L_1}(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n} |p_{ij} - q_{ij}| \tag{26}$$

$$D_{L_2}(P,Q) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} (p_{ij} - q_{ij})^2} \tag{27}$$

Now we optimize eight distance measures by eq.(20) using data for Sample 1 (from 1994 to 2010) and Sample 2 (from 2004 to 2010). The result is on Table 7. But with regard to the measures besides D_1, D_2, as we could not get the weight $w(t)$ satisfying $w(t) \in [0,1]$, we omitted them. And with regard to D_1, D_2 too, we could get some optimal values, but could not get optimal values for almost years in Sample 1. As a result, we show only the result for Sample 2. According to Table 7, optimal values are almost in a range from 0.7 to 1. It is obvious that Japanese corporates credits were severely affected by macroeconomic variables for this period.

Compared to the change of weight based on EDF data in previous section, as common points, an average for EDF below Moody's Ba is 0.79 and an average of weight in regression model is 0.75, hence both weights are very similar. In addition, though EDF once dropped in 2007, hereafter it increased in latter of 0.9.

Next we select the model which offers the optimal forecast for rating migration probability matrix using eight distance measures. The higher forecasting accuracy of model is, the smaller gap between forecasting rating migration probability matrix and actual rating migration probability matrix is. We use the regression model on original dataset, the regression model using HP decomposed dataset, and the two adjustment models for rating migration probability matrix proposed by Lando (1999)—Lando model and JLT model for comparison of their forecasting accuracies[3]. JLT model was an approach proposed in Jarrow et al. (1997). Absolute minimum distance on eight distance measures for four models are shown on Table 8. Judged from the definition in eq.(23), as D_1 and D_2 may be plus or minus, their minimum values are given as absolute minimum distances. On Table 8, D_2, L_1 and L_2 are all minimal in regression model based on original dataset and L_1, NSD_2, WAD_1 and WAD_2 are all minimal in regression model based on HP decomposed dataset. As a result, these two models are best in six out of eight distance measures. Especially as for five distance measures of L_1, L_2, NSD_2, WAD_1 and WAD_2, their values are much near zero and two regression models' forecast accuracies are much higher than the two other models' ones. As the model of which distance measure is smallest among four models give a minimum distance between actual rating migration probability matrix and forecasting one, two regression models respectively give the optimal weights for the credit cycle index on three or four distance measures.

3.3.3 Out-of-Sample Forecast of Rating Migration Probability

Based on the cumulative rating migration probability matrix for Sample 1 period (1994 to 2003) as in-sample period, we estimate the yearly cumulative rating migration probability matrix for Sample 2 period (2004 to 2010) as out-of-sample period. Namely we extend cumulative period of rating migration probability matrix on yearly basis (1994 to 2004, 1994 to 2005, $\cdots$, 1994 to 2010), calculate the actual cumulative rating migration probability matrix and rating thresholds by using their matrices. And when we re-estimate the

[3]Refer to Appendix A for the detail.

Table 7: Estimates of weight $w(t)$ for the credit cycle index by in-sample forecast

Dataset	Year	D_1	D_2	Dataset	Year	D_1	D_2
$^{\mathrm{zw}}$Original	2004	–	–	$^{\mathrm{zw}}$HP	2004	–	–
	2005	0.99348	0.99348	$^{\mathrm{zw}}$decomp.	2005	0.73134	0.73134
	2006	0.93724	0.92693		2006	0.99030	0.99040
	2007	0.99890	0.99889		2007	0.32673	0.23588
	2008	–	–		2008	–	–
	2009	0.85073	0.85065		2009	0.94708	0.93952
	2010	–	–		2010	0.77428	0.77428
	Average	0.94509	0.94249		Average	0.75395	0.73428

Note: Hyphen(−) indicates no minimum value exists.

weight $w(t)$ by optimization in eq.(20), we can get the forecast rating migration probability matrix as out-of-sample. The re-estimated weights are provided on Table 9. Looking at Table 9, there is not an obvious trend with regard to the change of the weight's estimate, compared to the in-sample estimate on Table 7. But as for the dataset after HP decomposition, out-of-sample estimates are higher than in-sample ones, excluding 2005.

Table 8: Absolute minimum distance by distance measure and rating migration probability matrix estimation model

Model	Original	HP decomp.	Lando	JLT
D_1	2.40455270	6.57061053	2.56366758	**1.74918841**
	(5.4854770)	(11.0474910)	(4.0955837)	(4.5494765)
D_2	**13.55883688**	37.00077452	17.82356664	14.35506750
	(36.6072340)	(70.0300658)	(28.3610118)	(27.3381453)
L_1	**0.00000401**	**0.00000401**	0.53441241	4.14887251
	(0.0000000)	(0.0000000)	(0.4840833)	(2.3544114)
L_2	**0.00000124**	0.27579759	0.22989649	1.30765011
	(0.0000001)	(0.7296886)	(0.2177961)	(0.5893528)
NSD_1	2.06044377	2.06044358	**2.01942810**	50.95213457
	(5.4510328)	(5.4510330)	(2.4282915)	(23.5362635)
NSD_2	0.00013681	**0.00013659**	13.46063992	(5.5869E+13)
	(0.0000418)	(0.0000418)	(4.5276188)	(8.3295E+13)
WAD_1	0.00000197	**0.00000195**	0.20054125	1.83700566
	(0.0000003)	(0.0000003)	(0.1886539)	(0.9510112)
WAD_2	0.00000167	**0.00000165**	0.20911490	1.36726086
	(0.0000008)	(0.0000008)	(0.2073474)	(0.6664059)

Note: The upper row in each model indicates an average value from 2004 to 2010, and the lower row (in parentheses) indicates a standard deviation in same period. Bold letters show the model which has a minimum value in each distance measure.

Table 9: Estimates of weight $w(t)$ for the credit cycle index by out-of-sample forecast

Dataset	Year	D_1	D_2
^{zw}Original	2004	–	–
	2005	0.99979	0.99980
	2006	0.98137	0.98006
	2007	0.98532	0.98574
	2008	–	–
	2009	0.85529	0.85073
	2010	0.00000	0.00000
	Average	0.95544	0.95408

Dataset	Year	D_1	D_2
^{zw}HP	2004	0.99999	0.99999
^{zw}decomp.	2005	0.79866	0.80258
	2006	0.99821	0.99819
	2007	0.46449	0.37892
	2008	–	–
	2009	0.94342	0.93956
	2010	0.98896	0.98651
	Average	0.86562	0.85096

Note: Hyphen(−) indicates no minimum value exists.

4 Conclusion

This research focuses on the influence of the business cycle in a corporate's credit valuation method and is one of the most advanced researches to incorporate the business cycle into a risk valuation model. Currently also in the bank's internal rating model, when we evaluate a corporate's credit, the main approach is the evaluation of an individual corporate's credit risk or a corporate group's one with financial indexes as major explanatory variables. However, the influence which macroeconomic variables affect on individual corporates is considerable, and our research contributes in the credit risk valuation in below points.

1. We formulated a new factor model considering macroeconomic variables.
2. We decomposed some major macroeconomic variables into the trend and cycle components by HP filter, using the data related to Japanese economy and corporates and selected the variables which affected on corporates' credits.
3. We calculated weight for the credit cycle index which is the systematic risk factor for Japanese corporates and analyzed what macroeconomic variables affected Japanese corporates in previous days.

But regrettably we could not get the enough explicit analysis output with related to the trend and cycle from Japanese corporates default data, which is different from European or American corporates default data. Hence in the near future we would like to analyze on European or American data and compare its output with Japanese one.

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Appendix A Adjustment method for rating migration probability matrix

We outline an adjustment method for a rating migration probability matrix by Lando (1999) and Jarrow et al. (1997). First of all, we express a discounted corporate bond's price by the following equation.

$$v^i(0,t) = \delta B(0,t) + (1-\delta)B(0,t)S^i(0,t) \tag{28}$$

where $\delta \in (0,1)$ is the recovery rate, $B(0,t)$ is the discounted JGB price with maturity t at time 0, and $S^i(0,t)$ is the probability in which corporate i exists till time t. Though the recovery rate is different depending on the attributes of instrument such as loan or corporate bond, default time, recovery period and evaluation time, it is said an average value is 50%.

From this equation, we get the following relation.

$$S^i(0,t) = \frac{v^i(0,t) - \delta B(0,t)}{(1-\delta)B(0,t)} \tag{29}$$

Hence we get the probability of default as $1 - S^i(0,t)$. Then we assume that there is a matrix matrix Λ which satisfy the following equation.

$$\Lambda = \begin{pmatrix} -\lambda_{1,1} & \lambda_{1,2} & \cdots & \lambda_{1,K} \\ \lambda_{2,1} & -\lambda_{2,2} & \cdots & \lambda_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{K,1} & \lambda_{K,2} & \cdots & -\lambda_{K-1,K} \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{30}$$

where,

$$\begin{aligned} \sum_{j=1, j\neq i}^{K} \lambda_{ij} &= \lambda_i \\ \lambda_i &\geq 0, \quad i = 1, \ldots, K \end{aligned}$$

And,

$$\exp(\Lambda) = P(0,1)$$

We assume that there exists one year rating migration probability matrix $P(0,1)$. The purpose is to generate a specified migration matrix $Q(0,1)$ by a modifying matrix $P(0,1)$ so as which last column's entries to be in accordance with corresponding ones. One year implied probability of default to the rating category i is $1 - S^i(0,1)$, finally we seek for a migration matrix $Q(0,1)$ satisfying $Q(0,1)_{iK} = 1 - S^i(0,1)$. Hence the entries other than the probabilities of default in $Q(0,1)$ don't correspond to ones in original migration matrix $P(0,1)$. And extending $Q(0,1)$ with high order, we can calculate a modified matrix $\bar{\Lambda}$ as follows.

$$\bar{\Lambda} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(Q(0,1) - I)^k}{k}, \quad n \in \mathbb{N} \tag{31}$$

where the modified procedure is different between Lando model and JLT model.

Lando model Multiplying default columns on the generator matrix by π_i ($i = 1, \ldots, K$) and modifying diagonal entries in a generator matrix $\bar{\Lambda}$ as follows, we get a modified generator matrix $\tilde{\Lambda} = (\tilde{\lambda}_{i,j})$ ($i = 1, \ldots, K; j = 1, \ldots, K-1$).

$$\tilde{\lambda}_{i,K} = \pi_i \cdot \lambda_{i,K}, \quad \tilde{\lambda}_{i,i} = \lambda_{i,i} - (\pi_i - 1) \cdot \lambda_{i,K} \tag{32}$$

JLT model This method is to multiply an each row in the generator matrix by π_i ($i = 1, \ldots, K-1$) and the modified generator matrix is expressed as follows.

$$\tilde{\Lambda} = \begin{pmatrix} \pi_1 \cdot \lambda_{1,1} & \pi_1 \cdot \lambda_{1,2} & \cdots & \pi_1 \cdot \lambda_{1,K} \\ \pi_2 \cdot \lambda_{2,1} & \pi_2 \cdot \lambda_{2,2} & \cdots & \pi_2 \cdot \lambda_{2,K} \\ \cdots & \cdots & \cdots & \cdots \\ \pi_{K-1} \cdot \lambda_{K-1,1} & \pi_{K-1} \cdot \lambda_{K-1,2} & \cdots & \pi_{K-1} \cdot \lambda_{K-1,K} \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{33}$$

Finally we can a modified migration matrix $\tilde{Q}(0,1)$ by the modified generator matrix $\tilde{\Lambda}$ by any of two models as follows.

$$\tilde{Q}(0,1) = \exp(\tilde{\Lambda}) = \sum_{k=0}^{\infty} \frac{\tilde{\Lambda}^k}{k!} \tag{34}$$

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