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Credit risk based on firm conduct-performance and bank lending decisions: A capped call approach

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Abstract

This paper models loan rate-setting behavior, taking into account the product pricing and performance of the borrowing firm, and also calculates the bank's loan-risk sensitive equity values. The lending function creates the need to model bank equity as a capped call option, which captures the credit risk directly related to management of a firm's operations. When the product price set by the borrowing firm is relatively high and the loan rate set by the bank is relatively low, a rise in the product price increases the loan amount at a reduced margin. A capped call as such makes the bank less prudent and more prone to risk-taking, thereby adversely affecting the stability of the banking system. We also show that the market-based estimates of bank equity, which ignore the cap, lead to significant overestimation.

JEL classification numbers: G21, G28
Keywords: Pricing, Credit risk, Firm conduct-performance, Bank lending, Capped call

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1 Introduction

One of the risks of making a bank loan is credit risk, the risk of default on the part of the borrowing firm. Psillaki et al. (2010) argue that credit risk is the most significant risk for banks, as the devastating effects of the current financial crisis will affirm, and postulate that a combination of financial and productive factors should enhance a bank's ability to predict a lending default more accurately than a model that relies solely on the use of financial indicators[3]. Their paper explains why less profitable borrowing firms are more likely to fail. Although considerable research on the market-based evaluation of bank equity has modeled the bank as a corporate firm with a call option on the bank's risky assets, no attempt has been made to explicitly analyze the credit risk characteristics of bank lending[4]. Through the effective management of credit risk exposure, banks not only support the viability and profitability of their own business but also contribute to market stability. Our contribution to literature is to extend the work of Psillaki et al. (2010) by explicitly modeling bank lending on the risk of default by the borrowing firm, related to product pricing and performance, and calculate the bank's loan-risk sensitive equity.

Recent relevant background to our paper includes the studies by Ravi Kumar and Ravi (2007), Ravi et al. (2008), Fethi and Pasiouras (2010), Psillaki et al. (2010), Brissimis and Delis (2011), and Lu (2012). The review of Ravi Kumar and Ravi (2007) presents the issue of bankruptcy prediction in banks and firms via statistical and intelligent techniques. This review concentrates on the use of financial factors, such as liquidity, profitability and capital structure, in risk evaluation. Ravi et al. (2008) also discuss applications of statistical and intelligent techniques in bankruptcy prediction. They primarily deal with productive firms. Fethi and Pasiouras (2010) present a survey which uses operational research and intelligent techniques to assess bank performance. Their survey also concentrates on the use of financial factors such as capital strength, asset quality, ownership and auditing. Psillaki et al. (2010) argue that the role of non-financial information remains largely unexplored, and then use the concepts of Färe et al. (2007) to evaluate credit risk based on firm performance. However, the existences of bank-level market power as pointed out by Brissimis and Delis (2011) and bank

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[3] We quote three studies from Wagner et al. (2009) to explain the important role played by productive factors in the postulation of Psillaki et al. (2010). Company default rates in the United States are at a record high (Keenan et al., 2000). The financial situation of many automotive suppliers has deteriorated (Murphy et al., 2005). Automotive News (2006) reports that "at least 38% of North America's auto parts makers are in fiscal danger and could face bankruptcy during the next two years, which means more bankruptcy filings by public and private suppliers and heightened risks for automakers."

[4] One exception in this literature is Dermine and Lajeri (2001) that explicitly models the risk characteristics of bank assets.

size as pointed out by Chang et al. (2011) are ignored in Psillaki et al. (2010). Lu (2012) adopts a continuous-time non-homogeneous mover-stayer model for the measurement of the credit risk associated with bank loans. While we also analyze credit risk related to bank equity evaluation based on firm performance, our focus on the inclusion behavioral modes of loan rate-setting and product pricing takes our analysis in a different direction.

The bank interest margin, the spread between the loan rate and deposit rate, is one of the principal elements of bank net cash flows and after-tax earnings. The margin is often used in the literature as a proxy for the efficiency of financial intermediation[5]. Based on previous work, the purpose of this paper is to develop an option-based model of the bank-borrowing firm behavior that integrates the productive information about borrower default in the product market with financial information about bank operations in the loan market. To this end, this paper calls attention to the fact that credit risk with productive information affects the distribution of bank asset returns, so that the standard Merton (1974) approach used to provide market-based estimation of bank equity returns needs to be adapted. The lending function of the bank creates specific risk characteristics and the necessity to model the bank's equity as a "capped" call option. The results are as follows: Our simulation exercise shows that market-estimates of the bank's equity value which ignore the cap lead to significant overestimation. We also show that when the product price chosen by the borrowing firm is relatively high and the loan rate set by the bank is relatively low, the risk of borrower default increases, the bank's equity value based on a capped call decreases and the overestimation of the bank's equity value increases with the product price and the margin. Under these circumstances, the effect of product pricing makes the bank increase its loan amount at a reduced margin. Cap structure, to an equal degree, forces the bank to be less prudent and more prone to risk-taking, thereby adversely affecting the stability of the banking system.

The paper is organized as follows: Section 2 develops the basic structure of the paper. Section 3 examines the marginal effect of the borrowing firm's product price on the optimal bank interest margin. Section 4 presents a numerical analysis. The final section contains the conclusion.

2 The model

We consider a bank-borrowing firm model where economic decisions are made and values are determined with only a one-period horizon, $t \in [0, 1]$. At $t = 0$, a firm is funding its production Q with a bank loan L and equity K_f. The production function is specified as $Q(L)$, where the marginal product is

[5] See, for example, Saunders and Schumacher (2000), and Wong (2011).

positive, $\partial Q / \partial L > 0$. The product market is assumed to be imperfectly competitive in the sense that the firm faces a demand curve, $P(Q)$, where $\partial P / \partial Q < 0$. The total revenue at $t = 1$ is expected to be $V_f = PQ$ and the loan payment to the bank is: $(1 + R_L)L$, where R_L is the loan rate. The market value of the firm's assets (total revenue, V_f) varies continuously over $t \in [0, 1]$ according to the stochastic process: $dV_f = \mu_f V_f dt + \sigma_f V_f dW_f$, where μ_f is the instantaneous expected rate of return on the asset, σ_f is the instantaneous standard deviation of the return, and W_f is a Wiener process. The stochastic process implies that V_f will follow a lognormal distribution.

Given the limited liability of the firm, the loan payment to the bank is reduced by a put option given to the borrower who can sell V_f; that is, the bank takes over the revenue of the firm when it defaults. As is well known from option theory (Merton, 1974), the put option is modeled as follows:

$$P_f = (1 + R_L)Le^{-R_L} N(-a_2) - V_f N(-a_1) \tag{1}$$

where

$$a_1 = \frac{1}{\sigma_f}(\ln \frac{V_f}{(1 + R_L)L} + R_L + \frac{\sigma_f^2}{2}),$$

$$a_2 = a_1 - \sigma_f,$$

and $N(\cdot)$ = the cumulative density function of the standard distribution.

The bank accepts D dollars of deposits at $t = 0$, and provides depositors with a market rate of return, R_D. Equity capital is denoted by K, which is assumed to be tied by regulation to a fixed proportion q of the bank's deposits, $K \geq qD$, to satisfy the capital adequacy requirement. The capital-to-deposits ratio q is further assumed to be an increasing function of L, $\partial q / \partial L = q' > 0$.[6] The bank makes term loans which mature at $t = 1$. Loan demand faced by the bank is $L(R_L)$ with $\partial L / \partial R_L < 0$, which implies that the bank exercises some monopoly power in lending activities (Mukuddem-Petersen et al., 2008). The bank also holds an amount B of liquid assets at $t \in [0, 1]$, which earns the security-market rate of R. When the capital constraint is binding, the bank's balance-sheet constraint is:

$$L + B = D + K = K(1/q + 1).$$

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[6] The capital-to-deposits ratio can capture bank operational risk (see, for example, Basel Committee on Banking Supervision, 2001).

With the explicit treatment of the credit risk from the borrowing firm, the market value of the bank's underlying asset $V = (1+R_L)L - P_f$ varies continuously over $t \in [0, 1]$ according to the stochastic process:

$$dV = \mu V dt + \sigma V dW ,$$

where μ is the instantaneous expected rate of return on the asset, σ is the instantaneous standard deviation of the return, and W is a Wiener process.[7] Z is the strike price of the call, which is specified as the book value of the net obligation: $(1+R_D)K/q-(1+R)[K(1/q+1)-L]$. The market value of the bank's C_b is given by the capped call option:

$$C_b = VN(d_1) - Ze^{-\delta} N(d_2) \tag{2}$$

where

$d_1 = \frac{1}{\sigma}(\ln \frac{V}{Z} + \delta + \frac{\sigma^2}{2})$,

$d_2 = d_1 - \sigma$,

and $\delta = R - R_D$.

Alternatively, if the credit risk from the borrowing firm is not explicitly considered, the market value of the bank's underlying asset $V_b = (1+R_L)L$ varies continuously over $t \in [0, 1]$ according to the stochastic process:

$$dV_b = \mu_b V_b dt + \sigma_b V_b dW_b ,$$

where μ_b is the instantaneous standard deviation of the return and W_b is a Wiener Process. Z_b is the strike price of the call, which is equal to Z. The market value of the bank's S_b is given by the "naked" call option:

$$S_b = V_b N(b_1) - Ze^{-\delta} N(b_2) \tag{3}$$

where

$b_1 = \frac{1}{\sigma_b}(\ln \frac{V_b}{Z_b} + \delta + \frac{\sigma_b^2}{2})$,

and $b_2 = b_1 - \sigma_b$.

Notice that although the value of the capped call option in (2) depends on P, the value of the naked call option in (3) does not, because the production of the

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[7] As pointed out by Bessler and Booth (1994), interest rate risk arises because the bank funds part of its fixed rate loans via variable rate deposits (for example, there is a mismatch in rate sensitivities of assets and liabilities). For simplicity, we do not ponder a situation with multiple sources of risk (both credit risk and interest rate risk).

firm is not explicitly taken into account in (3).

3 The impact of product price on optimal bank interest margins

First, the bank's objective is to set R_L to maximize the capped call. Partially differentiating Eq. (2) with respect to R_L, the first-order condition is given by:

$$\frac{\partial C_b}{\partial R_L} = \frac{\partial V}{\partial R_L} N(d_1) + V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} - \frac{\partial Z}{\partial R_L} e^{-\delta} N(d_2) - Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L} = 0 \qquad (4)$$

where

$$V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} = Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L}, \quad \frac{\partial d_1}{\partial R_L} = \frac{\partial d_2}{\partial R_L}$$

$$\begin{aligned}\frac{\partial V}{\partial R_L} &= (L + (1+R_L)\frac{\partial L}{\partial R_L}) \\ &\quad -[(\frac{\partial P}{\partial Q} Q + P)\frac{\partial Q}{\partial L}\frac{\partial L}{\partial R_L} N(a_1) - (L + (1+R_L)\frac{\partial L}{\partial R_L} - (1+R_L)Le^{-R_L} N(a_2)] \\ &\quad -[PQ \frac{\partial N(a_1)}{\partial a_1}\frac{\partial a_1}{\partial R_L} - (1+R_L)Le^{-R_L} \frac{\partial N(a_2)}{\partial a_2}\frac{\partial a_2}{\partial R_L}] \\ &\quad +[(\frac{\partial P}{\partial Q} Q + P)\frac{\partial Q}{\partial L}\frac{\partial L}{\partial R_L} - (L + (1+R_L)\frac{\partial L}{\partial R_L} - (1+R_L)Le^{-R_L}]\end{aligned}$$

$$PQ \frac{\partial N(a_1)}{\partial a_1}\frac{\partial a_1}{\partial R_L} = (1+R_L)Le^{-R_L} \frac{\partial N(a_2)}{\partial a_2}\frac{\partial a_2}{\partial R_L}, \quad \frac{\partial a_1}{\partial R_L} = \frac{\partial a_2}{\partial R_L}$$

$$\frac{\partial Z}{\partial R_L} = [\frac{(R - R_D)Kq'}{q^2} + (1+R)]\frac{\partial L}{\partial R_L} < 0$$

An inspection of equation (4) reveals that a necessary condition for the optimal loan rate based on the capped call is that the risk-adjusted value of the marginal loan repayment $(\partial V / \partial R_L) N(d_1)$ equals the risk-adjusted value of the marginal net obligation $(\partial Z / \partial R_L) e^{-\delta} N(d_2)$. The sufficient condition for the optimum is that: $\partial^2 C_b / \partial R_L^2 < 0$. Note that both the risk-adjusted values are negative in sign.

Alternatively, the bank's objective is to set R_L to maximize the naked call.

Partially differentiating (3) with respect to R_L, the first-order condition is given by:

$$\frac{\partial S_b}{\partial R_L} = \frac{\partial V_b}{\partial R_L} N(b_1) + V_b \frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial R_L} - \frac{\partial Z_b}{\partial R_L} e^{-\delta} N(b_2) - Ze^{-\delta}\frac{\partial N(b_2)}{\partial b_2}\frac{\partial b_2}{\partial R_L} = 0 \qquad (5)$$

where

$$V_b \frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial R_L} = Z_b e^{-\delta}\frac{\partial N(b_2)}{\partial b_2}\frac{\partial b_2}{\partial R_L}, \quad \frac{\partial b_1}{\partial R_L} = \frac{\partial b_2}{\partial R_L}$$

$$\frac{\partial V_b}{\partial R_L} = L + (1+R_L)\frac{\partial L}{\partial R_L} < 0$$

$$\frac{\partial Z_b}{\partial R_L} = \frac{\partial Z}{\partial R_L} < 0$$

A necessary condition for the optimal loan rate based on the naked call is that: $(\partial V_b / \partial R_L)N(b_1) = (\partial Z_b / \partial R_L)e^{-\delta}N(b_2)$. The sufficient condition for the optimum is that: $\partial^2 S_b / \partial R_L^2 < 0$.

The optimal bank interest margin is given by the difference between the optimal loan rate and the fixed deposit market rate. Since the deposit rate is not a choice variable of the bank, examining the impact of product price on the optimal bank interest margin is tantamount to examining that of the optimal loan rate. Consider next the impact on the bank's margin from changes in the firm's product price. Implicit differentiation of equation (4) with respect to P yields:

$$\frac{\partial R_L}{\partial P} = -\frac{\partial^2 C_b}{\partial R_L \partial P} / \frac{\partial^2 C_b}{\partial R_L^2} \qquad (6)$$

where

$$\frac{\partial^2 C_b}{\partial R_L \partial P} = \frac{\partial^2 V}{\partial R_L \partial P} N(d_1) - \frac{\partial V}{\partial R_L}\left(1 - \frac{VN(d_1)}{Ze^{-\delta}N(d_2)}\right)\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial P}$$

$$\frac{\partial^2 V}{\partial R_L \partial P} = -\Big[\frac{\partial Q}{\partial L}\frac{\partial L}{\partial R_L} N(a_1) + \left(\frac{\partial P}{\partial Q}Q + P\right)\frac{\partial Q}{\partial L}\frac{\partial L}{\partial R_L}\frac{\partial N(a_1)}{\partial a_1}\frac{\partial a_1}{\partial P}$$
$$-\left(L + (1+R_L)\frac{\partial L}{\partial R_L} - (1+R_L)L\right)e^{-R_L}\frac{\partial N(a_2)}{\partial a_2}\frac{\partial a_2}{\partial P}\Big] + \frac{\partial Q}{\partial R_L}\frac{\partial L}{\partial R_L}$$

$$\frac{\partial a_1}{\partial P} = \frac{1}{\sigma_f P} = \frac{\partial a_2}{\partial P} > 0$$

$$\frac{\partial d_1}{\partial P} = -\frac{1}{\sigma V}\left\{\left[QN(a_1) + PQ\frac{\partial N(a_1)}{\partial a_1}\frac{\partial a_1}{\partial P} - (1+R_L)Le^{-R_L}\frac{\partial N(a_2)}{\partial a_2}\frac{\partial a_2}{\partial P}\right] + Q\right\}$$

The sign of equation (6) is governed by $\partial^2 C_b / \partial R_L \partial P$. The first term on the right-hand side of $\partial^2 C_b / \partial R_L \partial P$ can be interpreted as the mean equity effect on $\partial V / \partial R_L$ from a change in P, while the second term can be interpreted as the variance or "risk" effect. Both the terms are indeterminate in sign. In general, the added complexity of the capped-call equity value does not always have clear results, but we can certainly speak of tendencies for reasonable parameter levels that roughly correspond to equation (6) with the bank's equity of equation (4). The numerical examples in the following section provide insight into the comparative static results of equation (6).

Table 1: Impact on R_L from changes in P denoted by P_f *.

(P, Q)	(R_L, L)				
	(3.75%, 240)	(4.00%, 236)	(4.25%, 232)	(4.50%, 228)	(4.75%, 224)
(10.0, 19)	49.907	45.932	41.980	38.063	34.196
(10.5, 18)	50.896	46.915	42.955	39.026	35.142
(11.0, 17)	52.879	48.889	44.914	40.964	37.051
(11.5, 16)	55.862	51.860	47.870	43.896	39.949
(12.0, 15)	59.848	55.838	51.834	47.839	43.860
(12.5, 14)	64.840	60.824	56.811	52.802	48.800
(13.0, 13)	70.837	66.818	62.801	58.784	54.770
(13.5, 12)	77.836	73.817	69.797	65.778	61.758
	(5.00%, 220)	(5.25%, 216)	(5.50%, 212)	(5.75%, 208)	(6.00%, 204)
(10.0, 19)	30.398	26.695	23.118	19.702	16.486
(10.5, 18)	31.322	27.589	23.973	20.509	17.235
(11.0, 17)	33.190	29.404	25.717	22.164	18.781
(11.5, 16)	36.039	32.186	28.411	24.742	21.215
(12.0, 15)	39.905	35.987	32.121	28.332	24.647
(12.5, 14)	44.811	40.841	36.902	33.008	29.180
(13.0, 13)	50.760	46.758	42.770	38.804	34.871
(13.5, 12)	57.740	53.724	49.712	45.708	41.716

* Parameter values, unless stated otherwise: $\sigma_f = 0.1$, $P = 19.5 - 0.5Q$, and $PQ < (1 + R_L)L$.

4 A numerical analysis

In bank interest margin management, computing changes in the option value to small changes in the constituent variable is essential for determining margins. Toward that end, we will compute the value functions of capped and naked call options. Starting from a set of assumptions on the value of the total revenue of the borrowing firm, the variance of its return, the size of the loan and the bank's net obligation, we can calculate the put of the borrowing firm, the capped call, and the naked call of the bank, which are consistent with equations (1) to (3). In a second step, a naked call is applied in a capped call to assess the extent of the bias by the difference in value between S_b and C_b.

In the first case reported in Table 1, we consider σ_f of 0.1 with the conditions of $P = 19.5 - 0.5Q$ and $PQ < (1 + R_L)L$ in Eq. (1). We let (R_L, L) change from (3.75%, 240) to (6.00%, 204) due to $\partial R_L / \partial L < 0$ and (P, Q) change from (10.0, 19) to (13.5, 12) due to $\partial P / \partial Q < 0$. Table 1 contains the put value. For a given level of R_L, the put value increases with the product price of the borrowing firm. This product price effect is unambiguously positive because an increase in the borrowing firm's product price decreases its revenue from sales. In response to this, the put value increases, ceteris paribus. For a given level of P, the put value decreases with the loan rate of the bank. This loan rate effect is negative because an increase in the bank's loan rate decreases the borrowing firm's liability payment to the bank. In response to this, the put value decreases, ceteris paribus. Since the positive product price effect is significantly offset by the negative loan rate effect when the product price set by the borrowing firm is relatively low and the loan rate set by the bank is relatively high, we have the result of $\partial R_L / \partial P < 0$ in the put valuation of P_f, for example, 49.907 at $(R_L, L) = (3.75\%, 240)$ and $(P, Q) = (10.0, 19)$; 46.915 at $(4.00\%, 236)$ and $(10.5, 18)$; 44.914 at $(4.25\%, 232)$ and $(11.0, 17)$; 43.896 at $(4.50\%, 228)$ and $(11.5, 16)$; and 43.860 at $(4.75\%, 224)$ and $(12.0, 15)$. Intuitively, as the non-performance from the borrowing firm is explicitly expressed as a capped put, the impact on the bank's interest margin from changes in the product price in the put depends on product price and loan rate levels. In particular, the borrowing firm's revenue is larger when the firm increases its product price at a relatively low level than at a relatively high level; the borrowing firm's liability payment is smaller when the bank increases its loan rate at a relatively high level than at a relatively low level. Thus, an increase in the product price denoted by the put value decreases the loan rate when the product price is low and the loan rate is high. However, the positive product price effect is only partially offset by the negative loan rate effect. When the product price is relatively high and the loan rate is relatively low, we have the result of $\partial R_L / \partial P > 0$ denoted by P_f, for example: 44.811 at $(5.00\%, 220)$ and $(12.5, 14)$; 46.758 at $(5.25\%, 216)$ and $(13.0, 13)$; and 49.712 at $(5.50\%, 212)$ and $(13.5, 12)$. These two results are used in a later subsection when the capped call is calculated. All positive put values provided in Table 1 are shown by the put value surface in Figure 1.

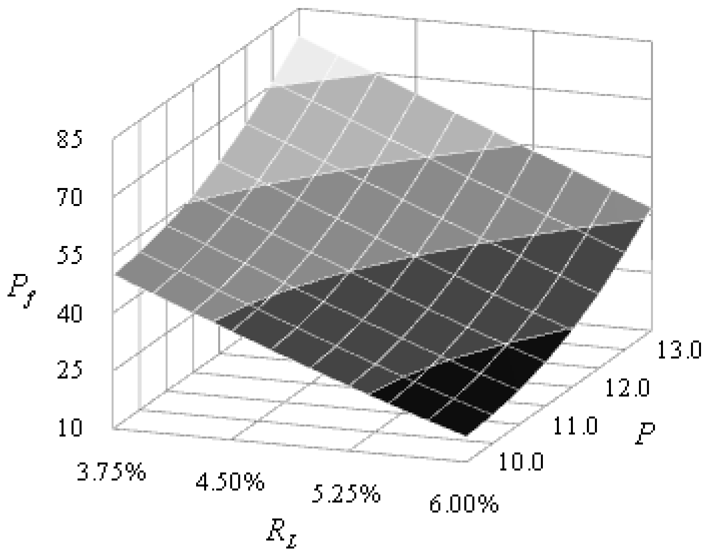

Figure 1: Impact on R_L from changes in P denoted by P_f

Table 2: Impact on R_L from changes in P denoted by $C_b{}^*$.

(P, Q)	(R_L, L)				
	(3.75%, 240)	(4.00%, 236)	(4.25%, 232)	(4.50%, 228)	(4.75%, 224)
(10.0, 19)	1.212	1.800	2.599	3.648	4.975
(10.5, 18)	1.091	1.634	2.379	3.365	4.626
(11.0, 17)	0.877	1.336	1.978	2.844	3.973
(11.5, 16)	0.620	0.969	1.472	2.170	3.108
(12.0, 15)	0.375	0.608	0.957	1.461	2.165
(12.5, 14)	0.187	0.318	0.524	0.838	1.299
(13.0, 13)	0.073	0.132	0.231	0.391	0.641
(13.5, 12)	0.021	0.040	0.076	0.138	0.244
	(5.00%, 220)	(5.25%, 216)	(5.50%, 212)	(5.75%, 208)	(6.00%, 204)
(10.0, 19)	6.595	8.499	10.654	13.003	15.466
(10.5, 18)	6.180	8.024	10.132	12.451	14.905
(11.0, 17)	5.392	7.109	9.111	11.355	13.775
(11.5, 16)	4.322	5.835	7.652	9.750	12.080
(12.0, 15)	3.114	4.346	5.885	7.737	9.877
(12.5, 14)	1.954	2.848	4.025	5.516	7.330
(13.0, 13)	1.020	1.572	2.347	3.393	4.751
(13.5, 12)	0.416	0.688	1.100	1.702	2.546

* Parameter values, unless stated otherwise: $R = 3\%$, $R_D = 2.5\%$, $D = 250$, $K = 20$, $\sigma = 0.1$, $\sigma_f = 0.1$, $B = 270 - L$, $P = 19.5 - 0.5Q$, and $PQ < (1 + R_L)L$.

In Table 2, we further consider $R = 3.00\%$, $R_D = 2.50\%$, $D = 250$, $K = 20$, $\sigma = 0.1$ and $B = 250 - L$ to explain the market value of the bank's equity in (2). At a given loan rate, the bank's equity value decreases with the borrowing firm's product price. At a given level of product price, the bank's equity value increases with its loan rate. We have the result of $\partial R_L / \partial P > 0$ denoted by the bank's equity value because the negative product price effect is only partially offset by the positive loan rate effect when the product price is relatively low and the loan rate is relatively high. An interesting result is that, as the borrowing firm increases its product price, the bank must now provide a return to a larger capped put base. One way the bank may attempt to augment its total returns is by shifting its investments to liquid assets and away from its loan portfolio at an increased margin. Capped call-put options as such make the bank more prudent and less prone to risk-taking. However, we have the result of $\partial R_L / \partial P < 0$ denoted by C_b when the product price is relatively high and the loan rate is relatively low. As such, the bank is less prudent and more prone to risk-taking. All provided positive equity values in Table 2 are shown by the equity value surface in Figure 2.

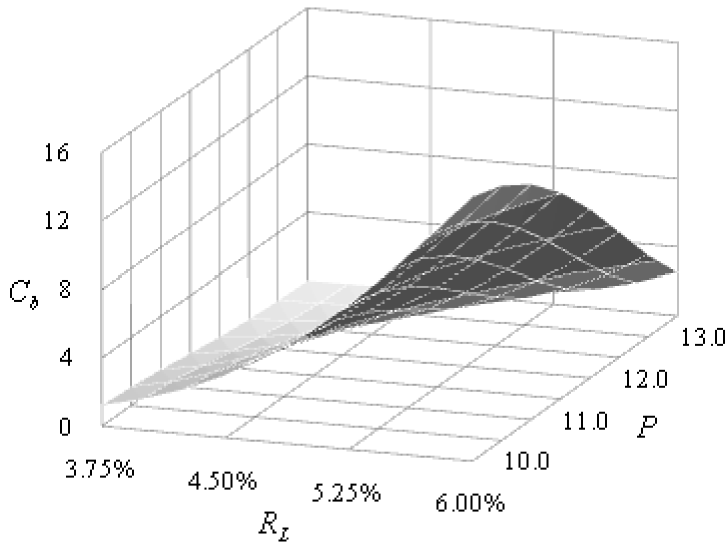

Figure 2: Impact on R_L from changes in P denoted by C_b

Table 3 contains the bank's equity values which are obtained by solving numerically Equation (3) for a set of values of (R_L, L). Let assign parameter values as $R = 3\%$, $R_D = 2.5\%$, $D = 250$, $K = 20$, $\sigma_b = 0.1$ and $B = 270 - L$. The equity values presented in Table 3 are based on a naked call assumption.

Table 3: Changes in R_L denoted by S_b *

(R_L, L)				
(3.75%, 240)	(4.00%, 236)	(4.25%, 232)	(4.50%, 228)	(4.75%, 224)
26.568	26.944	27.311	27.668	28.016
(5.00%, 220)	(5.25%, 216)	(5.50%, 212)	(5.75%, 208)	(6.00%, 204)
28.353	28.680	28.996	29.300	29.591

* Parameter values, unless stated otherwise: $R = 3\%$, $R_D = 2.5\%$, $D = 250$, $K = 20$, $\sigma_b = 0.1$, and $B = 270 - L$.

Table 4: Market-based estimates of the bank equity cap, $S_b - C_b$*

(P, Q)	(R_L, L)				
	(3.75%, 240)	(4.00%, 236)	(4.25%, 232)	(4.50%, 228)	(4.75%, 224)
(10.0, 19)	25.356	25.144	24.712	24.020	23.041
(10.5, 18)	25.477	25.310	24.932	24.303	23.390
(11.0, 17)	25.691	25.608	25.333	24.824	24.043
(11.5, 16)	25.948	25.975	25.839	25.498	24.908
(12.0, 15)	26.193	26.336	26.354	26.207	25.851
(12.5, 14)	26.381	26.626	26.787	26.830	26.717
(13.0, 13)	26.495	26.812	27.080	27.277	27.375
(13.5, 12)	26.547	26.904	27.235	27.530	27.772
	(5.00%, 220)	(5.25%, 216)	(5.50%, 212)	(5.75%, 208)	(6.00%, 204)
(10.0, 19)	21.758	20.181	18.342	16.297	14.125
(10.5, 18)	22.173	20.656	18.864	16.849	14.686
(11.0, 17)	22.961	21.571	19.885	17.945	15.816
(11.5, 16)	24.031	22.845	21.344	19.550	17.511
(12.0, 15)	25.239	24.334	23.111	21.563	19.714
(12.5, 14)	26.399	25.832	24.971	23.784	22.261
(13.0, 13)	27.333	27.108	26.649	25.907	24.840
(13.5, 12)	27.937	27.992	27.896	27.598	27.045

* Parameter values, unless stated otherwise: $R = 3\%$, $R_D = 2.5\%$, $D = 250$, $K = 20$, $\sigma = 0.1$, $\sigma_f = 0.1$, $\sigma_b = 0.1$, and $B = 270 - L$, $P = 19.5 - 0.5Q$, and $PQ < (1 + R_L)L$.

Table 4 contains bank equity cap values, defined as $S_b - C_b$,which are obtained from Tables 2 and 3. Three observations stand out. First, for a relatively low level of product price, the cap value decreases with the loan amount at an increased loan rate. For example, the first row of Table 4 reports the equity cap value of the bank which ranges from 25.356 to 14.125 when the product price is 10.0. For a relatively high level of product price, the cap value increases with the loan rate when the loan rate is relatively low, but decreases with the loan rate when the loan rate is relatively high. For example, the last row reports the cap value which ranges increasingly from 26.547 at $R_L = 3.75\%$ to 27.992 at $R_L = 5.25\%$, but ranges decreasingly from 27.896 at $R_L = 5.50\%$ to 27.045 at 6.00%.

Second, at a given level of the loan rate, the cap value decreases with the product quantity at an increased product price. For example, the first column reports the cap value which ranges from 25.356 to 26.547 at $R_L = 3.75\%$, the fifth column shows the cap value which ranges from 23.041 to 27.772 at $R_L = 4.75\%$ and the last column shows the cap value which ranges from 14.125 to 27.045 at $R_L = 6.00\%$. The cap value increases much more significantly at a relatively high loan rate than that at a relatively low loan rate.

Third, we have the result of $\partial R_L / \partial P < 0$ denoted by the cap value when product pricing is relatively low and loan rate-setting is relatively high, whereas $\partial R_L / \partial P > 0$ when product pricing is relatively high and loan rate-setting is relatively low. For example, the cap value is 21.758 at $R_L = 5.00\%$ and $P = 10.0$; 20.656 at $R_L = 5.25\%$ and $P = 10.5$; 19.885 at $R_L = 5.50\%$ and $P = 11.0$; and 19.550 at $R_L = 5.75\%$ and $P = 11.5$, that capture $\partial R_L / \partial P < 0$. The cap value is 26.193 at $R_L = 3.75\%$ and $P = 12.0$; 26.626 at $R_L = 4.00\%$ and $P = 12.5$; 27.08 at $R_L = 4.25\%$ and $P = 13.0$; and 27.530 at $R_L = 4.50\%$ and $P = 13.5$, that capture $\partial R_L / \partial P > 0$.

As stated earlier, the case of the naked call option is independent of the pricing behavior of the borrowing firm. The interpretation of these results follows a similar argument as in the case of the capped call option. Moreover, in the three cases reported, there is a systematic overvaluation of the bank's equity value under the naked call option relative to the capped call. In particular, we represent that market-based estimates of the bank's equity, which ignore the cap, lead to more significant overestimation when the product pricing is high and loan rate-setting is low, than when the product pricing is low and loan rate-setting is high. All provided overvaluations of the bank's equity returns are shown in Figure 3.

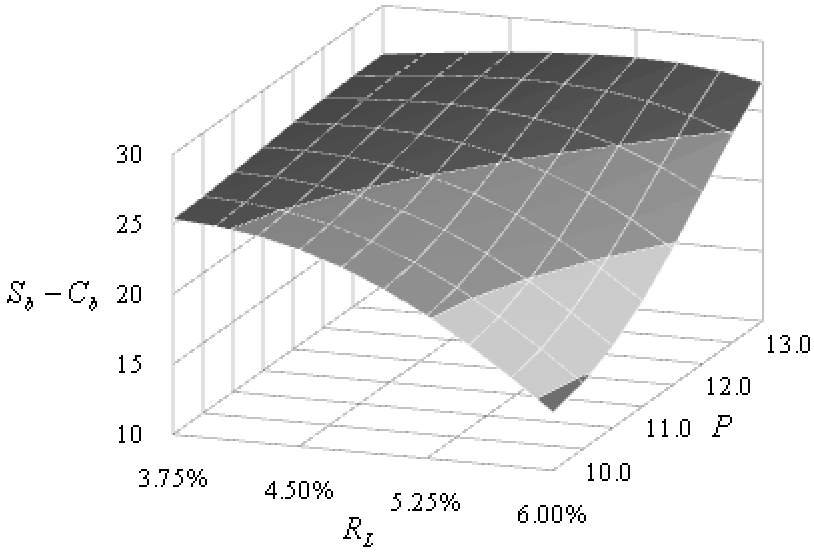

Figure 3: Impact on R_L from changes in P denoted by C_b

5 Conclusion

The objective of this paper was to show that bank lending and credit risk from the borrowing firm create a specific stochastic process for the assets of a bank. The credit risk is explicitly captured by the firm funding its production and then selling the product in an imperfect competitive market. The bank also exercises some monopoly power in lending. Bank equity is equivalent to a capped call when credit risk is a significant productive risk factor. We find that the market-based estimates of the bank's equity which ignore the cap lead to significant overestimation. This overestimation increases with product price and loan rate when the product price is relatively high and the loan rate is relatively low. Under these circumstances, an increase in the product price increases the loan amount at a reduced margin. Capped structure as such makes the bank less prudent and more prone to lending risk-taking, thereby adversely contributing to the stability of the banking system. One issue that has not been addressed is the bilateral monopoly in the one borrower-one bank case. In particular, is it the case that the results of this paper also apply to the bilateral monopoly case? The answer also depends on the cap structure. The aforementioned issue may provide ample opportunity for future research.

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