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The Equity Premium Puzzle: Pitfalls in Estimating the Coefficient of Relative Risk Aversion

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Abstract

Standard consumption-based models typically fail in pricing asset returns. In a famous seminal paper, Mehra and Prescott (1985), using a standard consumption model, prove the presence of a puzzle (i.e. equity premium puzzle). The recent financial literature still has to provide a convincing resolution to the well known puzzle. In contrast to this literature, which mainly focuses on the United States data, our paper simply replicates the closed form solution estimation, as in Mehra (2003), for a bunch of developed and emerging markets. On one side, our estimations confirm the existence of the puzzle and lead to bizarre values of the coefficient of relative risk aversion. On the other side, we claim that the key consumption model assumptions, the choice of a proper riskless asset and the lack of data, generate obstacles in finding robustness in the estimations of the CRRA coefficients, both in developed and emerging markets.

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1 Introduction

The financial literature is still trying to solve the Equity Premium Puzzle (EPP). The purpose of this paper is to show that the puzzle, under the standard model's assumptions, still exists, both in mature and emerging markets. Working on data representing both markets we first provide an ex-post simple time series analysis on the Equity Risk Premium (ERP). In line with the existing literature we find that emerging markets compensate investors with higher returns with respect to developed markets.[3] We observe that the time varying nature of the ERP, even in emerging economies, relates mainly to economic cycles, shocks and other macro phenomena. Following Jagannathan et. al (2000), we show that during the last decade this well-known premium shrunk, especially in advanced economies.

Given the Mehra & Prescott standard general equilibrium model in which individuals have an additively time separable utility function and a constant relative risk aversion coefficient, we then estimate the values of the relative risk aversion coefficients. While in developed economies a puzzle still exists in emerging ones a puzzle surges. To test the validity of the equilibrium model and justify the presence of "bizarre" coefficients, a "naïve" empirical asset pricing model is used. Usually investing in equities generates higher returns, thus providing investors with a return that exceeds the risk free rate of return. The difference between these two classes of securities represents the above mentioned and well-known ERP.

[3] See Bekaert (1995), Bekaert and Harvey (2003), Bekaert et al. (2003), Bekaert et al. (2007), Domowitz et al. (1997), Estrada (2000), and Salomons and Grootveld (2003) among others.

The concept of the ERP provides an intuitive measure of the extent to which the members of an economy, in aggregate, need to be compensated for the riskiness of the productive assets of that economy. More precisely, the ERP represents the amount that investors require to induce them to hold a portfolio composed by risky assets rather than riskless assets. The ERP is probably one of the most important and frequently used inputs in various financial and economic models. It is a key component in asset pricing, corporate finance and other financial fields. While the meaning and the usefulness of the EPP is unambiguous, attempts to empirically estimate it have faced much more problems. Most empirical studies report estimations mostly based on the US, and sporadically on other advanced economies. The effort put by economists to solve the puzzle has given rise to a big debate. To this extent, many solutions have been proposed. For example, Constantidines (1990), Jermann (1998) and more recently Barro (2009) have proposed different alternative models to solve the puzzle in the US economy.

In this paper we try to figure out the ERP values for a basket of emerging economies and compare these values to those of mature markets. In line with recent literature, our results confirm both the declining equity premium in mature markets and the presence of a higher equity premium in emerging economies.

An important theoretical result in economic literature shows that the average long-term ERP exceeds its desirable level, more specifically it does not reflect what equilibrium theory predicts. Mehra and Prescott (1985) and Mehra (2003) show that for the United States in the period 1889-1978 the ERP has been in excess of 6% per annum. According to their theoretical framework this leads to a level of relative risk aversion of 26, which does not exactly fit the normal range between 1 and 10 that the theory predicts. The difference between the return on stocks and the return on a risk-free asset is called the equity premium (or the equity risk premium, since it is thought to express the higher risk associated with risky securities). The fact that it is too large to be explained by standard economic models is called the e*quity premium puzzle*.[4]

The next sections are devoted to the analysis of the equity risk premium and the related puzzle in 4 developed and 8 emerging economies. In replicating the puzzle's setup, we discover that several theoretical and empirical drawbacks emerge.

2 Description of the Data

We use, both for developed and emerging equity markets, the Morgan Stanley Capital International (MSCI) Total Return Index.[5] Thus, the MSCI Total Return Index of the following countries has been downloaded: United States, Germany, Japan, Argentina, Brazil, China, India, Korea, Mexico, Russia, Singapore and Turkey. Since we are not interested in international investors' investment opportunities, our analysis is computed in local currencies. Therefore, market returns will not be influenced by an extra stochastic component (i.e. currency).

The choice of a proxy for the risk-free asset has been much debated in literature.[6] A desirable risk-free proxy should have zero risk of default, be traded in liquid markets, and have a duration similar to that of a risky investment. For these reasons the 10Y r Treasury bond is often used in empirical studies of the ERP in the United States. For example, Drew et al. (2004) use the Germany Benchmark bond 10Y yield for Germany and the 1-month interbank rate for the

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[4] A formal treatment of the puzzle is reviewed in Section 3.

[5] The MSCI Total Return Index measures the price performance of markets with income from constituent dividend payments. The MSCI Daily Total Return (DTR) Methodology reinvests an index constituent's dividends at the close of trading on the day the security is quoted ex-dividend (the ex-date). Source: MSCI Fundamental Data Methodology Report

[6] In Section 4 the debate is deeply discussed.

United Kingdom as risk-free rate of return. Damodaran (2008) argues that for an investment to have an actual return equal to its expected return there can be no reinvestment risk, that is, the risk-free rate. He further claims that even a 5Y Treasury bond is not risk free, since the coupons on the bond will be reinvested at rates that cannot be predicted today. According to his view, the risk-free rate for a 5Y time horizon has to be the expected return on a default-free (government) 5Y zero coupon bond. Summing up, Damodaram (2008) states that an investment can be risk-free only if it is issued by an entity with no default risk, and the specific instrument used to derive the risk-free rate will vary depending upon the period over which you want the return to be guaranteed. Therefore, our 10Y Treasury bond is in line with the common time horizon sample of Jan 2000 – Oct 2010 and with the AAA rating of the US public debt.

It is largely accepted in literature that the use of the rate that prevails on the money market is a feasible alternative and a suitable compromise for economies where a long-term treasury is not liquid or may not even exist. In such economies this is the best indicator of changes in the interest rate structure even if it is at the very short end of the yield curve. To define the ERP, a rate that most closely approaches the money market rate is used. More specifically, for each country we made use of the Central Bank's reference rate.[7] In finding values for the CRRA coefficient, as posed by the equity premium puzzle framework, the real private consumption quarterly time series is adopted (see Section 3). The next pages is devoted to a simple historical and behavioral analysis of the equity risk premium.

The ERP is defined as the return on equity minus the risk-free rate of return. In our specific case the rate that prevails on the money market (central bank reference rate) has been subtracted to the MSCI Total Return Index. Formally:

$$ERP_{t,t+1} = R_{t+1}^{e} - R_{t}^{f} \tag{2.1}$$

[7] To have data homogeneity, for all economies the money market rate is used.

Equity risk premium is estimated for each economy using a common time period (Jan 2000-Oct 2010).[8] A sophisticated reader could claim that the selected sample seems to be rationally "unfair", where for the latter we mean a sample with extraordinary shocks or affected by high volatility. Based on Figure 2.1-2.2 the reader's claim is well supported. Such a claim is further confirmed by two extraordinary events that hit the world economy within the last decade, the "9/11 terrorism attack" and the "sub-prime mortgage crisis". Even if the reader is right, we would like to stress the fact that the choice of the sample does not weaken the paper's main purpose, which is to show the instability and time-varying nature of the ERP, to prove the weakness of the consumption based general equilibrium model in explaining the high premium on stock markets and to study the presence of the EPP. In what follows some ERP standard statistics are illustrated.

The distributional characteristics of the time series are represented in Table 2.1-2.2. The higher returns and more volatile nature of emerging market returns are quite evident. For advanced economies we observe negative values, once the risk-free rate is subtracted to the MCSI Total Return Index. Over the period Jan 2000 - Oct 2010 we have negative risk premia. In the US we find an ERP of -0.15% over the entire sample period,[9] somewhat lower than the number documented in literature.[10]

[8] Given data availability, this is the largest possible interval.

[9] The ERP is estimated using another US risk-free rate proxy. The alternative proxy is borrowed from the Fama & French data library website, where the risk free-rate corresponds to the one-month Treasury Bill (from Ibbotson Associates).
The same number estimate is obtained.

[10] Mehra & Prescott (1985) display, on annual basis, an ERP of 6.18% over the period 1889-1978.

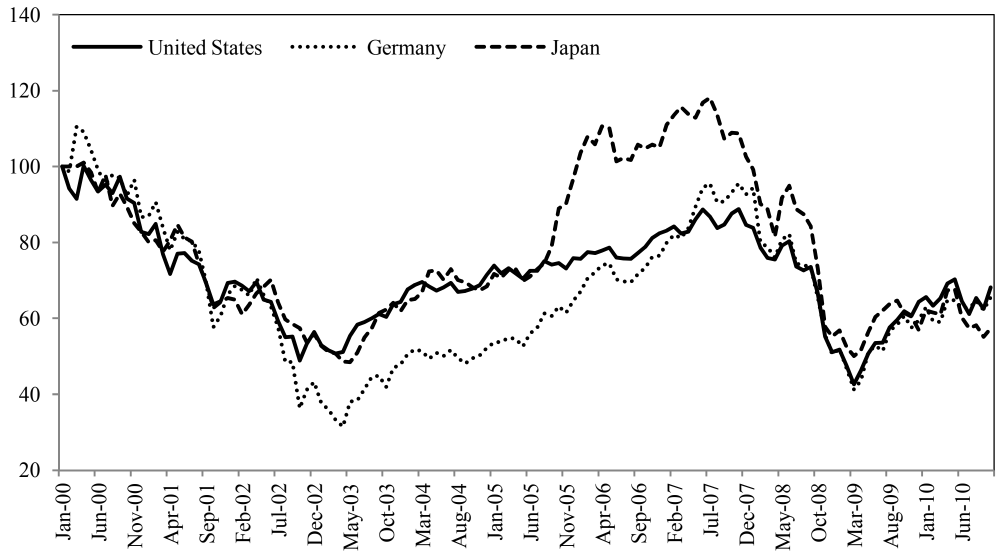

Figure 2.1: Advanced Economies (Equity Risk Premium Patterns)

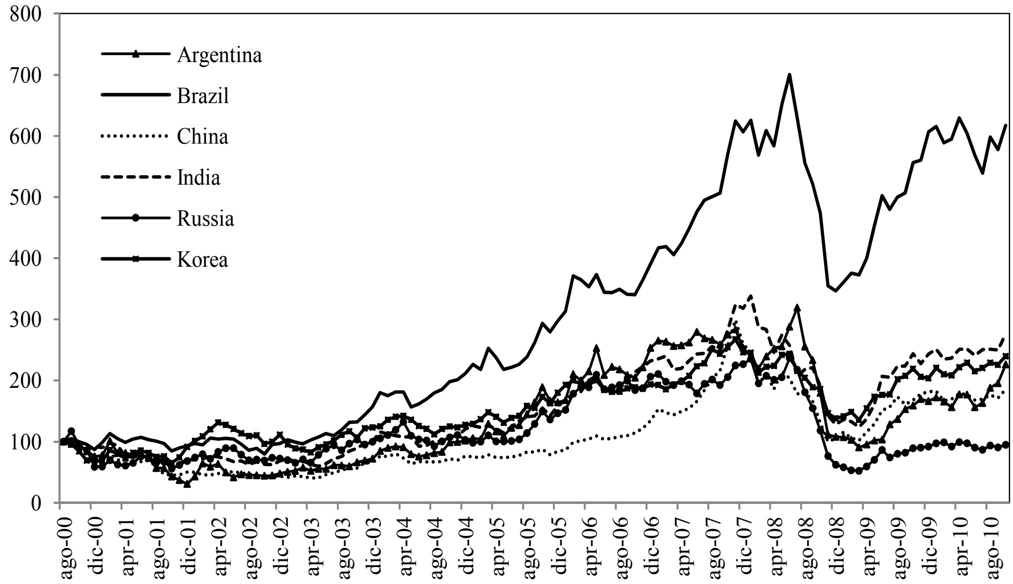

Figure 2.2: Emerging Markets (Equity Risk Premium Patterns)

The distributional characteristics of the time series are represented in Table 2.1-2.2. The higher returns and more volatile nature of emerging market returns are quite evident. For advanced economies we observe negative values, once the risk-free rate is subtracted to the MCSI Total Return Index. Over the period Jan 2000 - Oct 2010 we have negative risk premia. In the US we find an ERP of -0.15% over the entire sample period,[11] somewhat lower than the number documented in literature.[12]

Table 2.1: Developed Markets ERP Summary Statistics (sample: Jan 00 – Oct 10)

	United States	Germany	Japan	Singapore
Obs.	130	130	130	105
Mean	-0.15%	-0.05%	-0.30%	0.83%
Std. Dev.	4.78%	6.69%	5.19%	5.96%
Skewness	-0.4572	-0.4374	-0.3935	-0.6961
Kurtosis	3.5265	4.6587	4.2605	7.7423
Median	0.52%	0.35%	-0.21%	1.68%
Max	9.58%	20.74%	12.59%	22.13%
Min	-17.18%	-25.20%	-21.08%	-26.31%
J.Bera	6.031212	19.0484	11.9605	106.8709
	(0.0490)	(0.0000)	(0.0025)	(0.0000)

[11] The ERP is estimated using another US risk-free rate proxy. The alternative proxy is borrowed from the Fama & French data library website, where the risk free-rate corresponds to the one-month Treasury Bill (from Ibbotson Associates).
The same number estimate is obtained.

[12] Mehra & Prescott (1985) display, on annual basis, an ERP of 6.18% over the period 1889-1978.

Germany and Japan over the same period also display negative ERP (i.e. -0.05% and -0.3% respectively).[13] Among advanced economies only Singapore[14] generates a positive ERP (i.e. 0.83% on a monthly basis). On the contrary, emerging countries' stock markets generated on average high and positive equity excess returns. In line with other empirical studies (Salomons and Grootveld 2003) our results confirm the larger emerging markets ERP. On an annual basis emerging markets display an ERP ranging from a minimum 5.98% (Russia) to 19.95% (Brazil). These numbers are in line with those found by empirical estimates based on the United States' ERP in the 80's and 90's. Equity premia estimates in the US are found to be around 4% for the last two centuries (Siegel 1998) and around 7% for the 1926-1999 period (Center for Research in Security Prices). Those numbers strongly confirm the emerging markets' habit to offer higher compensation for risk.

A simple explanation for the extra performance of emerging markets' returns relies simply on the higher idiosyncratic risk associated to these countries. In order to show this, we provide an empirical analysis on distributional characteristics. In contrast to some past empirical studies, our data displays negative skewness in most of the cases. The presence of negative skewness over the entire sample is confirmed in all markets, both advanced and emerging, except for Argentina and Korea. As an implication the ERP seems to be not normally distributed. The statistics reported in Table 2.1-2.2 confirm the non normal distribution of the ERP. The null hypothesis of a normal distribution of the ERP is rejected at a 5% significance level in most of the markets.[15]

[13] Note that if we choose a sub-sample (Jan 00 – Jun 07), the ERPs still remain lower than those found three decades ago. Over this sub-sample United States, Germany and Japan display respectively an ERP equal to -0.87%, 2.08% and 3.33% (on an annual basis).

[14] The International Monetary Fund (IMF) classification where Singapore belongs to Advanced Economies, is considered.

[15] According to the Jarque-Bera test we accept the null hypothesis of a normal distribution in the following cases: Brazil, Korea and Turkey.

As suggested by Scott and Horvath (1980) rational investors should prefer positive skewness. They claim that, in a-priori economic terms, the investor would prefer an asset or portfolio with high probability of a return greater than the expected value compared to an investment with high probability that its return will be less than the expected value. Risk averse investors prefer positive skewness over no skewness and over negative skewness in the distribution of returns or wealth. Therefore in the presence of negative skewness investors will ask for compensation for bearing such risk.

Table 2.2: Emerging ERP summary statistics (sample: Jan 00 – Oct 10)

	Argent	Brazil	China	India	Korea	Mexico	Russia	Turkey
Obs.	130	130	130	123	130	63	130	99
Mean	1.36%	1.66%	0.87%	1.15%	0.80%	1.06%	0.50%	0.85%
Std. Dev.	12.06%	7.25%	8.67%	8.01%	8.02%	6.08%	10.81%	10.48%
Skewness	0.4007	-0.2735	-0.5058	-0.3046	0.0128	-0.7529	-0.2865	-0.1105
Kurtosis	5.4554	3.5706	3.2572	4.1071	3.4147	4.2939	3.8898	3.0403
Median	1.29%	1.64%	1.91%	1.37%	1.10%	1.99%	1.71%	2.01%
Max	49.84%	19.49%	19.21%	28.31%	26.29%	12.19%	35.16%	26.17%
Min	-38.14%	-25.08%	-23.24%	-25.41%	-21.33%	-20.69%	-35.26%	-25.66%
J. Bera	36.1367	3.3844	5.9018	8.1836	0.9352	10.3471	6.0671	0.208097
	(0.0000)	(0.1841)	(0.0523)	(0.0167)	(0.6265)	(0.0057)	(0.0481)	(0.9012)

A risk alert in finance is commonly given by standard deviation, which has one major drawback. Standard deviation measures uncertainty or variability of returns but in some cases this does not match the real magnitude of risk. This risk measure proxy is symmetric, large positive outcomes are treated as equally risky as large negative ones. Grootveld and Hallerbach (1999) argue that, in practice, positive outliers should be regarded as a bonus and not as a risk. Especially in the presence of non-symmetrical distributions it is better to look at some measure of downside risk. Statistics show that emerging markets, compared to developed markets, contain more downside risk. Economists and financial practitioners have long recognized that investors care differently about downside losses versus upside gains. Agents who place greater weight on downside risk demand additional compensation for holding stocks with high sensitivities to downside market movements. As pointed out by Estrada (2000), in general, investors are rewarded with higher returns, but if things go wrong, the damage can be severe and detrimental to performance. The structure of the emerging markets' excess return distribution is largely unstable. Distributional characteristics of ERP in Table 2.2 strongly confirm this behavior. The same argument, mostly due to the chosen period, can be partially sustained also for advanced economies. The financial literature supports the idea that the global business cycle is the main force behind the time varying nature of the ERP in emerging markets, as well as in mature markets. On this issue, Salomons and Grootveld (2003) emphasize the fact that emerging economies are heavily exposed to the global business cycles and that investors might see emerging markets as a "leveraged play" on the global cycle (i.e. high beta markets). In bearing this extra source of risk investors require higher returns. Donadelli and Prosperi (2011), via rolling window estimations of the standard CAPM, show the presence of an increasing risk factor path in six "ad hoc" macro-area emerging stock portfolios. In contrast to Donadelli and Prosperi (2011), here we concentrate on the standard consumption model presented by Mehra & Prescott (1985) where such a distinction between global and local risk is not presented and the CRRA coefficient plays a crucial role.

The analysis makes clear how the ERP is strongly constrained by the chosen sample period. In particular, in replicating the standard EPP analysis, one should consider the relevance of the time-varying component. Especially, if an entire century of data is not used. Due to lack of data, our time constrained analysis might be poor. However, the goal of this section is simply to capture the ERP patterns of a specific group of international markets. Differences across markets are thus exploited. Our main empirical results show that the emerging ERP's behavior differs from the developed one. These markets are different in paying premia to investors; in particular in the last decade when advanced economies have performed badly, emerging economies have offered increasing premia to investors. In the following sections, after a brief review of the theoretical set-up, we study the implication of this inconsistency in determining the relative risk aversion coefficient.

Table 2.3: Equity Risk Premia Risk Measures

Country	Sharpe Ratios	Standard Deviation	Downside Risk (Semi Deviation)
United States	-0.03	4.78%	40.73%
Germany	-0.01	6.69%	57.01%
Japan	-0.06	5.19%	43.27%
Argentina	0.11	12.06%	93.36%
Brazil	0.06	7.25%	60.03%
China	0.10	8.67%	74.41%
India	0.14	8.01%	65.66%
Korea	0.10	8.02%	64.88%
Mexico	0.17	6.08%	36.75%
Russia	0.05	10.81%	90.41%
Singapore	0.14	5.96%	46.27%
Turkey	0.08	10.48%	75.07%

3 The Model: A Review

It is largely accepted that stocks are riskier than Treasury bills, or than any other riskless proxy. Thus, a risky investment should provide high returns. Past empirical studies, as well as our estimates on emerging markets, strongly support the presence of high excess returns (i.e. equity premia). In line with our empirical analysis, the presence of a high premium in advanced markets cannot be confirmed. In Section 4 this point is treated deeply.

Is there any reasonable explanation for this premium? Mehra and Prescott (1985) were the first to declare the equity premium an official "puzzle". They use a standard general equilibrium model in which individuals have additively time separable utility functions (meaning that my utility of consumption this year does not depend on my consumption in other years) and constant relative risk aversion. The ERP comes down to be the differential between return to equity and return to riskless assets in excess of the premium that can be explained on the basis of a "reasonable" degree of risk aversion. Under certain assumptions on the agent's utility function, the theory predicts that the risk averse decision maker would require a higher rate of return on the risky asset than on the riskless asset in a portfolio selection. In what follows we briefly review the formal treatment of the EPP provided by Mehra and Prescott (1985) and Mehra (2003). In seeking for the existence of a puzzle, in advanced economies as well as in emerging markets, we use the standard theoretical framework developed in Mehra (2003), where a representative household maximizes an infinite sum of utility functions subject to a sequential budget constraint. Formally:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \tag{3.1}$$

where $0<\beta<1$ and $E_0(.)$ represents the expectation operator conditional on information available at time zero (initial period), β is the subjective discount factor, u is an increasing, continuous, differentiable, concave utility function and c_t is the per capita consumption at time t. The model requires the utility function to be of the CRRA class:

$$u(c,\alpha)=\frac{c^{1-\alpha}}{1-\alpha}, \quad 0<\alpha<\infty \tag{3.2}$$

where the parameter α measures the curvature of the utility function. At time t the representative agent carries a quantity y_t of the traded security from the previous period and chooses the amount, y_{t+1}, to be carried forward to period $t + 1$. Because in t the security yields dividend d_t, if it is priced (in terms of units of the consumption good) at p_t, the following budget constraint holds:

$$c_t + p_t y_{t+1} \leq p_t y_t + d_t y_t \tag{3.3}$$

In other words the representative agent maximize (3.1) under constraint (3.3). We end up with the following fundamental pricing relationship:

$$p_t u'(c_t) = \beta E_t\left[u'(c_{t+1})(p_{t+1} + d_{t+1})\right]. \tag{3.4}$$

Eq. 3.4 is used to price both stocks and riskless one-period bonds. For a risky security:

$$1 = \beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)} R_{e,t+1}\right],$$

where $R_{e,t+1} = (p_{t+1} + d_{t+1})/p_t$. For the riskless one period bonds, the fundamental pricing expression is

$$1 = \beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)} R_{f,t+1}\right].$$

A little algebra shows that the expected gross return on equity is defined in the following way:

$$E_t(R_{e,t+1}) - R_{f,t+1} = -\operatorname{cov}_t\left\{\frac{u'(c_{t+1}), R_{e,t+1}}{E_t u'(c_t)}\right\}. \tag{3.5}$$

The expected asset return equals the risk-free return plus a premium for bearing risk, which depends on the covariance of the asset return with the marginal utility of consumption. Next, a question arises: Is the magnitude of the covariance between the asset return and the marginal utility of consumption large enough to justify a 6% of equity premium in the US market?

In addressing the latter issue, some additional assumptions are required. More specifically:

- The growth rate of consumption $x_{t+1} = c_{t+1}/c_t$ is (i.i.d.)
- the growth rate of dividends $z_{t+1} = d_{t+1}/d_t$ is (i.i.d.)
- (x_t, z_t) are jointly log-normally distributed

As a consequence the gross return on equity $R_{e,t+1}$ is (i.i.d.) and $(x_t, R_{e,t})$ are jointly normally distributed. Under the assumption of a CRRA utility function, where the marginal utility is equal to $c_t^{-\alpha}$, and imposing the following :

- $\mu_x = E(\ln x)$
- $\sigma_x^2 = \text{var}(\ln x)$
- $\sigma_{x,z} = \text{cov}(\ln x, \ln z)$
- $ln\ x = continuosly\ compounded\ \ growth\ rate\ of\ \ consumption$

we end up with the following closed form equations

$$\ln R_f = -\ln \beta + \alpha\mu_x - \frac{1}{2}\alpha^2\sigma_x^2 \tag{3.6}$$

$$\ln E(R_e) - \ln R_f = \alpha\sigma_{x,R_e}, \tag{3.7}$$

where $\sigma_{x,R_e} = \text{cov}(\ln x, \ln R_e)$. The (log) equity premium in this model is the product of the coefficient of relative risk aversion and the covariance of the growth rate of consumption with the return on equity or the growth rate of dividends. The model's equilibrium condition imposes the identity $x = z$. It follows that the return on equity is perfectly correlated with the growth rate of consumption. Then equation (3.5) becomes:

$$\ln E(R_e) - \ln R_f = \alpha\sigma_x^2 \tag{3.8}$$

where the equity premium is the product of the coefficient of relative risk aversion α and the variance of the growth rate of consumption.[16] Using Eq. 3.6-3.8 and

[16] See Mehra (2003) for a detailed formalization of the model.

setting the coefficient of relative risk aversion α to be 10 and the discount factor β to be 0.99, Mehra and Prescott (1985) and Mehra (2003) find a risk-free rate equal to 12.7% and a return on equity of 14.1%. Such result implies an equity risk premium of 1.4%, far lower than the 6.18% historically observed. It can be easily recognized that the parameters α an β play a crucial role in determining the equity risk premium. If a lower value for β is considered, the risk-free rate is much higher. As a consequence, we face a lower premium. An analogous problem surges if we are supposed to set a lower value of α. Therefore, the 1.4% value represents the maximum ERP that can be obtained, given the constraints on α and β, in this class of models. Because the observed ERP was around 6%, Mehra and Prescott claimed to have a puzzle in their hands that risk considerations alone cannot account for. They also claim that $\alpha = 48$ and $\beta = 0.55$ would solve the puzzle. These numbers are obtained using US data over the 1889-1978 period. In synthesis, they show that the difference in the covariances of these returns with consumption growth is only large enough to explain the difference in the average returns if the typical investor is implausibly risk averse. In a quantitative sense this refers to the EPP, where stocks are not sufficiently riskier than riskless securities to explain the spread in their returns.

Philippe Weil (1989) shows that the data presents a second anomaly. According to a standard model of individual preferences, when individuals want consumption to be smooth over states (they dislike risk), they also desire smoothness of consumption over time (they dislike growth). Given that the large equity premium implies that investors are highly risk averse, the standard models of preferences would in turn imply that they do not like growth very much. Yet, although Treasury bills offer low return, individuals defer consumption (they save) at a sufficiently fast rate to generate an average per capita consumption growth of around 2% per year. Weil (1989) labels this phenomenon as the *risk-free rate puzzle*. A vast literature that seeks to solve these two puzzles exists. On one side researchers focus on more realistic preferences. On the other side, a considerable number of works attempt to solve the problem by using different risk-free rate proxies. For example, Harvey (1989) argues that the slope of the conditional mean-variance frontier, a measure of the price of risk, changes through time according to the business cycle.

Campbell and Cochrane (1999) claim that these puzzles can be understood with a simple modification of the standard representative-agent consumption-based asset pricing model. For them, the central ingredient is a slow-moving habit, or time-varying subsistence level, added to the basic power utility function.[17] In this way, as consumption declines toward the habit in a business cycle trough, the curvature of the utility function rises, so risky asset prices fall and expected returns rise. As pointed out in the introduction the choice of the real riskless rate is a key component within this framework. Empirical estimation becomes even more critical once we leave the US world. Our goal is to reproduce the standard theoretical framework so as to identify the value of α over a specific sample and for several economies, both advanced and emerging. In Section 4 results are illustrated and discussed. Adopting a "naïve" modified regression model we try to prove the model's weakness/robustness.

4 The EPP: Estimations and Key Drawbacks

Mehra and Prescott (1985) claim that it is difficult to reconcile certain empirical facts about equity and debt returns and the process of consumption growth with realistic values about the coefficient of relative risk aversion and the pure rate of time preference, in a conventional infinite-horizon model with an additively time separable CRRA utility function. In this model, the only parameter is the coefficient of relative risk aversion, α. The interpretation of α is that if consumption falls by 1 percent, then the marginal value of a dollar of income

[17] See Deaton and Muellbuer (1980), Deaton (1992), Ryder and Heal (1973), Sundaresan (1989), and Constantinides (1990), among others.

increases by α percent. The question Mehra and Prescott (1985) posed was this: what value of α is necessary to explain the historical equity premium? As pointed out in Section 3, they observe a value far above 40, i.e. too high to be reasonable. A high value of α implies that individuals should want desperately to smooth consumption over time, because consumption shortfalls deliver far more pain than surpluses given by pleasure. Since the economy becomes richer over time, individuals should all try to borrow from their richer future in order to improve their (relatively) impoverished present. But this common desire to borrow should lead to high real interest rates. Instead, the real rate of interest has been scarcely positive over long periods of time.

On top of that, Weil (1989) claims that the equity premium puzzle could be easily called the *risk-free rate puzzle*. While literature provides several alternatives to solve the latter, it provides only two rational explanations for the large equity premium: either investors are highly averse to consumption risk (i.e. $\alpha = 48$) or they find trading costs on stocks much higher than those on bonds or riskless securities. For example, Aiyagari (1993) suggests that incomplete markets and transaction costs are crucial for explaining the high equity premium and the low risk-free rate. In assessing the robustness of these two puzzles Korcherlakota (1996) emphasizes first that they are very important for macroeconomists.[18] He states that the risk-free rate puzzle points out the ignorance in understanding why agents persist to save even when returns are low. He also argues that the equity premium puzzle demonstrates that we do not know why agents are so averse to the risk associated with stock returns.

Thanks to this intuitive model and according to the above mentioned issues we are able to find easily a value of α for each economy (and for any sample period). Before estimating the parameters of interest, we want to check whether

[18] Korchelakota (1996) claims that these two puzzles are very robust. The models' robustness is implied by three assumptions about individual behavior and asset market structure. The assumptions are: i) agents have the same preferences (representative agent); ii) markets are complete; iii) asset trading is costless.

the i.i.d. assumptions on the rate of growth of consumption and dividends is feasible or not. A standard "first order" way to test i.i.d. assumptions is to check the autocorrelation function for our variables of interest. From Figure A.1 (see in Appendix) we can check that in most of the countries the autocorrelation function is inside the confidence bands, even if seasonality in our data is evident but this does not bias our results.[19] Furthermore Figure A.2 suggests that dividends have no serial autocorrelation 1 (see in Appendix). Even if this kind of test is not sufficient for testing independence, we can conclude that a certain degree of robustness has been achieved. Indeed, as suggested in the calibration of the model performed by Mehra & Prescott (1985) the existence of the puzzle does not depend on the i.i.d. assumptions.

Now, we can discuss accurately the model's major theoretical and empirical drawbacks. In contrast to standard literature we recognize that the analysis of exotic markets associated with the choice of specific sample periods leads to "bizarre values" of the coefficient α.[20] These values are easily determined by re-arranging Eq. 3.8 in terms of α,

$$\frac{\ln E(R_e) - \ln R_f}{\sigma_x^2} = \alpha \tag{4.1}$$

where σ_x^2 is the variance of the growth rate of per-capita consumption. Based on statistics reported in Table A.1 1 (see in Appendix), we end up with the following constant relative risk aversion coefficient values:

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19 Seasonality can be easily eliminated regressing our dependent variable on seasonal dummies.

20 In summarizing a number of studies Arrow (1971) concludes that relative risk aversion with respect to wealth is almost constant. He further argues on theoretical grounds that α should be approximately one. Friend and Blume (1975) find a larger value for α, whose estimates were around two. Kydland and Prescott (1982), in their studies of aggregate fluctuations, find that they needed a value between one and two to observe the relative variability of consumption and investment. Altug (1983) estimates the parameter to be near zero. All these studies constitute a justification for restricting the value of α to be a maximum of ten (Mehra and Prescott 1985).

Table 4.1: Constant Relative Risk Aversion Coefficients[21]

Emerging Markets	α	Advanced Economies	α
Argentina	8.53	Germany	**-21.88**
Brazil	31.87	Japan	**-116.29**
Korea	118.26	Singapore	29.75
Russia	2.98	United States	**-128.36**

Negative coefficients are due to negative mean equity premia. The presence of plausible coefficients (i.e. $2 < \alpha < 10$) - no puzzle - is confirmed only for Argentina and Russia. All other economies confirm the existence of a puzzle, with implausible values of the risk aversion coefficient. Advanced economies (i.e. Germany, Japan and the United States) display negative coefficients, while emerging ones (i.e. Argentina, Brazil, Korea and Russia) show high and positive values, meaning that agents are highly risk averse. The coefficient ranges from 2.98 (i.e. Russia) to 118.26 (i.e. Korea). Our estimates do not properly match those found in the literature on the US market; in particular our procedure provides bizarre values. The fact that we face a puzzle over the puzzle is mainly due to the “ad hoc” empirical setup we deal with. We remark that such "ad hoc" setup is forced by scarcity of data on exotic markets. On top of that we discover a high degree of heterogeneity across international agents, which generates unreasonable values for the CRRA coefficients.

On the surprising historical size of the equity premium the literature suggests that something else besides intrinsic risk is determining its size. Something that the model does not capture. One view in the finance literature is that this something is due to market imperfections. These imperfections are thought to decrease the willingness of investors to bear risk and so to increase the return they

[21] Due to data availability we could find robust statistics to compute the CRRA coefficients of China, India, Mexico and Turkey. In contrast to the first section of this paper for these estimations, via the CPI, all data are converted in real.

require to invest in the stock markets. This view about the reason for the large historical equity premium is consistent with the empirical analysis conducted in the 80's and 90's on the US market. According to this view, the premium should shrink when market imperfections are reduced. During the last twenty years the financial markets' innovation and development allowed investors to easily access information, communicate and transact with anyone and anywhere. All this leads to an essential reduction in trading costs. Although we cannot speak of perfection, mature markets are very close to reaching this status. Not surprisingly, premia in developed markets dropped significantly (Blanchard 1993, Cochrane 1997, Siegel 1999, Jagannathan et al. 2000). The declining in the advanced economies' equity premium is also confirmed by our data. Looking at the US market, the premium is negative over the entire sample and equal to 1.52% on an annual basis over the pre-crisis period. Numbers which are not fully comparable with those found by past researchers. The anomalies in equity premia behavior are due to the fact that variation in the realized premium depends heavily on the time horizon over which it is measured. Figure 4.1, on US quarterly data, well represents the declining nature of the ERP and its time dependence.

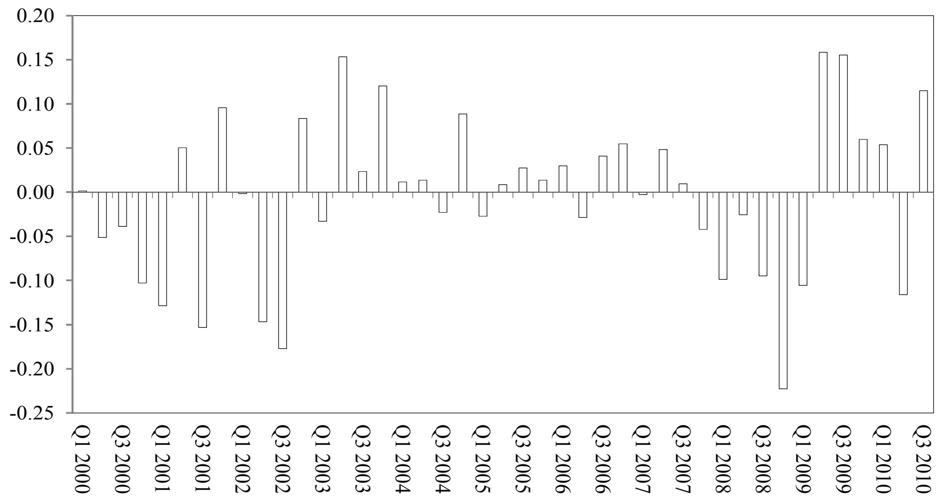

Figure 4.1: United States – Realized Equity Risk Premium per quarter (%)

In contrast to mature markets, emerging ones show on average higher ERPs and substantially higher CRRA coefficients. Furthermore, in some cases these values are bigger than those originally obtained by Mehra and Prescott on the US market. The degree of market imperfections is significantly higher in emerging markets compared to the developed ones and investors entering these markets will ask for higher returns. Is market imperfection enough to explain the “new” puzzle?

We believe that the issue of market imperfections is only a naïve way to interpret our empirical results. The model's additional features need to be discussed. For example, the theoretical set-up for our economy assumes that individuals live forever. Therefore, the time constrained analysis might be weak. In their 1985’s seminal paper, Mehra and Prescott use annual data over the sample 1889-1978. In contrast, our coefficients are computed, due to data availability, using a ten year time horizon, which is probably too short to explain individuals’ long-run market behavior.

Another important issue which causes us some robustness problems in replicating the EPP analysis is linked to the choice of the riskless security. Clearly, a proxy needs to be adopted. But what is the most realistic proxy? Do we need a unique world level proxy or a country specific one? Do we need to consider business cycles? As pointed out in Section 2, it is largely accepted that the use of the rate that prevails on the money market and/or the 10Y government bond represent feasible alternatives for the risk-free rate. In reality this approximation might be wrong. For example, in some markets a corporate bond can be less risky than a government bond. If we look at the current macro scenario, we are not so far from the assumption that many corporate bonds are safer than the debt issued by many European governments, such as Greece, Portugal, Spain and Italy. Thus the risk-free proxy should be updated through cycles. The idea is that we are not allowed to avoid the time varying component of our variables (i.e. consumption growth, equity returns and risk-free returns). In extreme cases it can be reasonable to use equity as risk-free rate proxy (e.g. Argentina's default of 2001). In the current international economic scenario, many economists believe that the only authentic riskless security is represented by the German debt.

In testing the validity of the model we relax the assumption of perfect correlation between the return on equity and the growth rate of consumption. Thanks to this assumption and using a simple algebraic trick, Eq. 3.7 can be rewritten in the following way:

$$\ln E(R_e) - \ln R_f = \alpha \frac{\sigma_{x,R_e}}{\sigma_x^2} \sigma_x^2 \quad (4.2)$$

where $\frac{\sigma_{x,R_e}}{\sigma_x^2} = \frac{\mathrm{cov}(x, R_e)}{\mathrm{var}(x)} \equiv \beta$. Under this formulation we can test the validity of the model by running the following regression:

$$[\ln E(R_e) - \ln R_f]_t = \gamma + \alpha \hat{\beta} \, \mathrm{var}(x)_t + \varepsilon_t \quad (4.3)$$

where the squared deviation from the mean has been used as proxy for the variance of the growth rate of consumption. The equilibrium model holds for $\gamma = 0$. In other words, a simple way to check if the equilibrium model holds is to run a test of significance on the intercept of this specific regression.[22] In contrast to the closed form solution, as for Eq. 3.8, this procedure allows us to test the statistical significance of the coefficient of relative risk aversion, thus the model's validity. Thanks to the regression in Eq. 4.3 we are allowed to answer the following questions: Does the CRRA coefficient predict the ERP? Is the model's validity preserved?

Our artificial regression, explained by Eq. 4.3, is subject to a couple of major drawbacks. First, the general framework refers to a closed economy. In our opinion an international asset pricing framework should be considered, where only country-specific agents can buy risky assets universally. Second, the trick

[22] To test the validity of our equilibrium model we are required to accept the null hypothesis against the alternative. Formally: $H_0: \ \gamma = 0, \ H_1: \ \gamma \neq 0$

developed in Eq. 4.2 falls into the error in the variable framework, where an estimated value (i.e. $\hat{\beta}$) is used to predict equity premia. In addition, our re-interpretation of Eq. 4.1, by means of Eq. 4.3 is theoretically wrong, since the original closed form solution is the direct result, with some additional assumptions, of the maximization problem, where (3.1) is maximized subject to (3.3). As a consequence, to apply any possible transformation, the problem's first order condition should be modified. Clearly, through the use of different preferences.

Following our naïve empirical setup, to capture the time-varying component of the coefficients, a rolling window empirical procedure is implemented. Figures A.1-A.4 show the rolling window estimations of the CRRA and Beta coefficients of the following countries: Argentina, Brazil, Korea, Russia, Germany, Japan, Singapore and the United States. Estimations confirm the time-varying nature of the coefficients. As expected, estimation errors are quite high. Across emerging countries, exploding confidence bands around the estimates are found.

As pointed out by Mehra and Prescott (2008), for all these reasons, over the past 20 years, attempts to solve the puzzle have become a major research impetus in finance and economics. Several generalizations of the Mehra and Prescott (1985) model have been proposed to better reconcile observation with theory. Some of them include alternative assumptions on preferences or modify probability distributions to capture extreme events, survival bias, incomplete markets and market imperfections. Campbell and Cochrane (1999) try to explain the puzzle by introducing habit in the basic power utility function. The general idea is that a representative agent will be more risk averse in a period of recession (i.e. when consumption is low or below the mean-trend level) and less risk averse as consumption and wealth increase during a boom. According to habit formation literature, risk aversion is time-varying, thus we need a model in which it changes with cycles. This theoretical drawback further weakens the regression in Eq. 4.3, which implicitly assumes alpha to be constant through time. Other works also include attempts at modeling limited participation of consumers in the stock market. It is largely known that all this research effort is mainly focused on the US market. EPP analysis based on "exotic" data are rare in literature. Being quantitatively and qualitatively constrained by data availability we discover that difficulties in estimations still exist. Once emerging markets are analyzed, additional empirical problems emerge and need to be discussed.

5 Main Results: A summary

To be more attractive, emerging markets are required to offer high equity premia. Our elementary empirical analysis confirms emerging stock markets' tendency to compensate investors for bearing extra risk. Compared to the past, we also show that mature markets offer lower returns. Moreover, over the sample period Jan 00–Oct 10, they display negative equity premia. As a consequence, in replicating the Mehra and Prescott (1985) and Mehra (2003) model, we end up with negative risk aversion coefficients (i.e. -21.88, -116.29 and -128.36 respectively for Germany, Japan and the United States). On the contrary, emerging markets' coefficients are positive and in a couple of markets bizarrely high. A reasonable coefficient (i.e. $2 < \alpha < 10$) is found in the following markets: Argentina and Russia (i.e. 8.53 and 2.98 respectively). The existence of such heterogeneity that derives from the estimation of our parameters of interest must be investigated.

Heterogeneity in the alphas comes directly from the fact that closed economies are analyzed. It is assumed that each economy works internally, meaning that each agent can consume only domestic goods and purchase only assets traded in the domestic financial market. In breaking the assumption of closed economies we allow investors to buy international consumption goods and to invest in international markets. Clearly, the assumption of free markets must hold (i.e. no transaction costs). For example, if a free market exists, each individual has an incentive to buy the same riskless security. Clearly, different coefficient values will be found. Even if our model is assumed to be theoretically correct, as for Eq. 4.3, results are found to be not robust. Standard errors of our estimates are too high. The inclusion of the global financial crisis in our sample further leads to unreasonable estimates.

A further explanation on the negativity of the risk premium is needed. On one side, we claim that an incorrect risk-free rate is used. On the other side, we simply point out that individuals are risk lovers. The idea of a risk lover individual is pretty unrealistic, especially if the analysis is developed over the last decade of data, which is our case. In line with the literature, the puzzle remains a solid problem to be solved. In studying it, many issues have to be considered. Once exotic markets are included in the estimation of the coefficient of relative risk aversion, many other problems emerge. To follow the standard framework we need to impose some strong assumptions, both theoretical and empirical, which lead to weak estimations.

6 Conclusions

During the last two decades many economists attempted to solve the asset pricing puzzle. Most of their work focused on US data. In this paper we simply replicate the general consumption problem, as in Mehra (2003), to understand if more recent data, either from the US or from other countries, is able to fit the baseline model. We show that many practical problems emerge. First, in analyzing more exotic countries, we are forced to restrict our sample. Real market data is often not available for more exotic economies. Second, we cannot rely on the assumption of a CRRA utility function. We show that CRRA coefficients change through time.

Furthermore we are not allowed to make any changes in the closed form solution, as in Eq. 4.2, unless the consumption maximization problem is re-stated, that is, the problem's first order condition has to be different.

To conclude, in trying to fit the standard consumption-based asset pricing model with more recent data for a heterogeneous bunch of countries, we show that many other empirical drawbacks occur.

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Appendix: Some Empirical Facts

Table A.1: International Economies Sample Statistics, 2000-2010 (quarterly data)

Economy	Mean Equity Premium, $E(Re) - Rf$	Variance of Growth Rate of Consumption, $\sigma^2(x)$	Standard Deviation of Growth Rate of Consumption, $\sigma(x)$
Argentina	2.87%	0.34%	5.80%
Brazil	2.45%	0.08%	2.77%
Germany	-0.14%	0.01%	0.79%
Korea	2.47%	0.02%	1.44%
Japan	-0.76%	0.01%	0.81%
Russia	1.69%	0.57%	7.54%
Singapore	1.18%	0.04%	1.99%
United States	-0.76%	0.01%	0.77%

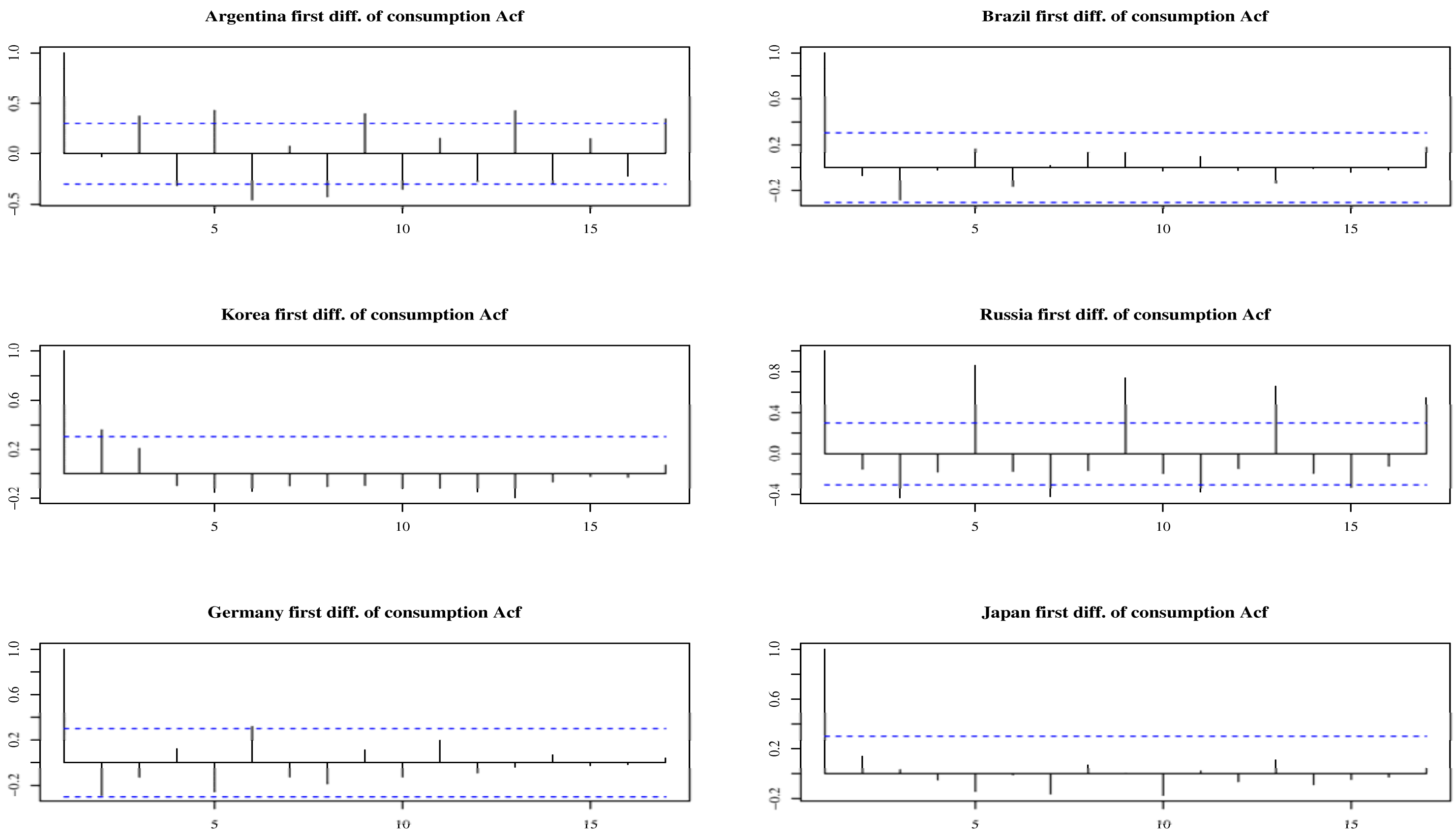

Figure A.1. Country-by-Country: Consumption Autocorrelation Function

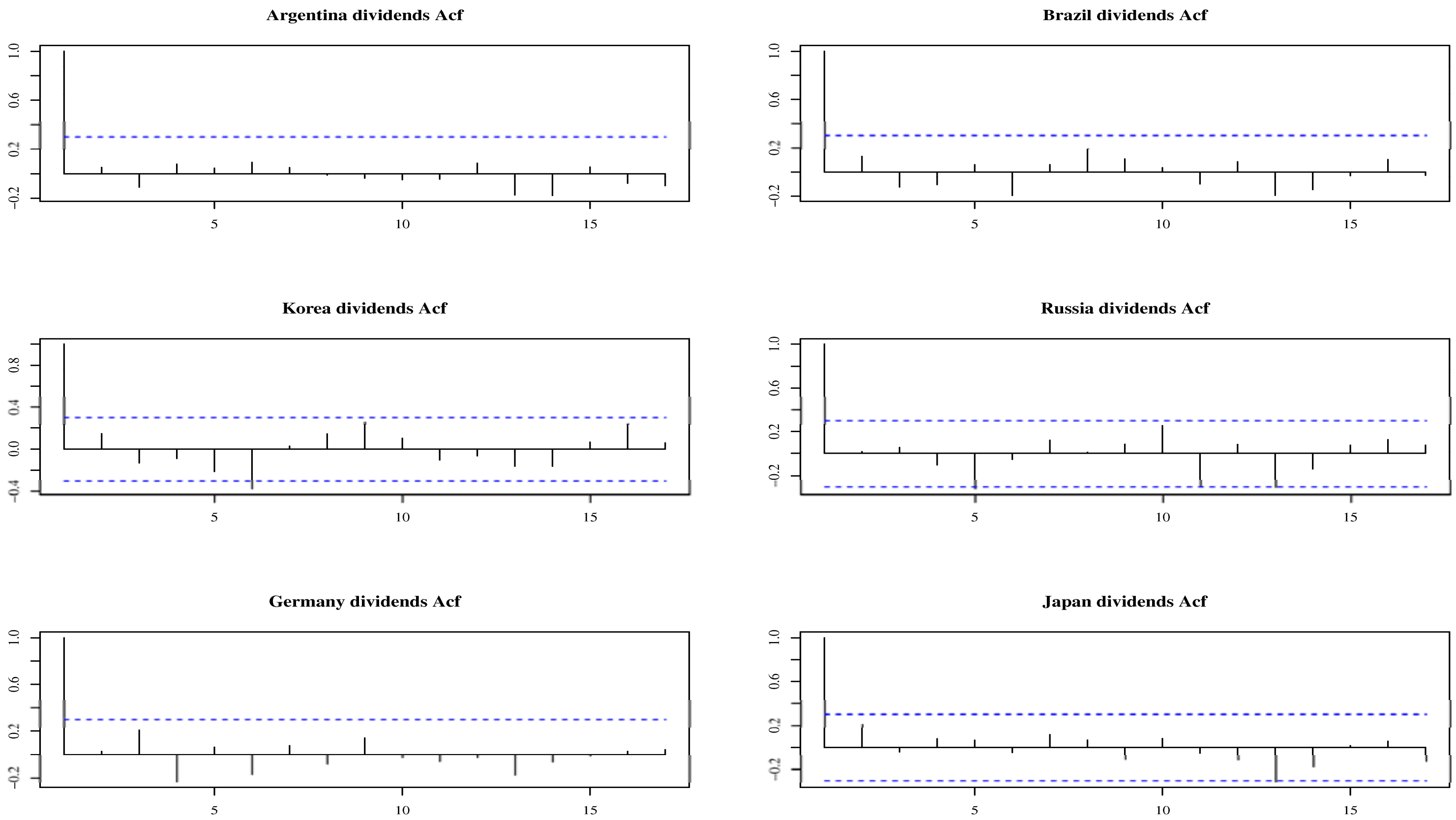

Figure A.2. Country-by-Country: Dividends Autocorrelation Function

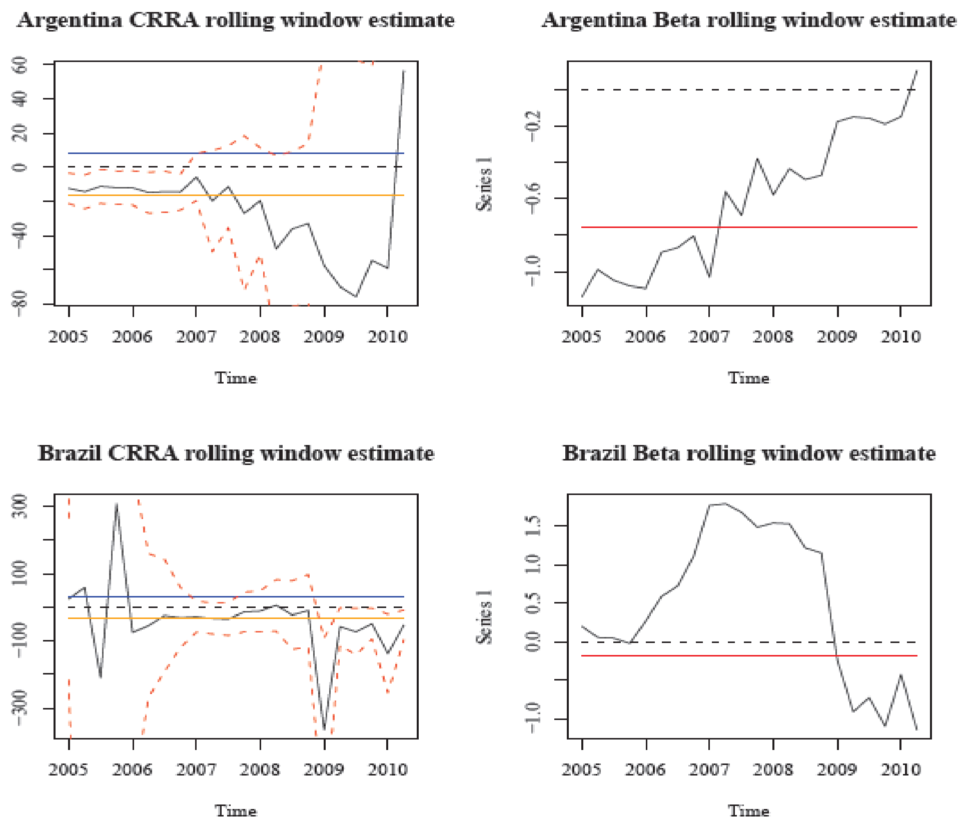

Figure A.3: Rolling window estimations. The black line corresponds to the rolling window estimates of the CRRA (left-panel) and Beta (right-panel) coefficients. The red dashed lines represent confidence intervals around the CRRA coefficient estimates. Sample: 2000–2010 (QoQ)

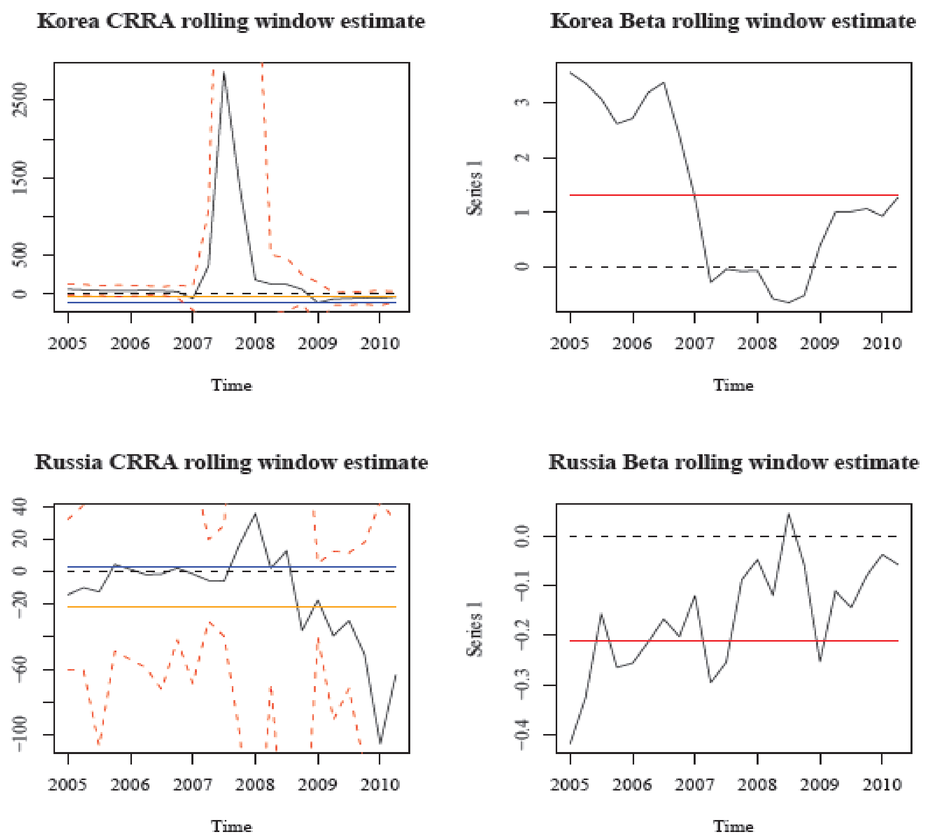

Figure A.4: Rolling window estimates. The black line corresponds to the rolling window estimates of the CRRA (left-panel) and Beta (right-panel) coefficients. The red dashed lines represent confidence intervals around the CRRA coefficient estimates. Sample: 2000-2010 (QoQ)

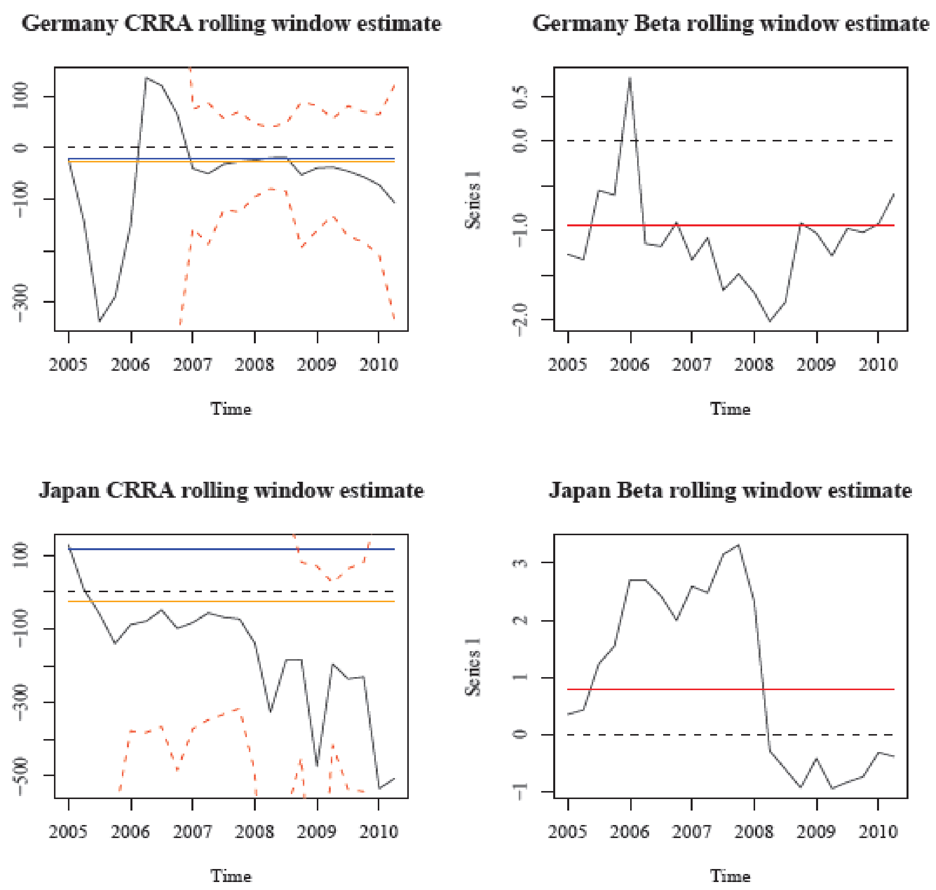

Figure A.5: Rolling window estimations. The black line corresponds to the rolling window estimates of the CRRA (left-panel) and Beta (right-panel) coefficients. The red dashed lines represent confidence intervals around the CRRA coefficient estimates. Sample: 2000-2010 (QoQ)

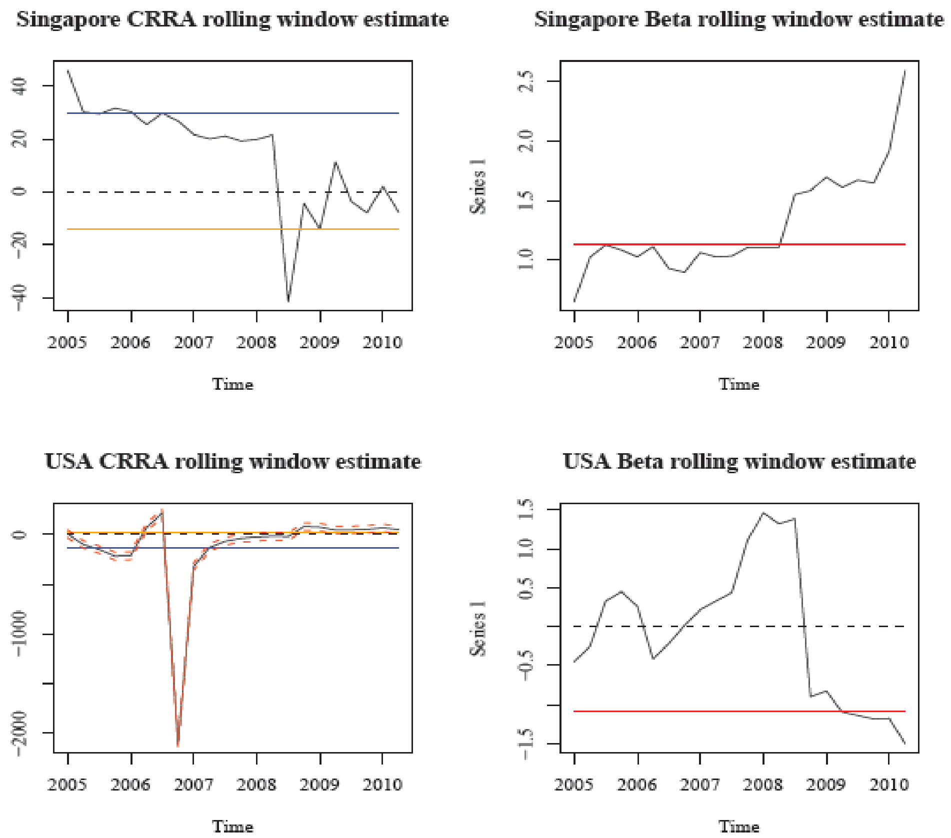

Figure A.6: Rolling window estimations. The black line corresponds to the rolling window estimates of the CRRA (left-panel) and Beta (right-panel) coefficients. The red dashed lines represent confidence intervals around the CRRA coefficient estimates. Sample: 2000-2010 (QoQ)