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Assessing the credit risk of bank loans using an extended Markov chain model

Su-Lien Lu[1]

Abstract

In this paper, we adopted a continuous-time non-homogeneous mover-stayer model for the measurement of the credit risk associated with bank loans. This model is an extension of a Markov chain model. Furthermore, we extracted the time varying risk premium to convert the mover-stayer model to a risk-neutral mover-stayer model.

This paper draws a number of conclusions and makes a number of important contributions. First, we determined that the mover-stayer model is better suited than the Markov chain model in estimating the credit risk of loans, according to likelihood ratio statistics. Second, we found that borrowers of investment grades are less likely to remain at their original rating. On the other hand, rating classes had a strong tendency to be downgraded, inferring the likelihood that downgrade momentum is an element of rating behavior. However, rating migration did not indicate the existence of upgrade momentum. Third, we estimated time-varying risk premium to transfer transition matrices to risk-neutral transition matrices.

[1] Graduate Institute of Finance, National Pingtung University of Science and Technology, e-mail: lotus-lynn@mail.npust.edu.tw

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Fourth, estimated default probabilities match business cycle indicators particularly well. Finally, estimation procedures are easy to follow and implement. Consequently, the findings in this study have important implications for the management of risk assumed by financial institutions.

JEL classification numbers: G10, G21

Keywords: credit risk, Markov chain model, mover-stayer model

1 Introduction

Over the past 10 years, major developments in financial markets have led to a more sophisticated approach to the management of credit risk. For the banking industry, the classic form of risk is credit risk, based on the relationship between the banker and the client. At its worst, credit can cause a financial institution to become insolvent or result in such a significant drain on capital and net worth that growth and the ability to compete with other institutions is adversely affected. Therefore, the management of credit risk has become a major concern for the banking industry as well as other financial intermediaries. This concern is reflected in the actions of the Basel Committee on Banking Supervision, which is instrumental in formalizing the universal approach to credit risk for financial institutions.[2]

Modeling the evolution of credit ratings or transition matrices is essential for

[2] The Basel Committee on Banking Supervision ("The Committee") produced guidelines for determining bank regulatory capital. The objective of this accord was to level the global playing field for financial institutions and protect all risks in the financial system. In 1988, the Committee issued "the International Convergence of Capital Standard" (or "Capital Accord of 1988"), establishing regulations regarding the amount of capital that banks should hold against credit risk. Furthermore, the treatment of the market and the operational risk were incorporated in 1996 and 2001, respectively. The final version of the Accord was published in 2004.

financial applications associated with risk management, including the assessment of portfolio risk, the pricing of credit derivatives, modeling the term structure of credit risk premiums, and the assessment of regulatory capital. Markov chain models are a stochastic process based on transition matrices and transition probability. More sophisticated examples of high-risk bond pricing methods, as outlined by Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997), require these matrices as an input. For example, Jarrow and Turnbull (1995) were the first to use matrices of historical transition probability from original ratings. A model that has since led to widespread commercial acceptance. Based on a risk-neutral probability valuation model to price securities, Jarrow, Lando, and Turnbull (1997) derived a risk premium for the dynamic credit rating process using a Markov chain process, whereupon they estimated the default probability according to a transition matrix. For credit derivatives, the works of Kijima and Komoribayashi (1998) and Acharya, Das, and Sundaram (2002) are based on transition matrices. Such a matrix is also used in risk management in credit portfolio models such as CreditMatrices to simulate the value distribution of a portfolio of credit assets (Gupton, Finger and Bhatia, 1997).

A number of researchers, such as Frydman, Kallberg, and Kao (1985) and Frydman and Schuermann (2007), proved the existence of two distinct Markov regimes governing the rate at which credit ratings move, suggesting a stochastic process integrating two Markov chains, namely the mover-stayer model. The mover-stayer model is an extension of the Markov chain model dealing with a very specific type of unobserved heterogeneity in the population. The model assumes a population comprises two unobserved groups: a stayer group with a zero probability of change, and a mover group following an ordinary Markov process.

The discrete-time mover-stayer model was first introduced by Blumen, Kogan, and McCarthy (1955). The model has been applied in many areas, such as modeling occupational mobility (Sampson, 1990), income dynamics (Dutta, Sefton, and Weale, 2001), consumer brand preferences switching (Chatterjee and Ramaswamy, 1996; Colombo and Morrison, 1989), bond rating migration (Altman and Kao, 1991), credit behavior (Frydman, Kallberg and Kao, 1985), tumor progression (Tabar et al., 1996; Chen, Duffy and Tabar, 1997), and labor mobility (Fougère and Kamionka, 2003). However, the model has not been applied to the investigation of credit risk.

In contrast with the discrete-time mover-stayer model, a number of researchers have employed the continuous-time mover-stayer model in many areas. Frydman and Kadam (2004) estimated the continuous-time mover-stayer model from continuous data by applying the model to the migration involved in bond rating. Cook, Kalbfleisch, and Yi (2002) developed a generalized mover-stayer model using panel data. Fougère and Kamionka (2003) applied the continuous-time mover-stayer model to the labor market using panel data. For a continuous-time mover-stayer model, movers evolve according to a continuous-time Markov chain model, whereas the stayers remain in their initial state.

The purpose of this paper was to measure credit risk using the continuous-time non-homogeneous mover-stayer model developed by Frydman and Kadam (2004). This paper contributes to the literature in the following aspects. First, the rating migration of borrowers is postulated according to a mixed Markov chain, mover-stayer model. Previous studies have always assumed that all ratings are homogeneous with respect to their movement among rating categories, and that their behavior would not change over time. However, heterogeneity in the rate of movement may be an important aspect of rating behavior. A number of implications related to observed heterogeneity in the migration of credit ratings must be discussed. Therefore, a continuous-time non-homogeneous mover-stayer model was used to assess and compare the credit risk of bank loans in this study. The model is illustrated using loans from a bank in Taiwan. Furthermore, using likelihood ratio statistics, we determined that the mover-stayer model is better suited than the Markov chain model to the measurement of credit risk associated with bank loans.

Second, we discovered that most stayers cluster in non-investment grades and borrowers of investment grades are less persistent in their adherence to original ratings. It is possible that downgrade momentum exists in rating behavior; however, no evidence has been provided of an upgrade momentum. As far as we know, this study is the first to adopt both a continuous-time non-homogeneous mover-stayer model for assessing and comparing the credit risk associated with bank loans.

Third, because the risk premium plays a crucial role in gauging the credit risk of bank loans, previous researchers have always ignored lending rates, preferring to handle the risk premium as time-invariant (Jarrow, Lando and Turnbull, 1997; Wei, 2003). Kijima and Komoribayashi (1998), Lu and Kuo (2006) and Lu (2007) suggested that the risk premium is not time-invariant; but is actually always time-variant. As a result, time-invariant risk premium was changed to time-variant risk premium making it more elaborate than in previous studies. Further, we transfer a mover-stayer model to risk-neutral mover-stayer model using the time-varying risk premium.

Fourth, the estimated default probability matches the business cycle. This means that default probability estimated by the mover-stayer model is credible and could help financial institutions to enhance the accuracy of their lending decisions.

Finally, the estimation procedures are easy to follow and implement. In this study, we adopted a maximum likelihood procedure for estimating the parameters of the continuous-time mover-stayer model using a recursive method.[3] The model could also be applied to value the credit risk of other financial institutions. On the whole, the proposed model not only provides an effective credit risk review mechanism for financial institutions, but also helps them to act in accordance with

[3] Frydman and Kadam (2004) suggest the maximum likelihood estimator of the continuous-time mover-stayer model; they proved the estimator to be strongly consistent.

the Basel Capital Accord.

This paper is structured as follows. Section 2 presents the model and experimental set-up. Section 3 provides the characterization of sample data. Section 4 presents the empirical results and analysis. The final section provides conclusions.

2 Model specification

2.1 The continuous-time non-homogeneous mover-stayer model

Let $X = \{x_t, t \geq 0\}$ is a stochastic process with a state space $\Phi = \{1, 2, \ldots, k\}$ where state 1 represents the highest class of credit class; state 2 the second highest, …, state (k-1) the lowest class of credit; state k designates the default. That is, x_t represents the individual credit rating of the borrower at time t and Φ represents all available classes of credit rating.

The continuous-time mover-stayer model is an integration of two independent Markov chains. The first chain degenerated into an identity matrix, I, i.e., its transition matrix is the identity matrix. The other chain is a non-degenerate transition matrix.

According to Cook, Kalbfleisch and Yi (2002), Lando and Skødeberg (2002), Fougère and Kamionka (2003), Christensen, Hansen and Lando (2004) and Jafry and Schuermann (2004), we defined M(t) as the $k \times k$ transition matrix of the continuous-time Markov chain model with elements, m_{ij}, and

$$m_{ij}(t) = \Pr\{x_t = j | x_s = i\}, \quad i \neq j, \quad i, j = 1, 2, \ldots, k, \quad \text{and} \quad \sum_{j=1}^{k} m_{ij}(t) = 1.$$

Then, we let the matrix $\Lambda(t)$ denote the $k \times k$ transition intensity matrix (generator matrix) with entry $\lambda_{ij}(t) = \lim_{\Delta t \to 0} \frac{m_{ij}(t, t+\Delta t)}{\Delta t}, \quad \Delta t \geq 0, i \neq j, i, j = 1, \ldots, k$.

The entries of the intensity matrix $\Lambda(t)$ satisfy $\lambda_{ij}(t) \geq 0, \text{for } i \neq j$, and $\lambda_{ii}(t) = -\sum_{j \neq i} \lambda_{ij}(t) \equiv \lambda_i$. Therefore, we generate matrix M(t) by matrix Λ as $M(t) = \exp(t\Lambda) \equiv \sum_{i=0}^{\infty} \frac{(t\Lambda)^i}{i!}$.[4]

We then define P(t) as the continuous-time transition matrix of the integration process of the Markov chain model, i.e., the mover-stayer model, as

$$P(t) = SI + (I - S)\exp(t\Lambda), \quad t \geq 0 \tag{1}$$

where $S = \text{diag}(s_1, s_2, \ldots, s_k)$, with s_i as the proportion of stayers in state i, $i \in \Phi$. The transition probability of the mover-stayer model are given by

$$p_{ij}(t) = p(x_t = j \mid x_0 = i) = \begin{cases} (1 - s_i)\{m_{ij}(t)\} & \text{if } i \neq j \\ s_i + (1 - s_i)\{m_{ij}(t)\} & \text{if } i = j \end{cases} \tag{2}$$

where $\{m_{ij}(t)\}$ is the element (i, j) of the matrix M(t). Thus, we rewrite the continuous-time mover-stayer transition matrix, P(t) as

$$P(t) = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1,k-1} & p_{1,k} \\ p_{21} & p_{22} & \cdots & p_{2,k-1} & p_{2,k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{k-1,k-1} & p_{k,k} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underset{((k-1)\times(k-1))}{A} & \underset{((k-1)\times 1)}{D} \\ \underset{(1\times(k-1))}{O} & \underset{(1\times 1)}{1} \end{bmatrix} \tag{3}$$

where $p_{ij} > 0$, $\forall\, i, j$, which are defined in equation (2). The row of equation (3) denotes the initial rating class, i=1,2,…, k, and the column denotes the rating class at the end, j=1,2,…, k. The submatrix $\underset{((k\times 1)\times(k\times 1))}{A}$ is defined on non-absorbing states $\hat{\Phi} = \{1, 2, ..., k-1\}$ excluding default state k. The submatrix $\underset{((k-1)\times 1)}{D}$ is the column vector with component $p_{i,k}$, representing the transition probability of the borrower for any credit class, i.e., i=1, 2, …,k-1, to switch to default class, i.e., j=k.

[4] A detailed exposition of analysis can be found in Lando and Skødeberg (2002), Jafry and Schuermann (2004) and Christensen, Hansen and Lando (2004).

In general, we would say that default state k is an absorbing state. Finally, the submatrix $O_{1\times(k-1)}$ is the zero column vector providing a probability of transition from the default state at the initial time until final time.

The assumption of the homogeneity of the generator and transition matrices is implausible over a longer period of time. It appears that the longer the horizon is, the more apparent the heterogeneity is. On the other hand, many sources of heterogeneity associated with rating behavior and estimation methods have to be modified to take non-homogeneity into account in the continuous-time mover-stayer model.

2.2 Maximum likelihood estimation

When all realizations are observed continuously, during a fixed period of time [0, T], we estimate by maximum likelihood function. In accordance with Frydman and Kadam (2004), we let A be the set of all realizations that remained continuously in the initial state and B be the set of all realizations with at least one transition. For estimation, we defined:

a_r = the number of realizations that remain continuously in state r;

b_r = the number of realizations with at least one transition that starts in state r;

τ_i^A = the total time in state i for histories with no transitions;

τ_i^B = the total time in state i for histories with at least one transition;

n_r = the total number of borrowers that begin in state r;

n_{ij} = the total number of i→j transitions in the sample.

Then, the likelihood function, $L_A(\Lambda, s)$, of the realization in set A is

$$L_A(\Lambda, s) = \prod_r \left[s_r + (1-s_r)\exp(-\lambda_r T)\right]^{a_r} \tag{4}$$

where λ_r denotes the entries of the intensity matrix, Λ, in state r. On the other hand, the likelihood function, $L_B(\Lambda, s)$, of the realization in set B is

$$L_B(\Lambda, s) = \prod_r (1-s_r)^{b_r} \prod_{j \neq i} \lambda_{ij}^{n_{ij}} \prod_i \exp(-\lambda_i \tau_i^B) \tag{5}$$

Thus, the overall log-likelihood function becomes

$$\begin{aligned} \log L(\Lambda, s) &= \log L_A(\Lambda, s) + \log L_B(\Lambda, s) \\ &= \sum_r a_r \log[s_r + (1-s_r)\exp(-\lambda_r T)] \\ &\quad + \sum_r b_r \log(1-s_r) + \sum_{i \neq j} n_{ij} \log \lambda_{ij} - \sum_i \lambda_i \tau_i^B \end{aligned} \tag{6}$$

By the score equation with respect to s_r, we obtain[5]

$$s_r = \frac{a_r - n_r \exp(-\lambda_r T)}{n_r - n_r \exp(-\lambda_r T)} \tag{7}$$

Taking the score equation with respect to λ_{ij} and we obtain

$$\lambda_{ij} = \frac{n_{ij}(\exp(\lambda_i T) - 1)}{Tb_i + \tau_i^B(\exp(\lambda_i T) - 1)} = \frac{n_{ij}}{Tb_i(\exp(\lambda_i T) - 1)^{-1} + \tau_i^B} \tag{8}$$

2.3 The likelihood ratio statistics

If we set all s_i equal to zero, the Markov chain can be obtained from the mover-stayer model. We then use likelihood ratio statistics to test whether the mover-stayer model is better suited to gauging credit risk than the Markov chain model. Therefore, we formulate the hypothesis test of the form $H_0 : s = 0$. The usual statistic for this test is the ratio of the two likelihoods:

$$LR = \sup_{\Lambda, s=0} L(\Lambda, s) / \sup_{\Lambda, s} L(\Lambda, s) = L(\hat{C}, 0)/L(\hat{\Lambda}, \hat{s}) \tag{9}$$

where $\hat{C}$ is the maximum likelihood estimate of the intensity matrix Λ under H_0, when the process is a assumed to be a Markov chain. On the other hand, $\hat{\Lambda}$

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[5] The detailed process of calculation can be found in Frydman and Kadam (2004).

and $\hat{s}$ are the estimates of maximum likelihood regarding the intensity matrix and fractions of stayers, respectively, in the mover-stayer model. The asymptotic distribution of $-2\log LR$ is χ^2 with the degrees of freedom equal to the number of nonzero parameters in vector s under H_1.

According to Frydmand and Kadam (2004), we compute $-2\log LR$ over fixed time horizon (0,T) as

$$-2\log LR = -2\left\{\sum_i n_i \log\left(\frac{\hat{c}_i}{\hat{\lambda}_i}\right) + \sum_i (\hat{\lambda}_i \tau_i^B - n_i) - \sum_i a_i \log(a_i/m_i)\right\} \tag{10}$$

where $\hat{c}_i$ and $\hat{\lambda}_i$ are elements of intensity matrices $\hat{C}$ and $\hat{\Lambda}$, respectively, in equation (9).

2.4 Risk premium

In addition, this paper uses the risk-neutral probability approach to assess the credit risk of bank loans. For some credit ratings, the historical default probability is nearly zero, but the observed loan rates almost always imply a non-zero default probability in the risk-neutral world. Moreover, a risk-neutral framework could contain unexpected probability of default by estimating the risk premium. Ideally, the implication is that we should match loan rates and transition matrices to obtain the risk premium.

For the pricing of bank loans, loan rates were incorporated into the proposed model using the risk-neutral probability approach. Consider the corresponding stochastic process, $\tilde{X} = \{\tilde{x}_t, t \geq 0\}$ of credit rating under the risk-neutral probability measure. Let $\tilde{P}$ denote the risk-neutral transition matrix of the mover-stayer model. On the other hand, let ℓ_{ij} represent the risk premium of the mover-stayer model. As shown in Jarrow, Lando, and Turnbull (1997), Lu and Kuo (2006) and Lu (2007), the risk-neutral probability is equal to the transition matrix multiplied by the corresponding risk premium. That is,

$$\tilde{p}_{ij}(t, t+1) = \ell_{ij}(t) \cdot p_{ij} \text{ , } i, j \in \Phi \text{ ,}$$

where $\tilde{p}_{ij}$ and p_{ij} are elements of matrix $\tilde{P}$ and P, respectively.

Because the pricing decision of bank loans, i.e., loan rate, plays a crucial role in measuring the credit risk of bank loans, consider the loan rate and risk-free rate to assess the risk premium. Let $V_0(t, T)$ be the time-t price of a risk-free bond maturing at time T, and let $V_i(t, T)$ be its higher risk (i.e., riskier) counterpart for the rating class, i. Because a loan does not lose all interest and principal if the borrower defaults, realistically consider that a bank will receive at least a partial repayment, even if the borrower goes into bankruptcy. Let δ be the proportions of the principal and interest of the loan, which is collectible on default, $0 < \delta \leq 1$, where in general δ is referred to as the recovery rate. If there is no collateral or asset backing, then δ =0.

First, we define

$$\tilde{A}(0, t+1) = \tilde{A}(0, t)\tilde{A}(t, t+1) \tag{11}$$

For equation (11), $\tilde{A}(t, t+1) = \Omega(t) \cdot A$, where A is ($(k-1) \times (k-1)$) submatrix of matrix P(t), which is defined in equation (3). $\Omega(t)$ is the ($(k-1) \times (k-1)$) diagonal matrix with diagonal components being the risk premium, $\ell_i(t)$. Then, according to Kijima and Komoribayashi (1998), we obtain the risk premium as

$$\ell_i(t) = \frac{1}{1 - p_{i,k}} \sum_{j=1}^{k-1} \tilde{a}_{ij}^{-1}(0, t) \frac{V_i(0, t) - \delta V_0(0, t)}{(1 - \delta) V_0(0, t)} \text{, i=1,2,…,k and t=1,…,T} \tag{12}$$

where $\tilde{a}_{ij}^{-1}(0, t)$ are the components of the inverse matrix $\tilde{A}^{-1}(0, t)$ with $\tilde{A}(0, t)$ is invertible. In addition, $p_{i,k}$ denotes the transition probability of borrowers of any credit class i switching to default class k as defined in equation (3). In particular, for t =1, we have

$$\ell_i(0) = \frac{1}{1 - p_{i,k}} \frac{V_0(0,1) - \delta V_i(0,1)}{(1 - \delta) V_0(0,1)} \text{, for i=1,2,…,k} \tag{13}$$

Thus, we transfer the transition matrix of the mover-stayer model into a risk-neutral transition matrix by incorporating time-varying risk premium. In this manner, we are able to estimate the risk premium by a recursive method from t = 1,…, T.[6]

3 Data

This study employed the loan data of Chiao Tung Bank, in Taiwan. Chiao Tung Bank was established in 1907 and is at present the second largest investment bank in Taiwan. Chiao Tung Bank also has a considerable and expanding business in pioneer and venture capital investments. However, the International Commercial Bank of China and Chiao Tung Bank formally merged into one bank under the name Mega International Commercial Bank on August 21, 2006.

The sample data is from two databases of the Taiwan Economic Journal (TEJ), including the Taiwan Corporate Risk Index (TCRI) and long and short-term bank loans. The sample period was from Quarter 1, 1998 to Quarter 4, 2006, comprising 16,571 observations of Chiao Tung Bank.

The TCRI provides a complete credit rating record for corporations in Taiwan. TEJ applies a numerical class from 1 to 9 and D for each rating classification, similar to the investment grade used by Standard & Poor's and Moody. The categories are defined in terms of default risk and the likelihood of payment for each individual borrower. Obligation rated number 1 is generally considered the lowest in terms of default risk. On the other hand, obligation number 9 is considered the most risky and rating class D denotes a default by the borrower.

The long- and short-term bank loan database supplied a full history of debt for all borrowers at Chiao Tung Bank, including the names of lenders, the names

[6] The recursive method for calculating the risk premiums can refer to Kijima and Komoribayashi (1998).

of borrowers, loan rates, debt issuance dates, and credit type. The credit risk of bank loans was investigated according to the lending structure of every borrower.

The government bond yield was taken as a proxy for the risk-free rate, published by the Central Bank in Taiwan. Because the maturity of bank loans and government bonds differ, the yield of government bonds had to be adjusted by interpolating the yield of the government bond whose maturity was the closest, for use as the risk-free rate.

The recovery rate served as security for bank loans with an influence on credit risk. In general, banks set a recovery rate according to the type, liquidity, and value of collateral prior to lending. Altman, Resti, and Sironi (2004) presented a detailed review of default probability, recovery rate, and their relationships. They found that most credit risk models treated recovery rate as an exogenous variable. For example, Kim, Ramaswamy, and Sundaresan (1989), Hull and White (1995), Longstaff and Schwartz (1995) assumed recovery rate to be exogenous and independent from the asset value of the firm. Fons (1987), Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), Briys and de Varenne (1997), Lando (1998), Duffee (1999) assumed an exogenous recovery rate that is either a constant or a stochastic variable independent of default probability. Because the recovery rate is a key characteristic for making lending decisions, banks set a recovery rate according to the type, liquidity, and value of collateral prior to lending. In general, a higher recovery rate means higher security for bank loans. According to previous studies, there is no clear definition of the recovery rate. Therefore, this study adopted the recovery rates from 0.1 to 0.9 assumed by Lu and Kuo [40, 41] and Lu (2007).

Finally, observations for short-term loans and incomplete data were excluded. Loans that had an overly low rate were also excluded because they were likely to have been the result of aggressive accounting politics, and would have biased the results. In short, this study investigated the credit risk of mid- and long-term secured loans of one bank, Chiao Tung bank, in Taiwan.

4 Empirical Results

First, Table 1 offers summary statistics of the loans made by Chiao Tung Bank with corresponding government bonds. From Table 1, the average loan rates and their corresponding government bond yields were 7.2669% and 6.4447%, respectively. The volatility of loan rates and risk-free rates were 1.6529% and 0.7729%. The average lending period was 7.2166 years. Generally, the rate and volatility of loans were higher than risk-free government bonds.

Table 1: Descriptive statistics

	Loan rate	***Corresponding government bond***	***Lending period (year)***
Minimum	0.25%	1.3492%	1
Maximum	13.625%	8.064%	25
Mean	7.2669%	6.4447%	7.2166
Standard deviation	1.6529%	0.7729%	3.6700
Kurtosis	1.1089	7.9922	0.4232
Skewness	-0.4617	-2.1542	-0.1057
Correlation	0.1221		

Because the proposed models depend on continuous-time observations, we calculate the intensity matrix, $\Lambda(t)$. In Table 2 we report the estimated intensity matrix based upon continuous-time observation.[7] Then, we construct a

[7] For example, we only record a migration from rating class 8 to the default when a firm begins in rating class 8 in the beginning of the year in which the default occurs by using discrete-time observations. However, many firms in the sample are downgraded to rating class 8 during the year and only stay there a short time prior to defaulting. These are not recorded as defaults from rating class 8 in the discrete-time method; they are recorded in the methods, based on the continuous observation.

continuous-time transition matrix of the mover-stayer model as equation (3). The average continuous-time transition matrix of the mover-stayer model is shown in Table 3. The last column in Table 3 presents default probability of every rating class. The diagonal numbers are the average probability of remaining at the initial rating. For speculative rating classes, such as rating classes 7-9, their default probabilities are higher than other rating classes were. On the other hand, these findings showed that the investment grades (rating classes 1-3) had a high probability of switching to other grades; and non-investment grades (rating classes 4-6) had a high probability of staying at the initial ratings.

Because the mover-stayer model considered heterogeneity in the population, whereas the Markov chain model only considered a single population, we conclude that the mover-stayer model is an extended Markov chain model. We then use likelihood ratio statistics to test whether the mover-stayer model is better suited than the Markov chain model for gauging credit risk. According to equation (10), we compute the likelihood ratio as 370.067. The test rejected the null hypothesis at less than 1% significant level.[8] Thus, the mover-stayer model better suited for gauging credit risk.

This paper uses the risk-neutral probability approach to assess the credit risk of bank loans. For higher credit ratings, the historical default probability is nearly zero, but the observed loan rates imply a non-zero default probability in the risk-neutral world. Moreover, a risk-neutral framework could contain unexpected default probability by estimating the risk premium

[8] Because there are 9 rating classes, the asymptotic distribution of test statistics is χ^2 with the degrees of freedom is nine. However, as discussed in Frydman and Kadam (2004), the distribution of the likelihood ratio statistic may not be valid because the null hypothesis lies on the boundary of the parameter space. Thus, we believe that a study of the asymptotic distribution under null hypothesis should be pursued in future work. The author thanks anonymous reviewers for this suggestion.

Table 2: The intensity matrix based upon continuous-time observations in the period 1998-2006

	Rating at the end of year									
Initial rating	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***D***
1	-0.0604	0.0370	0.0137	0.0079	0.0000	0.0018	0.0000	0.0000	0.0000	0.0000
2	0.0093	-0.0873	0.0583	0.0128	0.0069	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0107	-0.0794	0.0466	0.0187	0.0018	0.0008	0.0000	0.0008	0.0000
4	0.0000	0.0007	0.0139	-0.0746	0.0443	0.0124	0.0016	0.0011	0.0006	0.0000
5	0.0000	0.0000	0.0000	0.0217	-0.0848	0.0504	0.0098	0.0018	0.0000	0.0011
6	0.0000	0.0000	0.0000	0.0019	0.0362	-0.1065	0.0543	0.0084	0.0027	0.0030
7	0.0000	0.0000	0.0000	0.0000	0.0030	0.0475	-0.1227	0.0536	0.0079	0.0107
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0035	0.0617	-0.1484	0.0558	0.0274
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0011	0.0150	0.0378	-0.1059	0.0520
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

. The time-varying risk premium was then extracted and incorporated into the transition matrices of the mover-stayer model to transfer the transition matrices into a risk-neutral transition matrix. The risk premiums from 1998 to 2006 are listed in Table 4. We determined that when the risk premium was exactly unity, the default probability remained unchanged when the risk-neutral probability measurement was performed, i.e., $p_{ij} = \tilde{p}_{ij}, \forall i$. Therefore, the risk premium compensates for the increased discrepancy between default probabilities in the real and the risk-neutral world.

Table 3: Average transition matrix of mover-stayer model from 1998 to 2006

	Rating at the end of year									
Initial rating	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***D***
1	0.5977	0.1157	0.1146	0.0955	0.0382	0.0191	0.0000	0.0191	0.0000	0.0001
2	0.0189	0.6669	0.1082	0.0853	0.0853	0.0142	0.0212	0.0000	0.0000	0.0000
3	0.0000	0.0200	0.6141	0.1579	0.1460	0.0372	0.0186	0.0000	0.0000	0.0062
4	0.0000	0.0029	0.0154	0.8385	0.0774	0.0456	0.0116	0.0043	0.0029	0.0014
5	0.0000	0.0000	0.0009	0.0491	0.7482	0.1173	0.0566	0.0172	0.0000	0.0107
6	0.0000	0.0000	0.0000	0.0118	0.0732	0.7325	0.1268	0.0239	0.0181	0.0137
7	0.0000	0.0000	0.0013	0.0013	0.0172	0.1208	0.6689	0.1207	0.0392	0.0306
8	0.0000	0.0000	0.0000	0.0000	0.0135	0.0364	0.0942	0.5974	0.1998	0.0587
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0294	0.0750	0.0634	0.7395	0.0927

Table 4: Risk premium from 1998 to 2006

Year / Rating class	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	0.9995	0.9967	0.9646	1.0000	0.9189	0.7074
3	1.0000	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0006	1.0000	1.0000	1.0001	1.0001	1.0001	1.0001	1.0001	1.0001
6	1.0006	1.0000	1.0000	1.0000	1.0001	1.0001	0.9527	0.7423	0.5729
7	1.0005	1.0000	1.0002	1.0003	1.0003	1.0003	1.0003	1.0004	1.0003
8	1.0005	1.0002	1.0007	1.0006	1.0006	0.7290	0.4106	1.0007	1.0006
9	1.0011	1.0000	1.0000	1.0008	1.0009	1.0007	1.0015	1.0021	1.0012

Table 5: Average risk-neutral transition matrix of mover-stayer model from 1998 to 2006

	Rating at the end of year									
Initial rating	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***D***
1	0.5977	0.1157	0.1146	0.0955	0.0382	0.0191	0.0000	0.0191	0.0000	0.0001
2	0.0180	0.6363	0.1032	0.0814	0.0814	0.0136	0.0203	0.0000	0.0000	0.0458
3	0.0000	0.0200	0.6141	0.1579	0.1461	0.0372	0.0186	0.0000	0.0000	0.0061
4	0.0000	0.0029	0.0154	0.8385	0.0774	0.0456	0.0116	0.0043	0.0029	0.0014
5	0.0000	0.0000	0.0009	0.0491	0.7483	0.1173	0.0566	0.0172	0.0000	0.0106
6	0.0000	0.0000	0.0000	0.0109	0.0673	0.6730	0.1166	0.0220	0.0166	0.0936
7	0.0000	0.0000	0.0013	0.0013	0.0172	0.1208	0.6691	0.1207	0.0392	0.0304
8	0.0000	0.0000	0.0000	0.0000	0.0122	0.0329	0.0852	0.5406	0.1808	0.1483
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0294	0.0751	0.0635	0.7402	0.0918

From Table 5, we observe that the off-diagonal probabilities above the diagonal line were higher, implying a higher probability of downgrading to other grades. However, we did not observe the phenomenon of off-diagonal probabilities below the diagonal line.

As a result, we posit that downgrade momentum[9] might exist in the rating

[9] The momentum of downgrade is a type of non-Markov effect. One well known non-Markov effect is the effect of downward momentum on ratings. This means that firms that are downgraded to a lower class have a higher probability of experiencing a further downgrade from this class than companies that were not downgraded into the same class. Similar effects have been documented with respect to the effects of upward momentum. A previous studies of non-Markov effect was performed in Altman and Kao (1992), Lucas and Lonski (1992), Carty (1997), Kavvathas (2001), and Lando and Skødeberg (2002).

behavior with no evidence of an upgrade momentum.[10]

Rating momentum presupposes that prior changes in rating carry predictive power regarding the direction of future rating changes. That is, downgrade momentum would suggest that a borrower downgraded to a lower rating class is more likely to be downgraded again than upgraded.

To test the effects of momentum on credit rating transitions, we used logit models for credit ratings over 9 years. According to Fuertes and Kalotychou (2006), we define

$$UP_{it} = \begin{cases} 1 & \text{if borrower i was upgraded in year t} \\ 0 & \text{otherwise} \end{cases}$$

$$DW_{it} = \begin{cases} 1 & \text{if borrower i was downgraded in year t} \\ 0 & \text{otherwise} \end{cases}$$

$$UM_{it} = \begin{cases} 1 & \text{if borrower i was upgraded to the current rating over [t-1, t-9]} \\ 0 & \text{otherwise} \end{cases}$$

$$DM_{it} = \begin{cases} 1 & \text{if borrower i was downgraded to the current rating over [t-1, t-9]} \\ 0 & \text{otherwise} \end{cases}$$

The UM_{it} and DM_{it} are referred to as the upward and downward momentum indicators, respectively. We then estimate the upgrade logit regression as

$$y_{it} = \beta \upsilon_{it} + \varepsilon_{it}\ , \quad \varepsilon_{it} \sim iid(0, \sigma_i^2)\ , \quad i = 1, \cdots, N\ , \quad t = 1, \cdots, T$$

where $UP_{it} = 1$ for $y_{it} \geq 0$ and $UP_{it} = 0$ otherwise and $\upsilon_{it} \equiv UM_{it}$. A similar logit model is estimated to test for downgrade momentum.

The estimation results are presented in Table 6. The downgrade logit estimates provide strong support for the downgrade momentum. That is, we found that a downgrade in previous years significantly increased the current downgrade

[10] Fuertes and Kalotychou (2006) found significant downgrade momentum in sovereign ratings. However, their evidence did not provide upgrade momentum. Lando and Skødeberg (2002) also identified significant downgrade momentum.

probability. However, the upgrade logit model provided no evidence of upgrade momentum, at a 5% significant level. Therefore, the result supports the supposition that a downgraded borrower is more prone to a subsequent downgrade than an upgrade.

Table 6: Momentum effect

	Upgrade		Downgrade	
coefficient	estimate	p-value	estimate	p-value
β	0.081	0.098	0.309	0.000

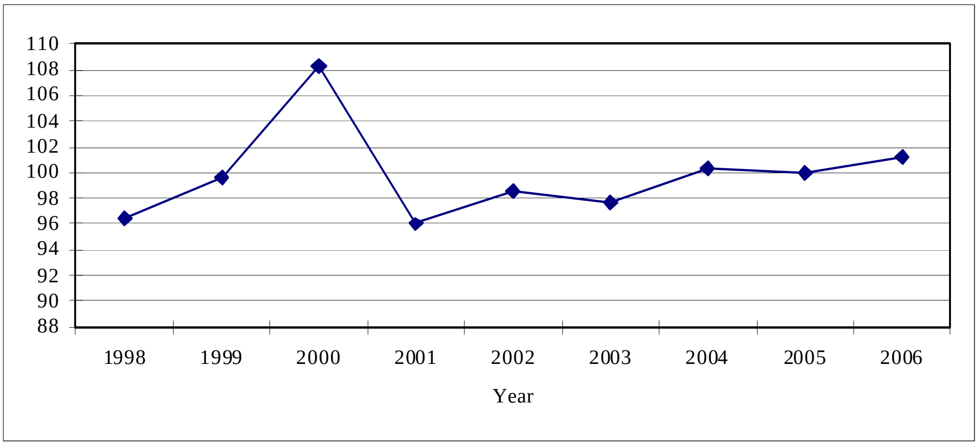

Figure 1: Trend-adjusted index of composite coincide index of Taiwan from 1998 to 2006

Feder and Just (1980), Kutty (1990), Athanassakos and Carayannopoulos (2001), and CreditPortfolio View suggested that economic performance is related to credit risk. Hence, the economic state is enormously important in the assessment of the probability of default (Belkin, Suchower, and Forest, 1998; Kim, 1999). In other words, credit risk may be related to economic state.

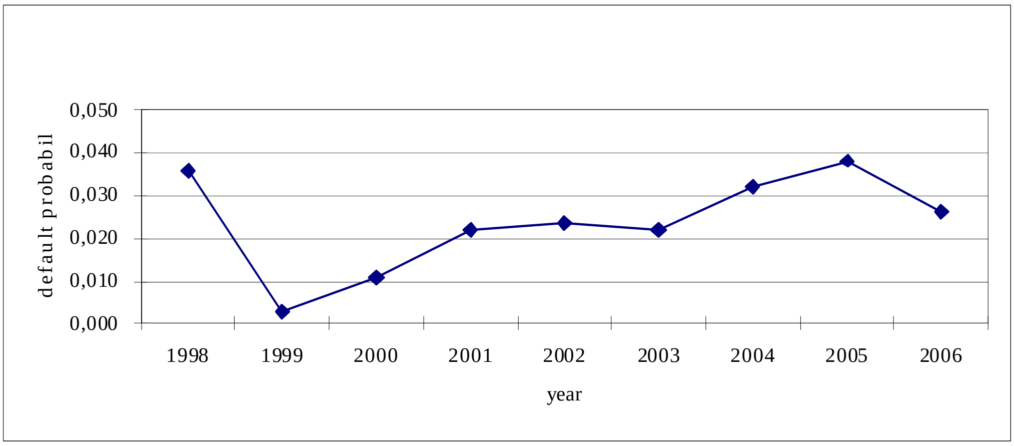

Figure 2: Default probabilities of mover-stayer model from 1998 to 2006

We then use the trend-adjusted index of the composite coincide index[11] to represent the economic state. Figure 1 shows the trend-adjusted index of the composite coincide index of Taiwan from 1998 to 2006. Figure 2 plots the default probability estimates from 1998 to 2006. As seen in Figure 1 and Figure 2, the default probability is relative high in 1998, due to the Asia financial crisis.[12] Because the economic state was a recession in 2001, the default probability in 2001 was relatively higher than that in 2000. In general, a higher trend-adjust

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[11] The data comes from the Council for Economic Planning and Development (CEPD) in Taiwan. The CEPD is responsible for drafting overall plans for national economic development; evaluating development projects; coordinating the economic policymaking activities of ministries and agencies; and monitoring the implementation of development projects, measures, and programs.

[12] The Asian financial crisis was initiated by two rounds of currency depreciation that have been occurring since early summer 1997. The first round was a precipitous drop in the value of the Thai baht, Malaysian ringgit, Philippine peso, and Indonesian rupiah. As these currencies stabilized, the second round began with downward pressures hitting the Taiwan dollar, South Korean won, Brazilian real, Singaporean dollar, and Hong Kong dollar.

index leads to a lower default probability; however, 2002-2004 were exceptions, due to the many shocks, such as SARS (Severe Acute Respiratory Syndrome) and the Iraq War. In 2004, the situation in Taiwan was confused due to the presidential election in Taiwan, and investors remained pessimistic due to these shocks. For these reasons, these shocks destroyed the inverse relationship between the trend-adjust index and default probabilities from 2002-2004.

Because economic state plays an important role in gauging the probability of default, financial institutions should be concerned with credit risk to guard against economic recession. According to Figure 1 and Figure 2, we find that probability estimated by the mover-stayer model was closely associated with the economic state. Consequently, we propose that the mover-stayer model could help financial institutions to gauge credit risk.

5 Conclusion

In this paper, we adopted a mover-stayer model for measuring the credit risk of bank loans. The mover-stayer model is an extension of a Markov chain, postulating heterogeneity in rating migration. That is, this model proposes that the population comprises two subpopulations, movers, and stayers. Furthermore, we performed continuous-time observations, whereupon the time-varying risk premium was incorporated into the mover-stayer model. The model proposed for evaluating credit risk is the continuous-time non-homogeneous mover-stayer model under risk-neutral probability measurement.

The main conclusions and contributions can be drawn as follows. First, we determined that this model is better suited to assessing credit risk than the Markov chain model. Second, from the rating properties of borrowers, we found that most movers clustered in investment grades, implying that borrowers in investment the grades are less likely to remain at their original grade level. On the other hand, there is a higher tendency to downgrade, implying the existence of downgrade momentum in rating behavior. However, no evidence suggested the existence of upgrade momentum for speculative grade investments. Third, we estimated time-varying risk premium by incorporating lending rate and risk-free rates that are usually ignored during the assessment of credit risk. Fourth, default probability estimated by the mover-stayer model immediately reacted to the business cycle. On the whole, we believe that the proposed model could help financial institutions to gauge their credit risk more effectively.

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