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The less likely the better: An empirical analysis of trading strategies accounting for the presence of stock market regimes

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Abstract

This contribution studies the out-of-sample performance of trading strategies applying 2-State-Markov-Switching models. Thereby, different probability thresholds are considered where the investor decides when to go in, respectively, out of the stock market. Furthermore, the investor may decide to invest in a risk free asset when a bear-market is expected to occur in the forecast period. In this study, the US-stock index S&P 500 is employed where the challenging period from January 2008-December 2010 is used for the out-of-sample experiment. The optimal trading strategies, given that the investor does not decide to invest in the risk free asset, suggest low probability thresholds of 0.075 and 0.050 concerning bull-market probabilities. Therefore, the investor is invested 69% respectively 88% of the investment horizon in the stock market. The optimal trading strategies exhibit total gross gains of 34.35% respectively 6.01% during the out-of-sample period, whereas the S&P 500 stock market return was -8.07% during the period under consideration.

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1 Introduction

During the last decade stock market participations faced three severe stock market crashes. Aroa and Buza [3] mention that the same mass psychology that created the boom in the stock market during 1995-1999, was accountable for the crash in the NASDAQ in January-March 2000. Stock markets all over the world followed and got into a severe bear-market. The same patterns were observable during the financial crises resulting in worldwide bear-market in 2008 and the current debt crises in 2011. The latest stock market crash was introduced by negative returns of the indices S&P 500, EuroStoxx 50, Hang Seng and Nikkei 225 of -12.08%, -15,40%, -22.39% and -12.69% within only two months (i.e. August 01, 2011 – September 30, 2011). Claessens, Koese and Torrones [5] who study a large data set of recessions, equity price declines and credit contractions within OECD countries suggest that the duration of equity price declines (i.e. bear-markets) is on average 26.56 months whereby the average duration of bull-markets is estimated at 47.16 months. As negative returns especially occur within bear-markets, a rational investor may attempt to time the market which means to be invested in stocks only within bull-market regimes and not to be invested in the stock market when the regimes switches to a bearish one. Several studies employ Markov-Switching models (MSM) providing estimates of state probabilities, given each point in time. As stock market regimes are in line with Guidolin and Timmernann [7], [8] and Grobys [6] among others persistent, the investor may decide to be invested in the stock market only when a certain probability level which acts as indicatorfunction of the current market regime is exceeded. But what is the optimal probability level? This contribution throws light on this issue while distinguishing between different models. These studies show that the optimal probability threshold depends on both the data frequency being used and the investment alternative. If the investor has not the possibility to invest in a risk free asset during bearish markets, the highest Sharpe ratio (i.e. 0.7061) exhibits a strategy where the probability threshold indicating bull-market regimes is chosen to be $PL = 0.050$, while taking into account weekly frequented input data.

2 Background

Investors, academics and practitioners establish that low frequency trends in stock markets do exist which is often referred to as bull- and bear-market regime respectively. This common conclusion has been established by studies of Perez-Quiros and Timmermann [10], Ang and Bekaert [2], Guidolin and Timmernann [7] & [8] and Grobys [6] among others. Guidolin and Timmermann [8] employ a multivariate 4-State-MSM where the first state is a low return, highly volatile crash/bear state, state 2 and 3 are low-volatility, bullish states, while regime 4 is a high-volatility, recovery state which tends to follow crash regimes. They argue that regimes change frequently although the states are quite persistent. Their findings suggest that optimal asset allocations vary considerably across these states and change over time as investors revise their estimates of the state probabilities. Furthermore, Alexander and Dimitriu [1] employ a 2-State-MSM to assess structural breaks in the relationship between abnormal returns and stock price dispersion. They argue that regime switching models provide a systematic approach allowing for modeling multiple breaks and regime shifts in the data generating process. They test the predictability of their model by employing operational criteria respectively trading rules while the trading rule is constructed solely on the sign of the change in dispersion but not on its magnitude. A similar approach is applied in studies by Grobys [6] who analyzes stock market linkages in bull- and bear-markets regimes. Thereby, a multivariate 2-State-MSM is estimated capturing low frequented trends in different stock markets simultaneously. In an out-of-sample experiment a probability threshold is employed that indicates the current stock markets' regime and thereupon the decision rests in which stock market to be invested. All these studies suggest predictability of regimes even though Markov-Switching models are not appropriate to forecast actual returns as shown by studies of Bessec and Bouabdallah [4] for instance. The following contribution uses a simple 2-State-MSM where two different data frequencies are taken into account namely monthly and weekly data. These studies suggest that a model involving weekly frequented data basically exhibits higher gains and trading activity whereas both models are dominated by a strategy where the investor can invest in a risk free rate when the stock market is expected to remain in a bear-market regime.

3 Econometric Methodology

Following Hamilton [9], it will be supposed that the stock market's mean and volatility are driven by a state variable S_t that may take integer values from $1,...,k$

$$\log(R_t) = \mu_{S_t} + \varepsilon_t \,, \tag{1}$$

where $\log(R_t)$ denotes the log-returns on time t, μ_{S_t} denotes the mean of the stochastic process depending on the state S_t, and ε_t denotes the error which is assumed to be distributed with $N\left(0,\sigma_{S_t}^2\right)$. Furthermore, the regime switching in the variable S_t (i.e. from "bear-market" to "bull-market" or vice versa) are governed by the transition probability matrix S, where S is a $\left(k \times k\right)$ matrix with elements

$$\Pr\left(S_t = i \middle| S_{t-1} = j\right) = p_{ji}\ , \tag{2}$$

with $i, j = 1,...,k$. Each regime is in line with Hamilton [9] the realization of a first-order Markov chain with constant transition probabilities as also applied in studies by Guidolin and Timmermann [8] and Grobys [6]. As the state variable S_t is hidden, respectively, unobservable a filtered estimate has to be computed from the datavector R_t. Estimation will be performed by maximizing the likelihood function being associated with (1)-(2). As S_t is assumed to be unobservable, it has to be treated as latent variable.

The estimates $\left(\hat{\mu}_{S_1},...,\hat{\mu}_{S_k}\right)$, $\left(\hat{\sigma}^2_{S_1},...,\hat{\sigma}^2_{S_k}\right)$ and $\hat{S}$ being used to assess the marginal state probabilities are restricted to the information set Ω. The latter takes only into account the in-sample-data $t = 1,...,T_1$. As regimes are assumed to be highly persistent, the current state probabilities $\left(\hat{p}_{1t},...,\hat{p}_{kt}\right)$ being associated with the last observation are used as best forecasts for the one-step-ahead forecast that means $\left(\hat{p}_{1,t},...,\hat{p}_{k,t}\right) \simeq \left(\hat{p}_{1,t+1},...,\hat{p}_{k,t+1}\right)$ whereby the out-of-sample period runs from $T_1 + 1,...,T_2$ [2]. The investor decides whether in the next period $t+1$ to invest in the stock market or not if a regime where a positive return can be expected (i.e. bull-market) exceeds a certain probability threshold $PT \in (0,1)$ in the current period t such that the gain function is given by

$$\Pi\left(PT, R_{t+1}\right) = \sum_{t=T_1}^{T_2} d_{PT,t} \cdot R_{t+1}$$

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[2] This operational criterion makes only sense if the regimes are highly persistent and if the deviation between the marginal probability that takes only into account a certain information set and the actual estimated state probability of the following period is close to zero. The empirical p-value of testing the null hypothesis $\left(\hat{p}_{bull,T_1} - \hat{p}_{bull,T_1+1}\right) = 0$ is 0.4231 based on weekly frequented data suggesting that the approximation $\hat{p}_{bull,t} \simeq \hat{p}_{bull,t+1}$ can be employed as an estimator.

where

$$d_{PT,t}=1 \text{ if } \hat{p}_{t,bull}>PT \quad \text{and} \quad d_{PT,t}=0 \text{ if } \hat{p}_{t,bull}\le PT .$$

As an alternative to the decision whether to invest or not, the investor may decide to invest in a risk free asset when deciding to be not invested in stocks. The risk free asset is assumed to exhibit returns of rf/t where the gain function is given by

$$\Pi\left(PT,R_{t+1}\right)=\sum_{t=T_1}^{T_2} d_{PT,t}\cdot R_{t+1}+\sum_{t=T_1}^{T_2} d_{rf,t}\cdot rf/t$$

with

$$d_{rf,t}=1-d_{PT,t} \quad \forall \quad t\in\left(T_2-T_1\right).$$

4 Results

Following Guidolin and Timmermann [8] stock market data from the US-stock index S&P 500 is taken into account. Thereby the model is estimated for two different frequencies namely monthly and weekly data. The in sample period runs from January 1954-December 2007 being in line with Guidolin and Timmermann [8] whereby the in-sample estimates for $\left(\hat{\mu}_{S_1},...,\hat{\mu}_{S_k}\right)$, $\left(\hat{\sigma}^2_{S_1},...,\hat{\sigma}^2_{S_k}\right)$ and $\hat{S}$ are used to restrict the model whereupon out-of-sample estimates for $\left(\hat{p}_{1t},...,\hat{p}_{kt}\right)$ are based. The model is estimated for $k=2$, where $k=1$ denotes the bull state and $k=2$ denotes the bear state. The matlab package MS_Regress from Perlin [11] is used to estimate the 2-State-Markov-Switching models whereby the package commands adv.Opt are used to restrict the iterative estimates for $\hat{p}_{1,t+i-1}\simeq\hat{p}_{1,t+i}$ where $i=1,...,T_2$. Equation (3) shows the estimated model employing monthly data whereas equation (4) shows the estimates for the model being based on a weekly frequency:

$$\begin{pmatrix} \hat{\mu}_{S_1} \\ \hat{\mu}_{S_2} \end{pmatrix} = \begin{pmatrix} \underset{(0.0008)}{0.0045} \\ \underset{(0.0025)}{-0.0031} \end{pmatrix} \text{ with } \begin{pmatrix} \hat{\sigma}^2_{S_1} \\ \hat{\sigma}^2_{S_2} \end{pmatrix} = \begin{pmatrix} \underset{(0.0000)}{2.00e-04} \\ \underset{(0.0001)}{6.73e-04} \end{pmatrix} \quad \text{and} \quad \hat{S} = \begin{pmatrix} \underset{(0.04)}{0.96} & \underset{(0.07)}{0.14} \\ \underset{(0.02)}{0.04} & \underset{(0.07)}{0.86} \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \hat{\mu}_{S_1} \\ \hat{\mu}_{S_2} \end{pmatrix} = \begin{pmatrix} \underset{(0.0002)}{0.0011} \\ \underset{(0.0005)}{-0.0005} \end{pmatrix} \text{ with } \begin{pmatrix} \hat{\sigma}^2_{S_1} \\ \hat{\sigma}^2_{S_2} \end{pmatrix} = \begin{pmatrix} \underset{(0.0000)}{3.80e-05} \\ \underset{(0.0000)}{1.58e-04} \end{pmatrix} \quad \text{and} \quad \hat{S} = \begin{pmatrix} \underset{(0.02)}{0.98} & \underset{(0.01)}{0.04} \\ \underset{(0.00)}{0.02} & \underset{(0.01)}{0.96} \end{pmatrix} \tag{4}$$

The expected bull- respectively bear-market duration is 23.48 months and 7.72 months for the model of equation (3), whereas the corresponding durations are 15.53 months and 7.48 months for the model of equation (4).

The probability thresholds $PT \in \{0.975, 0.950, ..., 0.050, 0.025\}$ are held constant for the out-of-sample period running from January 2008-December 2010 corresponding to 36 and 156 out-of-sample observations in monthly and weekly terms respectively. Figure 1 shows the Sharpe-ratios of the different strategies depending on the probability threshold (horizontal axis) and therewith the investment strategy.

The performance of the investment strategies depends on both the alternative decision whether to hold the risk free asset when not invested in the stock market and the data frequency. Given that the investor does not invest in the risk free asset, the Sharpe ratios of the model taking into account weekly data outperforms the 2-State-MSM model employing monthly data for $PL = [0.975, 0.725]$ and $PL = [0.600, 0.025]$. The maximum Sharpe ratio (i.e. 0.7061) exhibits a strategy (A) where the probability threshold is chosen to be $PL = 0.050$ while taking into account weekly frequented data. Thereby the investor is 69% of the investment time (i.e. January 2008 – December 2010) invested in the S&P 500 stock market. This strategy exhibits an annual expected gross return of 11.45% and a corresponding volatility of 16.21% where the investor trades 19 times (i.e. in- and out-of the market). The maximum Sharpe ratio for the 2-State-MSM model taking into account monthly data is 0.3440 (strategy B) and achieved when the investor chooses a probability threshold of $PL = 0.075$ whereby the investor is invested in 86% of the investment time in the stock market. Thereby the investor trades twice the position in- and again out-of the stock market.

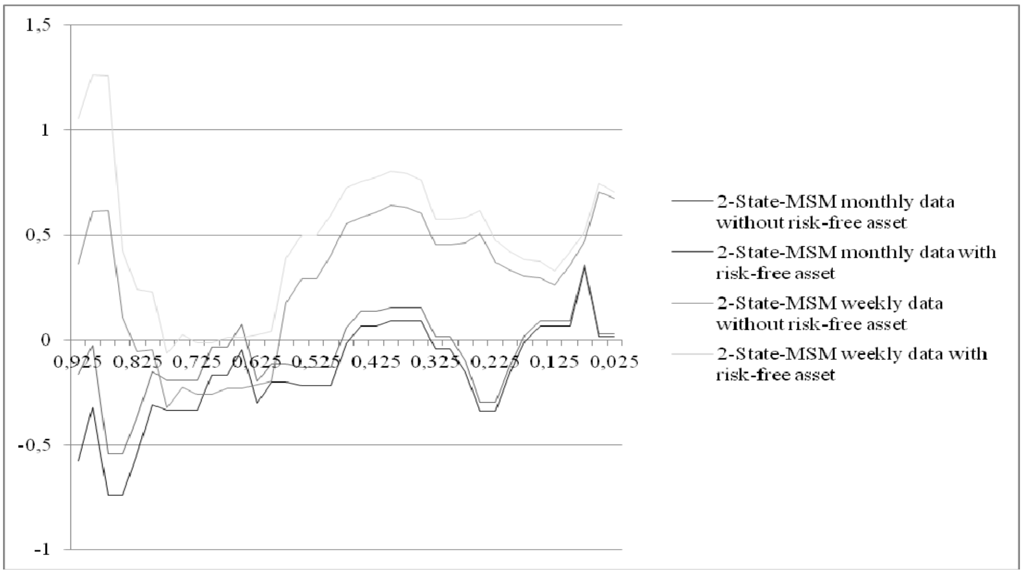

Figure 1: Trading strategies' Sharpe-ratios and related probability thresholds

If the investor takes into account the alternative that is to invest in the risk free rate when not invested in the stock market, the optimal thresholds change. In the following the risk free interest rate is assumed to be 2% p.a. Then, for $PL = \{0.975, 0.950\}$ the Sharpe ratios are infinity as the investor would not invest in stocks at all, but holds the risk free asset only. Combined investment strategies where the investor is supposed to be invested in the stock market for at least one time period exhibit maximum Sharpe ratios of 1.2637 (strategy C) corresponding to $PL = 0.900$ and 0.3601 (strategy D) corresponding to $PL = 0.075$ for the weekly and monthly frequented model respectively.

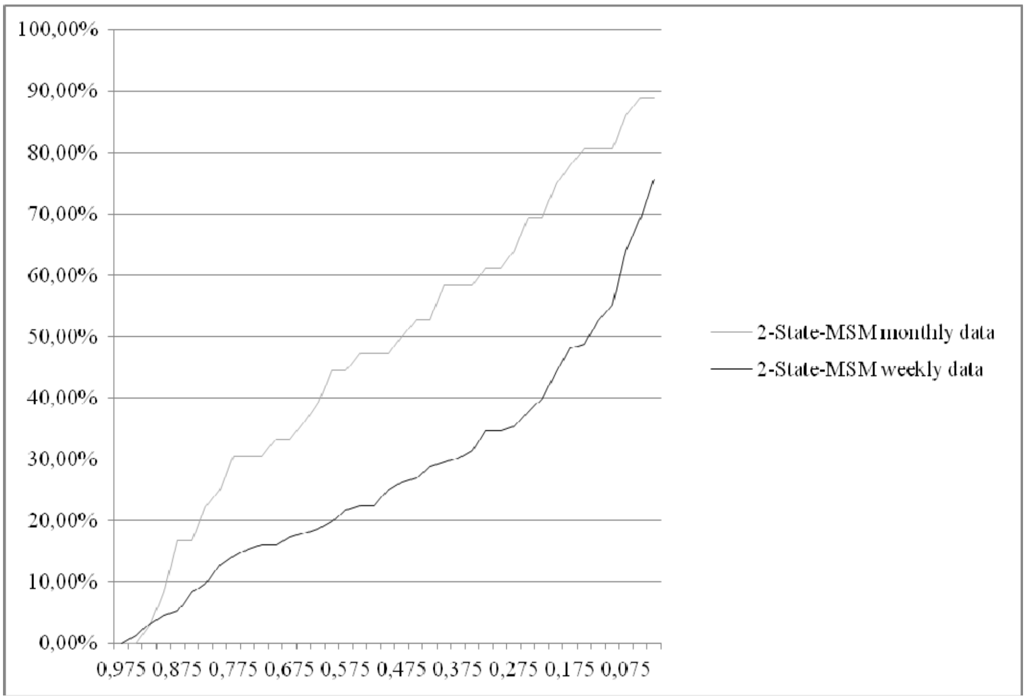

Figure 2: Time invested in the S&P 500 (in %) and probability thresholds

5 Discussion

If the investor does not invest in a risk free asset when the chosen probability of bull-market regime falls below the probability threshold, both models suggest an optimal probability threshold below 0.1. In other words, the investor decides to go out of the stock market only when the probability of bear-market is already above 0.9275 or 0.950 in the previous period. The optimal trading strategies exhibit an out-of-sample return of 11.45% p.a. and 6.01% p.a. respectively whereas the S&P 500 returns sum up to -8.07% from January 2, 2008-December 1, 2010. Consequently, both trading strategies (A and B) outperform compared to the benchmark. However, taking into account the possibility to invest in a risk free asset changes the results concerning the model accounting for weekly data while suggesting a higher probability threshold of $PT = 0.875$ which can be explained by the nonlinear volatility decrease as the time being not invested in the stock market decreases.

In contrast to Grobys [6] studies where the model is re-estimated for all out-of-sample observations, the parameter estimates for the models in (3) and (4) are hold constant. Thus, the probability estimates for the out-of-sample period are restricted to follow the same process as in the sample. The in-sample data accounts for the period from January 1954-December 2007 which is also used in the studies by Guidolin and Timmermann [8]. Consequently, 648 monthly and 2816 high frequented weekly observations could be used to estimate the models ensuring stable parameter estimates. The estimated durations of bull- and bear-markets regarding the model where weekly data is taken into account (i.e. 15.53 and 7.48 months) are close to the estimates in Grobys' [6] multivariate-MSM where the durations are estimated at 15.67 and 8.46 months for bull- and bear-markets respectively. In contrast to this, Guidolin and Timmermann [8] estimate the bear-market to be low persistent as only two months are spent in this regime. The estimated mean of bear-market returns is in amount lower than the estimated mean for bull-markets (see equations (3) & (4)) which is also in line with Grobys [6] studies whereas the volatilities in bear-market regimes is 3.4 respectively 4.2 times higher than in bull-market regimes.

6 Conclusion

Unlike others studies this empirical out-of-sample experiment shows that the lower the probability threshold concerning the bull-market regime is chosen the higher the trading strategies' gains as long as the investor does not invest in the risk free asset when going out of the stock market. This result hold even for the model that accounts for monthly data, given the investor decides to invest in the risk free asset when expecting bear-market regimes in the next period. The strategies based on monthly data model are invested larger periods of time of the overall investment horizon in the stock market. As the Sharpe ratios of the latter are basically lower in comparison to the model accounting for higher frequented data, it can be concluded that it is less accurate to for market timing purposes. The simple probability threshold framework can be expanded where the probability thresholds for entrance in the market and exist from the market may differ. Moreover, the 2-State-MSM can be replaced by a MSM that accounts for more regimes which requires though more complex trading rules. These issues may be areas for future research.

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