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Market timing and statistical arbitrage: Which market timing opportunities arise from equity price busts coinciding with recessions?

The Swedish stock market in the financial crises 2008

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Abstract

Even though a random walk process is from a statistical point of view not predictable, some movements can be correlated with specific events concerning other variables. Then, predictable patterns may arise being dependent on this joint event. There is evidence given that equity price busts being associated with recessions continue until the economy switches from the state of recession to an economic pick-up. The following contribution takes into account the Swedish stock index OMX 30 and 25 preselected stocks. The out-of-sample period runs from September 12, 2008 – March 12, 2009, whereas on September 11, 2008 the official press release was issued that European economies face a recession. This study suggests a market timing opportunity resulting in a maximum statistical arbitrage opportunity corresponding to a profit of 19% p.a. with an empirical probability of 50.14%. The optimal defensive strategies, however, exhibit excess

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returns of 15.12% p.a. above the benchmark with a marginal lower volatility as the benchmark, respectively, 28.08% p.a. with 7.99 percent units higher volatility as the benchmark.

JEL classification numbers: C13, C22, G11, G12

Keywords: Statistical arbitrage, Financial crises, equity price busts, Cointegration

1 Introduction

The perception that stock prices already reflect all available information being often referred to as the efficient market hypothesis is widely discussed in the academic literature and rests upon studies by Kendall (1953). As soon as there is any news available indicating that a stock is underpriced, Bodie, Kane and Marcus (2008) highlight that rational investors would buy this stock immediately and, hence, bid up its price to a level where only ordinary returns can be expected to gain. Consequently, the efficient market hypothesis suggests that stock prices follow random walk processes involving that prices changes are random and thus unpredictable. Studies by Chan, Gup and Pan (1997) who examine eighteen national stock markets by using unit root tests figure out that the world equity markets are weak-form efficient and, hence, support the efficient market hypothesis. Unit-root tests as applied by Chan, Gup and Pan (1997) though take only information into account which is involved in the univariate data generating processes.

Against this, Claessens, Kose and Terrones (2009) provide a comprehensive empirical characterization of linkages between key macroeconomic and financial variables around business and financial cycles. Their studies involve 21 OECD countries and cover over 47 years from 1960-2007. Thereby, they take into account 122 recessions, 113 credit contractions and 245 episodes of equity prices declines, whereas 61 of these equity price declines are referred to as price busts. Equity price busts are in accordance to the definition of Claessens, Kose and

Terrones (2009) peak-to-trough declines in equity prices which fall within the top quartile of all price declines. Their findings show that equity price busts overlap about one-third of the recession episode. Furthermore, given the event that the economy faces a recession, in 60% of all cases equity price declines occur at the same time.

Moreover, Claessens, Kose and Terrones (2009) conclude that recessions tend to coincide with contractions in domestic credit and declines in asset prices and in most advanced countries. Thereby, the typical duration of en equity price bust is twice that of a recession, but ends at the same time when the associated recession is ending. But what implications do equity price busts offer concerning asset allocations if the bust coincides with a recession? If market participators expect stock prices to fall in further periods, they will rebalance their stock portfolios such that the expected loss will be minimized. As an alternative, investors could construct arbitrage portfolios while going short on the index and long on an equity portfolio exhibiting defensive properties and therewith outperforms if the stock markets declines in future periods.

In the following contribution the equity price bust being associated with the financial crises in 2008 is analyzed with respect to market-timing opportunities. Furthermore, the optimization problem being associated with an advantageous asset allocation conditional on the state of the economy is examined. Thereby, the Swedish leading stock index OMX 30 is taken into account. 50 different portfolios are estimated which track artificial indices corresponding to defensive investment strategies. The optimization procedure accounts for 20 stocks corresponding to the companies exhibiting the highest market capitalization being line with Alexander and Dimitriu (2005). While holding the optimal weights constant within a six-month period out-of-sample (i.e. September 12, 2008-March 12, 2009), evidence is given for statistical arbitrage opportunities. The estimated optimal asset allocations as suggested here dominate the index in both, the Reward-to-Risk ratio and the Reward-to-Risk-Difference ratio. During the six months period being examined both events occur at the same time, the economy faces a recession and the financial market faces an equity price bust. Consequently, a rational investor who expects the equity prices to fall in future periods will select a defensive asset allocation in order to minimize expected losses as soon as the recession is ascertained. Optimal defensive strategies, as suggested here, exhibit, given the considered Swedish stock market conditions, returns of -19.19% p.a. and -6.13% p.a. involving a volatility of 53.50%, respectively, 61.50% p.a. The OMX 30 though had a return of -34.16% p.a. and exhibited a volatility being equal to 53.51% under the same period of consideration (i.e. September 12, 2008-March 12, 2009).

2 Background

Even though the efficient market hypothesis holds when testing stock markets price movements of most advanced countries whether the event "equity price bust" occurs, given the event that the economy faces a recession, predictable patterns may evolve. Claessens, Kose and Terrones (2009) who consider a large data set of recessions, equity price declines and credit contractions within OECD countries, argue that a typical episode of an equity price decline, respectively, an equity price bust tends to result in a 24% and 51% fall in equity prices. Thereby, the duration's mean is 6.64 and, respectively 11.79 quarters where the latter figure is statistically significant even on a 1% significance level. Furthermore, recessions that coincide with equity price busts last for 3.79 quarters on average, whereas recessions that do not coincide with equity price busts last 3.49 quarters on average.

Claessens, Kose and Terrones (2009) conclude that equity price declines overlap with about one in three recessions. If a recession coincides with an equity price bust, the recession can start as late as four to five quarters after the asset bust has started. However, the equity price bust typically ends with the end of its corresponding recession but can continue for two to nine quarters after the recession has ended. Against it, the minimum duration of a recession is in accordance to Claessens, Kose and Terrones (2009) two quarters, whereas a typical recession lasts about four quarters. The latter fact clearly exhibits market timing potential: A rational investor who expects stock prices to fall during the next two quarters will choose a defensive asset allocation in order to minimize the expected loss in future periods. Furthermore, investors could exploit from equity price declines by shorting the index and taking a position in stocks which response defensively.

Once the recession is ascertained, a rational market participant will rebalance the equity portfolio in order to anticipate a further fall in equity prices. Even though Aroa and Buza (2003) mention that recessions are not periodic and that they differ in duration, intensity and occurrence, there are still similarities in the sequence of events and circumstances that typically occur over the course of a business cycle. The same is the case with respect to stock market crashes. Each stock market crash is preceded by a bubble formation as argued by Aroa and Buza (2003) where bubbles, respectively, bull markets are usually associated with a period of prosperity, when the future seems bright and investors have easy access to money. Against this, excessive pessimism follows this exuberance and creates as a consequence the stock market crash, respectively, the bear market. In accordance to Aroa and Buza (2003), the same mass psychology evoking the expectation that every dot-com company will be profitable and, hence, created the boom in the stock market during 1995-1999, was accountable for the crash in the NASDAQ in January-March 2000. The U.S economy began to slow down during the second half of the year 2000, and the rest of the world followed, resulting in a worldwide recession. If the market stands in a bear market, crisis events which can be generalized as bad news exacerbate the stock market's downturn movements as mentioned by Aroa and Buza (2003). Of course, the downturn will not end as long as the majority of news which arrive the market will be evaluated as good news from the market participitiants' point of view. Hence, press releases such as issued on September 11, 2008 from the German Insitute for Worldeconomics (IfW Kiel) declaring that European countries face a recession will consequently be associated with an expectation that stock prices will continue to decline even in future periods.[2]

For instance, the Swedish leading stock index OMX 30 lost already 36.91% compared to its peak on July 16, 2007 on the day where the official press release was issued (i.e. September 11, 2008). Considering a period of two quarters thereafter it could be observed that the OMX 30 fell by additional 17.08% (i.e. from September 12, 2008 – March 12, 2009). Hence, the equity price bust began more about 14 months before the recession was ascertained and continued afterwards. The same patterns could be observed in 2001. On November 26, 2001, the National Bureau of Economic Research issued a press release declaring the recession began in March 2001.[3]

Market observers recognized similar patterns: From November 26, 2001 until May 26, 2002 the Swedish leading stock index OMX 30 fell by 18.85%. However, the recession was also anticipated by an equity price bust where the OMX 30 lost already 44.27% from its peak on March 7, 2000 until the day where the recession was declared by the National Bureau of Economic Research (i.e. on November 26, 2001). The same patterns could be investigated concerning other European stock markets. For instance, the German's leading stock index DAX fell in the period November 26, 2001 until May 26, 2002 by 36.89%, whereas the stock index additionally fell by 4.20% from between November 26, 2001 and May 26, 2002. Considering the financial crises in 2008 and the associated equity price bust which again anticipated the recession, the DAX lost 23.77% from July 16, 2007 until September 11, 2008 and only additional 1.54% from September 12, 2008 – March 12, 2009.

2 See http://www.ifw-kiel.de/media/press-releases/2008/pr11-09-08b.

However, not all stocks participate in booms, respectively, bull markets. In accordance to Aroa and Buza (2003) railroad stocks were excluded from the boom of 1928-1929, whereas overinvesting in utilities caused this speculative bubble formation. The bubble formation during 1995-1999 showed an overpricing of the telecommunication and internet sector as studied by Jensen (2005) and Harmantzis (2004), whereas a similar mass psychology caused the overpricing concerning the financial sector during 2004-2008 as described by Baker (2008) and Soros (2008). Poterba and Summers (1989) and Cecchetti, Lam and Mark (1990) point out that the sector will adjust the stronger the more excessive the speculative bubble has been.

Therefore, a rational investor who expects the market to decline in further periods will allocate the assets to a portfolio exhibiting defensive properties during the equity price decline. As a consequence of the equity price bust which started March 2000, Aroa and Buza (2003) mention that investors had moved the money into energy and health care company stocks during 2000 and 2001 since these sectors were expected to response defensively in bear markets. But do defensive asset allocation strategies being built on historical stochastic movements of artificial indices exhibit robustness within the out-of-sample period? This contribution throws light on the following issues: First, 50 different asset allocations will be estimated which track constructed artificial indices assuming to exhibit defensive properties if the investors expect the market to decline in further periods. Thereby, the Swedish stock index OMX 30 will be employed in order to construct artificial indices and a set of 20 preselected stocks will be used in order to estimate optimal asset allocation weights. Second, based on these portfolios tracking defensive artificial indices it will be determined which would be the optimal asset allocation, given the out-of-sample risk-return estimates. Thereby, two different optimization calculi will be taken into account. The third issue is that

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[3] See http://www.nber.org/cycles/november2001/ (accessed on December 02, 2010 21:25).

it will be discussed how this market-timing approach can be applied for both, funds management and hedge funds management.

3 Econometric Methodology

In order to estimate weight allocations exhibiting defensive stock portfolios, three years of historical daily data is taken into account. Following Alexander and Dimitriu (2005), the cointegration approach is employed where three years of daily data is necessary to estimate robust cointegration optimal allocation weights. The day where the press release is issued will in the following be denoted as t^{rec}, whereas the day where the stock index exhibits the highest notation during the latest bubble formation will in the following be denoted as $t^{\max}$. In line with Grobys (2010) a linear trend is added to the historical index returns that switch the direction on the day where the price bust begins. Since the exact day is unknown, it will be assumed that the price bust takes place on day $t^{\max}$ since on the latter day the stock market notation shows the maximum difference between $t^{\max}$ and t^{rec} within the last three years. Arora and Buza (2003) report that not all stocks participate in bubble formations. Hence, estimating portfolios that do not follow the market's exaggeration are expected to decline less than the market during the crash and can consequently be employed to estimate portfolios involving defensive asset allocations. In line with Grobys (2010) the linear trend is first subtracted to the market returns and switches at time point $t^{\max}$ the direction. Subtracting a linear trend term until $t^{\max}$ and adding the term from observation $t^{\max}$ onwards results in an artificial index being below the benchmark until $t^{\max}$ and exhibiting higher returns from $t^{\max}$ onwards as the bubble disperses. Then, the integrated time series corresponding to the artificial indices are given by

$$p_{\delta t} = c + \sum_{i=1}^{t} R_t^{OMX} - \sum_{i=1}^{t} \delta \cdot i \qquad \text{for } t = 1,\ldots,t^{\max} \tag{1}$$

$$p_{\delta t} = p_{\delta t^{\max}} + \sum_{i=t^{\max}}^{t} R_t^{OMX} + \sum_{i=t^{\max}}^{t} \delta \cdot i \qquad \text{for } t = t^{\max}+1,\ldots,t^{rec}, \tag{2}$$

where δ denotes the factor that is subtracted, respectively, added to the index in daily terms and R_t^{OMX} denotes ordinary index returns at time t. Hence, for each $\delta = \delta_1, ..., \delta_M$ different integrated artificial indices $p_{\delta t}$ can be generated. Figure (1) shows the index and the artificial index for the factor $\delta = 0.10$ (i.e. 25% in annual terms) for the in-sample period. The integrated time series of stocks being employed to track the artificial indices are in line with Grobys (2010) calculated such that

$$p_{kt} = c + \sum_{i=1}^{t^{rec}} R_{kt} , \tag{3}$$

where R_{kt} denotes the return of stock k at time t and c is a constant term.

In order to estimate cointegration optimal weight allocations, the maximum-likelihood optimization procedure is employed being in line with Grobys (2010) and given by

$$\log L(\theta, t, \delta) = -\frac{T}{2}\log(2 \cdot \pi) - \frac{T}{2}\log \sigma^2 - \frac{1}{2}\sum_{t \in T}\left(\frac{(\varepsilon_{\delta t})^2}{\sigma^2}\right) \tag{4}$$

where $\varepsilon_{\delta t} = p_{\delta t} - \sum_{k=1}^{K} a_{\delta k} \cdot p_{\delta kt}$. In accordance to van Montefort, Visser and Fijn van Draat (2008) it is usual to impose weight restrictions. In the following it is sufficient though to restrict the weights to sum up to one and to be positive being given by

$$\sum_{i=k}^{K} a_{\delta k} = 1 \tag{5}$$

$$a_{\delta k} > 0 \qquad \text{for} \qquad k = 1, ..., K . \tag{6}$$

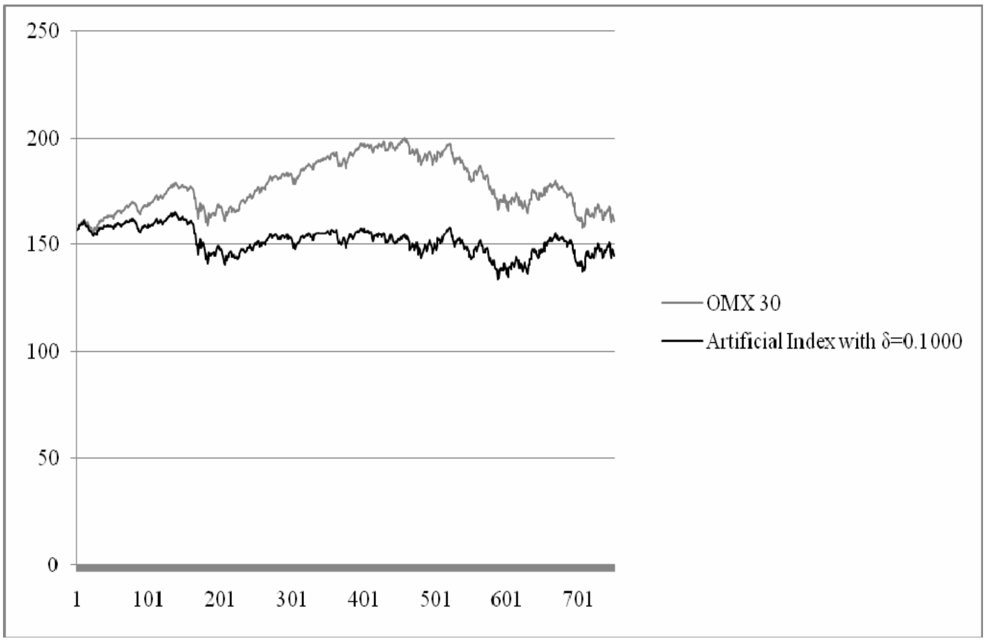

Figure 1: The OMX 30 and the an artificial index within the in-sample period

The weights being estimated at day t^{rec} are hold constant two quarters ahead as the market decline is in accordance to Claessens, Kose and Terrones (2009) expected to end with the recession while the minimum recession takes per definition two quarters. Each optimal weight allocation is stored in a vector and employed to estimate M different out-of-sample portfolio processes depending on $\delta = \delta_1, ..., \delta_M$. The optimal defensive strategies can be determined by optimizing the two following optimization problems being different from each other: First, the Reward-to-Risk ratio can be maximized, given by

$$\max_{\delta} \left\{ \frac{\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX} \right)}{1/N \sqrt{\sum_{t=t^{rec}}^{T} R_{\delta t} - 1/N \cdot \sum_{t=t^{rec}}^{T} R_{\delta t}}} \right\}, \tag{7}$$

where $R_{\delta t} = \hat{a}_{\delta 1} \cdot R_{1t} + ... + \hat{a}_{\delta K} \cdot R_{\delta Kt}$ denotes the estimated returns of the portfolio δ that tracks the artificial index $p_{\delta t}$, R_t^{OMX} denotes the ordinary index returns and the out-of-sample window runs from $t = t^{rec} + 1, ..., T$ while $N = T - t^{rec}$

denotes the trading days within the out-of-sample period. In equation (7) it is calculated how much does an additional return above the benchmark cost in terms of volatility and rests upon the Reward-to-Risk ratio being introduced by Sharpe (1964). The maximum value is optimal in the sense that it depicts the asset allocation that generates the highest return for each unit portfolio volatility with respect to the out-of-sample time window. However, the optimal asset allocation can be another one if the excess returns are related to the increase of volatility: In this case a rational investor would prefer to invest in portfolio p_{lt} instead of p_{mt} if the increase of excess returns exceeds the increase in portfolio volatility. Then, the optimization problem is in contrast to equation (7) given by

$$\max_{\delta}\left\{\left|\frac{\left(\sum_{t=t^{rec}}^{T} R_{\delta t}-\sum_{t=t^{rec}}^{T} R_t^{OMX}\right)}{1/N\left(\sqrt{\sum_{t=t^{rec}}^{T} R_{\delta t}-1/N\cdot\sum_{t=t^{rec}}^{T} R_{\delta t}}-\sqrt{\sum_{t=t^{rec}}^{N} R_t^{OMX}-1/N\cdot\sum_{t=t^{rec}}^{T} R_t^{OMX}}\right)}\right|\right\} \quad (8)$$

As the volatility of the optimal portfolio can be lower compared to the stock market's volatility, the absolute amount has to be maximized. The constructed portfolios are tested whether the maximum likelihood estimation provides weight allocations that exhibit a cointegration relationship with the artificial indices being tracked concerning the in-sample period. In line with Alexander and Dimitriu (2005) the ADF-test will be employed, given by

$$\Delta\hat{\varepsilon}_{\delta t}=\gamma_{\delta}\hat{\varepsilon}_{\delta t-1}+\sum_{l=1}^{L}\alpha_{\delta l}\Delta\hat{\varepsilon}_{\delta t-l}+u_{\delta t} \quad (11)$$

Thereby, the null hypothesis tested is of no cointegration, i.e. $\gamma_{\delta}=0$., against the alternative of $\gamma_{\delta}<0$.[4] Whether the null hypothesis of no cointegration is rejected, the cointegration-optimal tracking portfoliod based on the maximum likelihood procedure of equation (4) is expected to have similar stochastic patterns

[4]The critical values for the t-statistic of y are obtained using the response surfaces provided by MacKinnon (1991).

as the artificial indices concerning the in-sample period. The error vector $\hat{\varepsilon}_{\delta t}$ comes from an auxiliary regression from the integrated portfolio time series on the integrated artificial market index times series such that $\hat{\varepsilon}_{\delta t} = p_{\delta kt} - p_{\delta t} \cdot \hat{\beta}_{\delta}$ where, $\hat{\beta}_{\delta} = \left(p_{\delta t}' \cdot p_{\delta t} \right)^{-1} p_{\delta t}' \cdot p_{\delta kt}$ (see equations (1) and (2)). If $\hat{\varepsilon}_{\delta t}$ is stationary, the estimated portfolios are said to be cointegration optimal. Since the artificial indices are via construction cointegrated with the benchmark, the portfolios can be considered as being cointegrated with the ordinary benchmark, too.

Furthermore, the out-of-sample portfolios are priced first of all by running OLS-regressions as following:

$$R_{\delta t} = \alpha_{\delta} + \beta_{\delta} R_t^{OMX} + u_{\delta t}, \tag{11}$$

where $u_{\delta t}$ is assumed to be a white noise process and $R_{\delta t} = \left(p_{\delta t} - p_{\delta t-1} \right) \cdot 100 / p_{\delta t-1}$. Equation (11) is often referred to as ordinary index model (see Bodie, Kane and Marcus 2008) and is usually employed to determine whether the portfolio beta (i.e. β_{δ}) is above or below the market beta being equal to one. Thereby, a beta being larger than one indicates an offensive asset allocation while a beta below one usually indicates a defensive one. Furthermore, if the portfolio alpha (i.e. α_{δ}) is statistically significant higher than zero, the portfolio is said to generate abnormal returns and, hence, involves statistical arbitrage opportunities. However, Grobys (2010) mentions that the results of regressions such as formalized by equation (11) can be misleading as the statistical arbitrage is cached in the trend-stationary stochastic process being integrated in the portfolio processes. Moreover, regressions that take into account only the detrended series, such as the portfolio returns, do not account for this issue as mentioned by Alexander (1999). In line with Bondarenko (2003) a statistical arbitrage opportunity arises when the expected payoff of a zero-cost trading strategy is positive and negative returns occur only stochastically. Therefore, it will be analyzed how much the empirical probability is that an estimated portfolio exhibits returns being above the benchmark, that is

$$P\left(E_t\left(R_{\delta t}\right) \geq \left(1+\delta\right) E_t\left(R_t^{OMX}\right) \middle| t = t^{rec}+1,...,T\right).$$

Finally, a regression is performed in order to figure out how well the enhancement factors predict the out-of-sample excess returns, respectively, out-of-sample performance:

$$1/N\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX}\right) = c + \delta + u_\delta \tag{12}$$

where u_δ is assumed to be a white noise error term with $u_\delta \sim \left(0, \sigma_u^2\right)$, c is a constant term and $\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX}\right)$ denotes the excess return of portfolio corresponding to the enhancement factors $\delta = \delta_1,...,\delta_M$ within the out-of-sample period. If the data suggest a breakpoint, equation (12) is augmented by a dummy variable accounting for a break in the parameters given by

$$1/N\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX}\right) = c_1 + c_2 d + \delta_1 + \delta_2 d + \upsilon_\delta \tag{13}$$

where υ_δ is assumed to be a white noise error term $d = 0$ before the break and $d = 1$ otherwise.

4 Results

In this work, the OMX 30 is employed which is the leading stock index in Sweden and accounts for stocks of the largest 30 companies in accordance to their market capitalization. The data concerning the in-sample and out-of-sample periods can be downloaded for free on the index provider's website www.nasdaqomxnordic.com. In order to track the constructed artificial indices, 25 stocks exhibiting the highest market capitalization (see the appendix) on September 2008 are preselected in order to estimate the maximum likelihood functions. This stock selection approach is in line with Alexander and Dimitriu (2005) who also select stock in accordance to their market capitalizations. The

German Research Insitute for Worldeconomis (IfW, Kiel) issued on September 11, 2008 an official press release where it was reported that the Euro area faces a recession[5]. At the same time the Swedish leading stock index OMX 30 lost already 36.91% compared to its peak on July 16, 2007 (i.e. $OMX_{t^{\max}}$ =1.311,87). Since t^{rec} is in this study September 11, 2008, 750 days before the latter date have to be taken into account in order to estimate the maximum-likelihood function corresponding to high frequented daily data from September 21, 2005 – September, 11, 2008. The asset allocation takes place on September 12, 2008 and the allocation weights are held constant from September 12, 2008 until March 12, 2009 corresponding to $N = 124$ trading days out-of-sample. Claessens, Kose and Terrones (2009) denote such price declines such as the OMX 30 exhibited during the in-sample period as busts. Equity price busts which anticipate recessions are much stronger compared to ordinary prices declines and end with the recession, earliest though two quarters afterwards (see Claessens, Kose and Terrones (2009) for detailed information).

The artificial indices are in accordance to equations (1)-(2) constructed with $\delta = 0.0040, 0.0080, ..., 0.2000$ (i.e. corresponding to $\delta_{annual} = 1\%, 2\%, ..., 50\%$ in annual terms) uniformly distributed over time so that 50 different asset allocations could be estimated which is also in line with Alexander and Dimitriu (2005). Exhibit 1 gives an overview concerning the statistical properties, whereas in the appendix, the asset allocations are given with respect to all estimated portfolios. The optimization procedure concerning equation (7) suggest an asset allocation corresponding to portfolio 29 (see tables 1a-d and figure 2) which tracks an artificial index being constructed with $\delta = 0.1160$ (i.e. 29% in annual terms). The three main positions join 94.40% of the overall weight allocation and are invested in the industrial machinery sector, heavy electrical equipment industry

[5] See IfW Press Release September 11, 2008.

and the clothing industry. However, optimizing with respect to equation (8) suggests a more diversified asset allocation. Portfolio 14 tracking an artificial index with $\delta = 0.0560$ (i.e. 14% in annual terms, see tables 1a-d) invests only 40.53% in the same business sectors as portfolio 29.

The OMX 30 declined from September 12, 2008 until March 12, 2009 by 17.08%, whereas portfolio 14 declined only by 9.52% and performed consequently 7.56% better in comparison to the benchmark while exhibiting a marginal lower volatility of 53.50% p.a. compared to the benchmark's volatility being 53.51%. Exhibit 1 shows that the beta is close to the market beta (i.e. $\hat{\beta}_{14} = 0.98$). Portfolio 29 which is optimal with respect to the optimization procedure concerning equation (7) exhibits within the out-of-sample window a loss of only 3.04% (i.e. corresponding to excess returns of 28.08% p.a.) while the volatility is 7.99 per cent units higher in comparison to the benchmark's volatility. Again, the OLS regression being in line with the ordinary index model indicates that the beta (i.e. $\hat{\beta}_{29} = 1.03$) is quite close to the market beta.

Testing for cointegration shows that all estimated weight allocations exhibit a cointegration relationship with the artificial indices up to an enhancement factor equal to $\delta = 0.1320$ (see table 2 in the appendix) on a 5% significance level. Against it, portfolios tracking artificial indices involving trends being larger $\delta_{annual} = 35\%$ do not exhibit a cointegration relationship with the artificial benchmark. Exhibit 1 shows that the abnormal returns being estimated in accordance to the ordinary index model (see equation (11)) are statistically not significant concerning the out-of-sample period. As the integrated artificial indices are via construction cointegrated with the benchmark (see equations (1) and ((2)), it can be concluded that the estimated portfolio exhibit a cointegration relationship with the ordinary benchmark, too, while involving a stationary trend switching at $t^{\max}$ the direction. Figure 2 shows clearly the statistical arbitrage opportunity since on 77% of all days, the cointegration optimal portfolio 29 outperforms the index while exhibiting an excess return equal to 14.04% after 124 trading days out-of-sample. Furthermore portfolios 23 and 25 exhibit maximum statistical arbitrage opportunities as their returns are 19% above the index returns or higher with an empirical probability of 50.14%. Even if portfolio 29 which tracks an artificial index being enhanced by the factor $\delta = 0.1160$ (i.e. 29% in annual terms) is optimal with respect to its Reward-to-Risk ratio, it generates returns of 29% p.a. above the index or higher with an empirical probability of 45.53%. The statistical arbitrage opportunities are in accordance to the definition of Bondarenko (2003) with respect to portfolio 29 limited up to excess returns of 9% as the empirical probability that the portfolio generates returns of 9% or higher than the index is 50% for the latter figure.

Figure 3 plots the Reward-to-Risk ratios and shows an increasing trend on a decreasing rate while after the maximum, corresponding to portfolio 29, the ratio is declining. Estimating the forecast adequacy concerning the maximum likelihood optimal weight allocations gives the results of equation (12). Thereby, equation (12) takes only the elements 1,…,33 into account, as first, a visual inspection of the vector δ clearly shows a changed slope between $\delta = 0.1320$ and $\delta = 0.1360$. The second indicator for a break is that cointegration optimality does only hold for the sample $\delta = 0.0040$ until $\delta = 0.1320$ (see table 2 in the appendix). Therefore, equation (13) takes also into account the break in the slope parameter (t-statistics in parenthesis):

$$1/N\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX}\right) = \underset{(-6.21)}{-7.96} + \underset{(18.64)}{1.35} \cdot \delta_{annual}, \quad \text{for } \delta_{annual} = 1,...,33 \quad (15)$$

$$1/N\left(\sum_{t=t^{rec}}^{T} R_{\delta t} - \sum_{t=t^{rec}}^{T} R_t^{OMX}\right) = \underset{(-6.48)}{-7.96} + \underset{(6.98)}{37.30} \cdot d + \underset{(19.45)}{1.35} \cdot \delta_{annual} \underset{(-10.49)}{-1.52} \cdot \delta_{annual} \cdot d \quad (16)$$

for $\delta_{annual} = 1,...,50$, with $d = 0$ for $\delta_{annual} < 33$ and $d = 1$ else. Equations (15) and (16) show that all parameter estimates are statistically significant. The R-squared of 0.9254 concerning equation (15) and 0.9124 with respect to equation (16) suggest that the forecast capability of cointegration optimal weight allocations is high. Equations (15) and (16) estimate for portfolio 29 an excess return of 15.60% after the six month period out-of-sample whereas the realized excess return was only 1.56 percent units lower (i.e. 14.04%) in the end of the forecast period. In other words, cointegration optimal portfolios which track these artificial indices are relatively stable even within the out-of-sample period up to a premium of $\delta = 0.1160$ in daily (i.e. equation (15) and (16) take into account the annual premium being added to the index).

5 Discussion

If a recession is anticipated by an equity price bust, investors would expect the equity prices to fall further in future periods as longs as the economy faces the state of recession. If the majority of news that arrive the market participators do not change the market participators' mind such that the future seems bright again, there may be no rational reasons for a breakup concerning the equity price decline.

Table 1a: Statistical properties of the portfolios out-of-sample

Factor p.a. in %	Annual mean in %	Annual volatility in %	Reward-to-Risk ratio	Reward-to ΔRisk-ratio	Beta	t-statistic of beta
1.00	-37.25	57.38	-0.05	-0.73	1.07	8.52
2.00	-43.29	58.32	-0.15	-1.84	1.08	7.29
3.00	-43.75	58.12	-0.16	-2.02	1.07	6.76
4.00	-42.05	57.36	-0.13	-1.98	1.06	7.50
5.00	-42.07	57.59	-0.13	-1.87	1.07	7.24
6.00	-39.98	57.18	-0.10	-1.51	1.06	7.39
7.00	-32.47	55.39	0.04	1.05	1.03	9.65
8.00	-32.04	55.27	0.04	1.36	1.03	8.22
9.00	-28.28	54.52	0.11	6.08	1.01	7.95
10.00	-25.88	54.18	0.16	12.84	1.00	7.12
11.00	-25.27	54.11	0.17	15.28	1.00	6.27
12.00	-21.89	53.61	0.23	128.33	0.99	5.76
13.00	-20.28	53.34	0.27	-85.26	0.98	5.19
14.00	**-19.19**	**53.50**	**0.29**	**-1099.85**	**0.98**	**4.66**
15.00	-18.24	53.55	0.30	377.68	0.97	4.15

16.00	-17.55	53.83	0.31	52.06	0.97	3.78
17.00	-16.96	54.83	0.32	13.20	0.99	3.60
18.00	-16.18	55.46	0.33	9.36	0.99	3.35
19.00	-14.54	55.41	0.36	10.45	0.99	3.19
20.00	-14.10	56.23	0.36	7.47	1.00	3.04
21.00	-13.36	57.39	0.37	5.44	1.01	2.85
22.00	-12.56	57.73	0.38	5.18	1.01	2.74
23.00	-12.10	58.93	0.38	4.12	1.03	2.57
24.00	-9.99	59.25	0.41	4.26	1.02	2.42
25.00	-9.56	59.41	0.42	4.21	1.02	2.37

Table 1b: Statistical properties of the portfolios out-of-sample

Factor p.a. in %	Annual mean in %	Annual volatility in %	Reward-to-Risk ratio	Reward-to ΔRisk-ratio	Beta	t-statistic of beta
26.00	-9.65	60.36	0.41	3.62	1.03	2.27
27.00	-7.53	60.46	0.44	3.87	1.03	2.17
28.00	-6.75	60.67	0.46	3.87	1.03	2.12
29.00	**-6.13**	**61.50**	**0.46**	**3.54**	**1.03**	**2.02**
30.00	-7.46	62.84	0.43	2.89	1.05	1.98
31.00	-11.35	65.89	0.35	1.86	1.09	1.91
32.00	-6.83	62.48	0.44	3.08	1.04	1.95
33.00	-7.33	63.25	0.43	2.78	1.05	1.91
34.00	-9.41	64.84	0.39	2.21	1.07	1.89
35.00	-11.47	66.78	0.34	1.73	1.10	1.85
36.00	-12.60	66.74	0.33	1.65	1.10	1.86
37.00	-15.20	65.81	0.29	1.56	1.09	1.90
38.00	-17.82	65.54	0.25	1.38	1.09	1.95
39.00	-18.70	65.56	0.24	1.31	1.09	1.95
40.00	-9.58	69.17	0.36	1.59	1.10	1.63
41.00	-9.10	68.70	0.37	1.67	1.10	1.67
42.00	-10.42	67.48	0.36	1.72	1.10	1.78
43.00	-11.42	67.84	0.34	1.61	1.10	1.73
44.00	-12.23	66.92	0.33	1.66	1.10	1.81
45.00	-13.11	67.08	0.32	1.57	1.09	1.78
46.00	-14.01	67.03	0.30	1.51	1.09	1.77
47.00	-12.11	67.27	0.33	1.62	1.10	1.78

48.00	-10.04	67.79	0.36	1.71	1.10	1.75
49.00	-14.44	68.05	0.29	1.38	1.09	1.66
50.00	-15.06	67.54	0.29	1.38	1.09	1.70

Table 1c: Statistical properties of the portfolios out-of-sample

Factor p.a. in %	Annual alpha in %	t-statistic of alpha	Tracking-Error volatility p.a. in %
1.00	-0.57	-0.06	73.88
2.00	-6.11	-0.54	87.59
3.00	-6.74	-0.56	93.96
4.00	-5.46	-0.51	83.71
5.00	-5.36	-0.48	87.00
6.00	-3.51	-0.32	84.73
7.00	2.99	0.37	63.05
8.00	3.27	0.35	73.71
9.00	6.53	0.68	75.20
10.00	8.65	0.81	83.22
11.00	9.12	0.75	94.11
12.00	12.10	0.93	101.26
13.00	13.44	0.94	111.40
14.00	**14.48**	0.91	**123.90**
15.00	15.27	0.86	138.64
16.00	15.94	0.81	152.12
17.00	17.04	0.82	162.17
18.00	18.03	0.80	175.02
19.00	19.49	0.83	183.13
20.00	20.28	0.81	193.91
21.00	21.49	0.79	210.11
22.00	22.35	0.79	218.76
23.00	23.24	0.76	236.20
24.00	25.26	0.79	249.49
25.00	25.68	0.78	254.69

Table 1d: Statistical properties of the portfolios out-of-sample

Factor p.a. in %	Annual alpha in %	t-statistic of alpha	Tracking-Error volatility p.a. in %
26.00	25.91	0.75	268.48
27.00	27.81	0.77	279.18
28.00	28.55	0.78	286.32
29.00	**29.35**	0.76	**301.11**
30.00	28.63	0.71	313.33
31.00	26.23	0.61	336.98
32.00	28.95	0.71	315.15
33.00	28.74	0.69	323.84
34.00	27.47	0.64	334.90
35.00	26.34	0.58	350.68
36.00	25.21	0.56	349.72
37.00	22.30	0.51	337.86
38.00	19.71	0.46	330.12
39.00	18.86	0.44	330.13
40.00	28.34	0.55	400.31
41.00	28.86	0.58	389.05
42.00	27.42	0.58	365.56
43.00	26.39	0.55	374.95
44.00	25.48	0.55	356.99
45.00	24.54	0.53	362.62
46.00	23.55	0.50	364.17
47.00	25.63	0.55	364.08
48.00	27.85	0.58	371.30
49.00	23.10	0.46	387.28
50.00	22.41	0.46	378.17

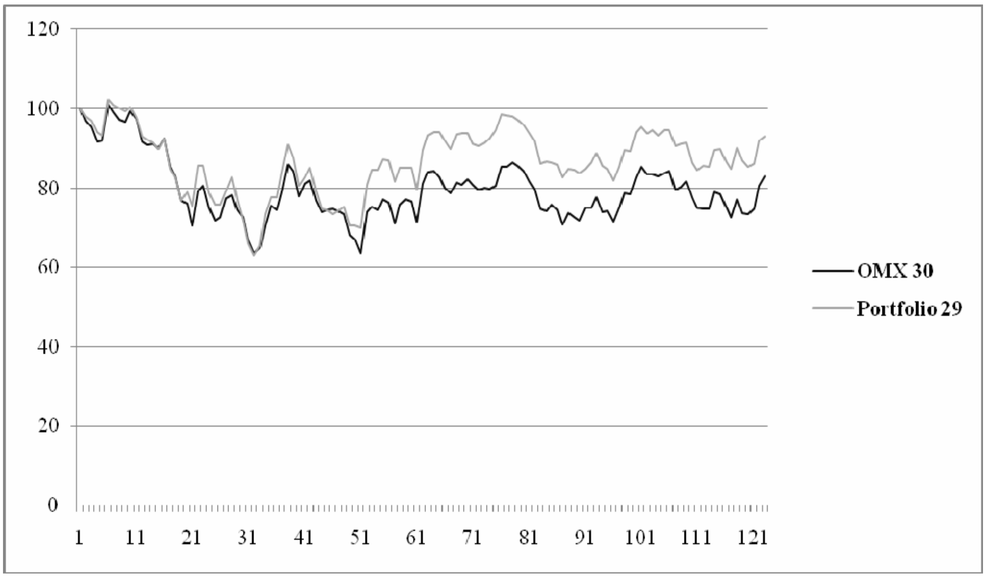

Figure 2: The OMX 30 and a cointegration optimal portfolio within the out-of-sample

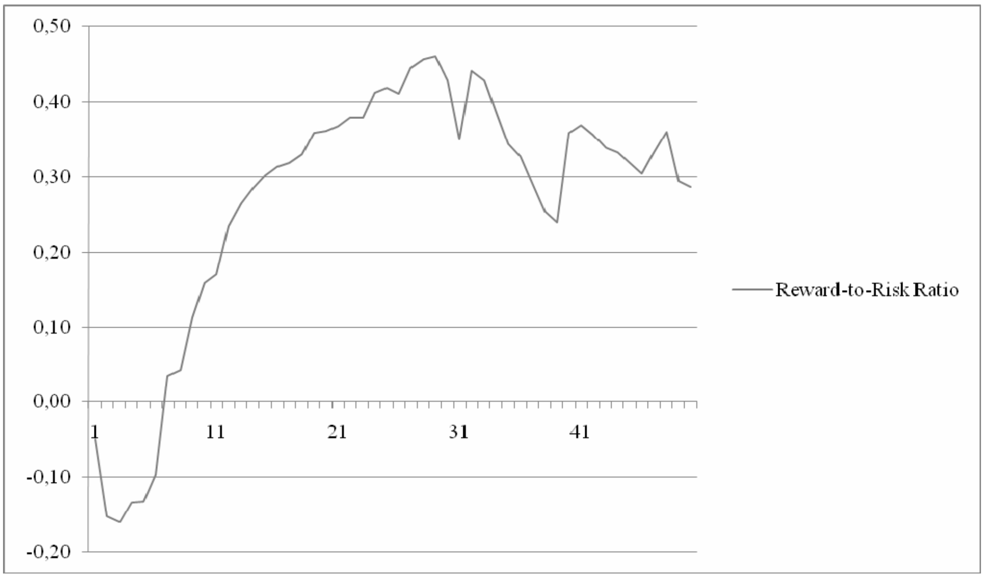

Figure 3: Reward-to-Risk Ratio depending on δ_{annual}

As a consequence, rational investors expecting equity prices to fall will rebalance their portfolios and choose a more defensive asset allocation strategy in order to minimize the expected loss in the nearer future. Thereby, the optimization procedures concerning the asset allocation depends on the investors individual Reward-to-Risk preferences. Two different optimization procedures are considered. Both optimal asset allocation strategies dominate the underlying index and exhibit within the out-of-sample period excess returns with low volatilities. Given the data set being employed, maximum-likelihood estimation gives robust parameter estimates providing adequate forecasts concerning the portfolios out-of-sample performance. However, the forecast adequacy depends on the stock data set being employed. The higher the artificial indices are enhanced, the less stable will be the forecast reliability of the parameter estimates as fewer stocks are available in order to mimic the constructed stochastic processes, respectively, to track these defensive strategies.

The market timing opportunity as suggested here depends exclusively on the information being provided by public institutes. The major price decline though is hardly predictable and occurred before the press release was issued. The latter empirical fact holds for both, equity price busts being mentioned earlier (i.e. 2000-2001 and 2008-2009). Moreover, the German stock market, for instance, showed only marginal additional price declines after the press releases (i.e. on November 26, 2001 and, respectively, on September 11, 208) were issued. However, the S&P 500 showed similar patterns like the OMX 30 and fell by 17.13% from March 7, 2000 until November 26, 2001, whereas the price decline was additional 6.36% between November 26, 2001 and May 26, 2002. During the financial crises in 2008, the S&P 500 fell by 19.31% between July 16, 2007 and September 11, 2008 and additional 39.88% between September 11, 2008 and March 12, 2009. Thus, the U.S. Index exhibited the same good market timing opportunities like the Swedish stock market during the financial crises period. However, Aroa and Buza (2003) who consider a large data set accounting for 20 bear markets within the last 102 years figured out that bear market durations have been reduced from 25.89 months for the first 15 bear markets to 14.28 months for the last 5 bear markets (excluding the last bear market in 2008).

Furthermore, market timing strategies as introduced here can be employed for active funds management who is aiming at minimizing losses in down-market movements. However, market-timing is two-sided: Maintaining a defensive asset allocation in bull markets can result in lower portfolio returns compared to the underlying stock index. Hence, the funds management faces the problem to define a rule when the defensive asset allocation strategy should be changed to an offensive one, for instance.

Moreover, the German Institute for Worldeconomics in Kiel (IfW) issued already on March 13, 2008 a press release where it was reported that the world economic growth has slowed significantly towards the end of 2007 in response to the housing market crisis in the US. Besides it was mentioned that the problems in the financial sector would continue to weigh on the real economy and that the risk of the US slipping into recession was substantial.[6] At this time the OMX 30 lost already 28.61% (i.e. between July 16, 2007 and March 13, 2008) and a price bust could have already been ascertained. Since the latter price bust took eight months (i.e. from July 2007 until March 2008) market participators could have expected the bust to continue for at least additional six months on average.

Aroa and Buza (2003) mention that the stock market crashes in October 1929, October 1987 and March 2000 had in common, that all three periods were preceded by periods of increased volatilities. Considering the financial crises in 2008 though, an increase in volatility could also be ascertained. Significant changes in stock market volatility could therefore also act as an indicator for a forthcoming equity price bust. As a consequence, the day when the recession is officially declared may be the last chance for an active funds-management to minimize losses during the continuing bear market.

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[6] See http://www.ifw-kiel.de/media/press-releases/2008/pr13-03-08a .

Alexander and Dimitriu (2005) construct six plus/minus benchmarks by adding and subtracting annual returns of 5%, 10% and 15% to and from the reconstructed DJIA returns, uniformly distributed over time. Their findings that cointegration optimal portfolios can be found even if the artificial benchmarks diverge significantly from the benchmark can be supported in this study, too. The constructed portfolios exhibit a cointegration relationship with the artificial index up to an enhancement factor of 33% in annual terms. In contrast to Alexander and Dimitriu (2005) who argue that returns are significantly more volatile without any compensation of additional returns as the spread between the benchmarks tracked widens, it is shown here that the portfolio tracking an artificial index being enhanced by 29% in annual terms exhibits the highest Reward-to-Risk ratio. Alexander and Dimitriu's (2005) study suggest, however, that the best performance is produced by strategies tracking narrow spreads such as 5% hedged with the portfolio tracking the artificial benchmark. The latter issue cannot be supported in this study: The results (see table 1a-d) show that all strategies tracking narrow spreads such as 1%-5% hedged with the portfolio tracking the artificial indices exhibit returns below the benchmark and volatilities above the benchmark's volatility.

However, statistical arbitrage opportunities as defined by Bondarenko (2003) should exhibit an expected payoff being nonnegative. Given a required excess return of 10% above the stock market, an investor has to select a portfolio tracking an enhancement factor of 25% in order to gain the required excess returns with the highest probability (i.e. 52.85%) with respect to each trading day out-of-sample. As portfolio 25 exhibits a beta of 1.02 which is close to the market beta, it can be employed to construct a zero-cost trading strategy while going short on the index and long on portfolio 25, six months ahead while the expected profit is 17% p.a. (corresponding to the empirical probability of 50.14% which means that the portfolio returns are expected to be 17% above the index or higher on each trading day out-of-sample). However, the realized excess return of portfolio 25 was

12.30% (i.e. 24.60 % p.a.) above the OMX 30 on the last day of the out-of-sample period under consideration (i.e. September 12, 2008 - March 12, 2009). The latter fact holds even though the empirical probability that the portfolio returns exceed the enhanced benchmark returns with the daily enhancement factor $\delta = 0.1000$, corresponding to $\delta_{annual} = 25\%$, is 47.15%.

Grobys (2010) constructs artificial indices irrespective of the underlying economy's state. Thereby, four years of daily data are taken into account in order to construct artificial indices while the underlying stock market's stochastic process is modified (i.e. S&P 500) by subtracting a linear trend within the first two years in-sample and adding a linear trend with the same slope parameter within the third and fourth year concerning the in-sample period. The idea is in that study to mimic trends in different business sectors while assuming that business sectors behave different during a business cycle. This methodology results in a stable cointegration relationship concerning the out-of-sample period and statistical significant abnormal returns of 6.83% p.a. while the volatility is one third lower in comparison to the benchmark. In contrast to Grobys' (2010) studies where eleven different mutual funds are taken into account, in the studies presented here, stocks are employed, only. The market timing opportunity does not rest upon an assumption about cyclical patterns concerning some business sectors but on official press releases and the empirical fact that an equity price bust that coincides with a recession ends the earliest when the associated recession is ending but can even continue afterwards. Consequently, this market timing opportunity cannot be exploited by iterative working computer programs but requires a critical observation of the stock market as well as the state of the economies. Unlike Grobys (2010) the out-of-sample portfolio processes cannot be priced by applying Vector-Error-Correction models. Therefore, empirical probabilities are estimated in this study for each portfolio, given different enhancement factors. Statistical arbitrage opportunities could be ascertained. However, the empirical probabilities suggest that the statistical arbitrage opportunities are below the expected mean. In other words, portfolio 29, for instance, may be expected to generate out-of-sample abnormal returns in line with the enhancement factor being equal to 9% in annual terms. The empirical probability that abnormal returns are gained with a probability of at least 50% holds only with respect to excess returns being equal to 9%.

6 Concluding Remarks

Even though a random walk process is from a statistical point of view not predictable, some movements can be correlated with some specific events concerning other variables. Then, predictable patterns may arise being dependent on this joint event. There is evidence given that equity price busts being associated with recessions continue until the economy switches from the state of recession to an economic pick-up. Such joint events can be exploited for both, rebalancing equity portfolios in order to select defensive weight allocations or to exploit this market conditions to construct a statistical arbitrage portfolio. There are incentives from an investor's point of view to short the index and take a defensive portfolio position being evened up after six months, for instance.

In the literature is reported that stock market crashes often happen in news vacuum (see Arora and Buza (2003), for instance). Anticipating a stock market crash is hardly possible even though an increasing volatility can be considered as one kind of indicator that some market conditions can be changing. However, increasing volatilities can also be observed in bull markets. It remains still unclear when the market is changing from a bull to bear market and vice versa. Investing in a portfolio exhibiting defensive properties may be a good choice if in fact the market continues to decline further. In contrast, excess returns can be diminished if the market moves upwards earlier than expected. Thus, there is still need of research concerning the optimal rebalancing moment. Not all stock markets are falling during recessions to the same extent as the Swedish index OMX 30. However, stock market integration drives international stock markets of most advanced countries to exhibit similar properties. Advantageous asset allocations do not only require to employ algorithmic optimization methods being based on historical data but to include also psychological effects driving the market such as herding behavior as well as speculative attacks and accounting for market participators' expectations concerning future periods. The latter items though require a good understanding of the linkages between macroeconomic, financial and psychological variables

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Appendix

Table 2: Testing for cointegration

Factor p.a. in %	ADF-statistic in-sample	Factor p.a. in %	ADF-statistic in-sample
1	-16.00*	26	-3.49*
2	-14.97*	27	-3.36*
3	-15.14*	28	-3.23*
4	-15.58*	29	-3.07*
5	-14.88*	30	-2.94*
6	-14.74*	31	-2.88*
7	-14.18*	32	-2.76*
8	-13.36*	33	-2.76*
9	-12.26*	34	-2.55*
10	-11.20*	35	-2.36
11	-10.24*	36	-2.33
12	-9.42*	37	-2.10
13	-8.68*	38	-1.87
14	-8.06*	39	-1.86
15	-7.56*	40	-2.28
16	-7.09*	41	-2.22
17	-6.57*	42	-2.17
18	-6.14*	43	-1.97
19	-5.73*	44	-1.94
20	-5.27*	45	-1.98
21	-4.83*	46	-1.94
22	-4.52*	47	-1.89
23	-4.20*	48	-1.91
24	-3.94*	49	-1.99
25	-3.72*	50	-1.88