Construction of Investment Risk Measure by the Dispersion Degree of Estimation Errors of Working Capital

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Abstract

This study is based on investors’ viewpoint and adopts the accruals quality model [1] as proxy variable of earning quality. By applying the process capability concept in engineering application, we establish capability index of basis accrual quality and transform it into the investment risk assessment. It provides investors an effective way to control investment risk and to improve the investment decision-making process.

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1 Introduction

Financial statements play an important role when investors make investment decisions. Investors are able to effectively evaluate business performance by interpreting information through financial statement analysis [10]. There are some uncertain factors in operating activities of a company. Therefore, there is room for the administration to do within-GAAP adjustment. These uncertain factors will appear in financial information eventually and make it have information risk [2]. When investors use the risk information provided by the administration, it can cause investors investment loss.

After Enron scandal, investors learn that financial statements are unable to reveal the true value and potential risks of a company. They also question the soundness of the financial reporting. A comparison of financial statements of Enron before and after bankruptcy revealed major inconsistencies between the operating cash flow and net income after taxes. Thus, the accrual quality reported was suspicious. Dechow and Daichev [1] pointed out the bases of measures of earning are divided into accrual basis and cash basis. The difference of the two bases is the estimate of the administration for assuming and recognizing future cash flow under accrual basis. The accrual quality model, DD model [1], is used as the proxy variable of earning quality.

Recently, the studies of information risk and have increased a lot [3, 8]. Other studies suggest D&D accruals quality model as the measurement of information quality to discuss the relation with earnings [4, 5].
The DD model uses the regression model to make the accrual changes of working capital correspond to the cash flows of operating activities from previous, current, and future period and uses regression residual standard deviation as reverse measure. As a result, when the quality of accruals is poor, the residual standard deviation of the DD model is larger and information risk investors face will rise.

However, the viewpoint of the DD model does not consider individual decision-making criteria of investors. It thinks both estimation error and error correction influence accrual quality but does not take antipathy and tolerance of investors’ decision-making criteria toward estimation error and error correction into account. The first question raised by our study is “Is it proper to take DD as accrual quality indicator?” Besides, our study brings up how we can measure the estimation error of the chosen investment object not within investors’ tolerance when investors estimate cash flow from information of uncertain accruals within tolerance of estimation error.

Therefore, our study addresses investor first-order loss function in accordance with investors’ antipathy level and tolerance toward DD and includes the concept of construction quality to build the accrual quality capability index as substitution variable for information risk. Then, we convert the accrual quality capability index into the loss probability of investors. Since this probability is caused by the uncertainty of accrual quality, we define it as “Investment Risk.”

In application, first, we derive the statistic of the accrual quality ability indicator as base of statistical confidence. If the accrual quality ability indicator of the investment object is greater than or equal to one, the investment object has accrual quality ability. We continue to the second step. Otherwise, we stop. Second, we develop the statistic of homogeneity test and run homogeneity test on investment objects passing the ability test on the same basis. If investment objects meet the requirement of the homogeneity test, there is no significant difference among investment objects; conversely, if not meet the requirement, the investment
objects continue to the final step. Third, if the investment objects do not meet the requirement of the homogeneity test, we have to compare the accrual quality ability of investment objects two by two. Therefore, on the relative basis, our study develops the statistics of accrual quality ability of investment objects in two by two comparison test and applies the test statistics to run the test. By this application, investors can clearly choose better investment objects when facing different accrual quality ability of them.

The main contribution of our study is to extend DD’s accrual quality to accrual quality ability indicator, and convert it into investment risk under accrual quality measure. Further, we get a series of test statistics to help investors choose better investment objects. The remainder of this paper is organized as follows. The second section discusses analysis model. The third section probes into data resource and estimator. The fourth section is empirical results. The fifth section is conclusion.

2 Analysis Models

We aim at building a measure of investment risk. The relating discussions are as follows:

2.1 Accrual Quality

In financial statements, there are two accounting bases to assess the business performance. One is net profit after tax in accrual basis and the other is cash flow from operating activities in cash basis. If there is a large difference between net profit after tax and cash flow from operating activities in the company’s financial statements, which means the reported earning cannot be recovered in cash. This may lead the company to bankruptcy crisis because of the shortage of cash.
course, this will also decrease the reliability of the accrual quality [9, 11]. Therefore, we adopt the DD model to assess accrual quality. It is as follows:

\[
\Delta WC_t = \alpha_0 + \beta_1 CFO_{t-1} + \beta_2 CFO_t + \beta_3 CFO_{t+1} + \epsilon_t
\]  

(1)

\(\Delta WC_t\): Change in working capital, \(- [\Delta A/R \text{ (Change in accounts receivable)} + \Delta INV \text{ (Change in inventory)} - \Delta A/P \text{ (Change in accounts payable)} - \Delta T/P \text{ (Change in taxes payable)} + \Delta OA \text{ (Change in other assets (net))}];\)

\(CFO_{t-1}\): Cash flow from \(t-1\) operating activities;

\(CFO_t\): Cash flow from \(t\) operating activities;

\(CFO_{t+1}\): Cash flow from \(t+1\) operating activities;

\(\epsilon_t\): Residuals (estimation errors) from Eq. (1).

Dechow and Dichev [1] pointed out that \(\epsilon_t\) from Eq. (1) stands for the estimation errors of working capital, whereas standard deviation of \(\epsilon_t\) is the index of accrual quality. When \(\sigma (\epsilon_t)\) is larger, the accrual quality is worse; on the contrary, when the \(\sigma (\epsilon_t)\) is smaller, the accrual quality is better.

### 2.2 Investment Loss

The concept of quality loss is a measure model of consumer loss to judge if quality of merchandise is good or bad. The criteria for judgment are based on the qualification of specific requirements. There are only two results – either to accept or to reject the merchandise. This concept is applied to the measure of accrual quality. The investment loss function (Figure 1) from accrual quality is as follows:

\[\text{Dechow and Dichev [1] believed that accruals are also affected by firm and industry’s attributes. To eliminate the scale factor, the mentioned variables are the deflator of the total assets of the current year. However, the residuals of the DD model include intentional and random estimation errors effects. The residual in McNichols (2002) adding the effect of Jones model is smaller than adding the effect of the DD model. The connotations are so different, and it could be discussed in future studies.}\]
\[ L = \begin{cases} 
0, & \text{if } |\varepsilon_i - T| \leq S \\
A, & \text{other} 
\end{cases} \quad (2) \]

$L$: Investment loss of accrual quality; when the estimation errors of working capital for the investment objects are within the range of $S$, the investment loss is 0. On the contrary, when the estimation errors of working capital for the investment objects are outside the range of $S$, the investment loss is $A$;

$S$: An acceptable range of the estimation errors of working capital; the range is set by standard deviation of estimation errors of working capital for potential investment objects;

$\varepsilon_i$: Estimation errors of working capital of investment objects;

$T$: Target value of estimation errors of working capital of potential investment objects;

$USL$ (Upper Specification Limit): Maximum acceptable specification limit of estimation errors of working capital of investment objects;


Figure 1: Investment Loss Function of Accrual Quality in a Specific Period
In general, investors refer to financial statements of investment objects when they are making investment decisions. Further, they build an investment portfolio. As mentioned earlier, higher standard deviation from the Eq. (1) shows lower accrual quality and thus high investment risk. For the long-term, the losses of investors will be affected by accrual quality of investment objects. This implies the investment loss for bad accrual quality of investment objects will be bigger. The expected investment loss is expressed as follows:

\[ E(Loss) = \sum_{i=1}^{N} L_i \cdot P_i \]  

*E (Loss)* : expected losses from decline of accrual quality on investment objects ;  
*L_i* : the investment losses from decline of accrual quality on investment objecti ;  
*N* : the number of investment objects ;  
*P_i* : the probability of investment losses from investment object i.

Investors are unable to expect and avoid investment losses resulting from decline of accrual quality on investment objects in advance when establishing their investment portfolio. Given constant investment losses *L_i*, the higher the probability of investment losses *P_i*, the higher the expected losses *E(Loss)*. Moreover, to lower *E(Loss)*, investors need to focus on the reduction of *P_i* under risk aversion. Therefore, this study regards *P_i* as investment risk from the decline of accrual quality. The question would be how to reduce the investment risk caused by the lowered accrual quality.

### 2.3 Basic Capability Index of Accrual Quality

We exploit the Process Capability Index, proposed by Kane [7], to construct a basic capability index of accrual quality, *C_{BAQ}*, and further infer it to investment risk *P_i*. That is, we introduce the concept of process capability used in industrial
engineering into the error term derived from Eq. (1). Under the normal distribution assumption, we establish the tolerable upper bound and lower bound on the basis of 90% probability distribution of residuals for all potential investment objects. Assuming a normal distribution, 3.29 times potential investment objects $\sigma(\varepsilon_t)$ is the tolerance as well as the numerator of $C_{BAQ}$.

We assume that $\sigma(\varepsilon_t)$ of investment objects is equal to $\sigma(\varepsilon_t)$ of the potential investment objects at most. In this circumstances, the corresponding tolerance forms 3.29 times $\sigma(\varepsilon_t)$ of investment objects, which is also the denominator of $C_{BAQ}$. When all potential investment objects are equal to $\sigma(\varepsilon_t)$ of investment objects, $C_{BAQ}$ is 1. The investment risk is 10% (=1-90%). Then the equality of basic capability index of accrual quality for portfolio in our study is:

$$C_{BAQ} = \frac{USL - LSL}{3.29\sigma_p} \quad (4)$$

$USL$ (Upper Specification Limit): Maximum acceptable specification limit of $\varepsilon_t$ of potential investment objects (i.e. $USL = \varepsilon_T + Z_{(1-\alpha/2)}\sigma_T$);

$LSL$ (Lower Specification Limit): Minimum acceptable specification limit of $\varepsilon_t$ of potential investment objects. (i.e. $LSL = \varepsilon_T - Z_{(1-\alpha/2)}\sigma_T$);

$\varepsilon_T$: Mean of $\varepsilon_t$ of the potential investment objects$^7$;

$\sigma_T$: Standard deviation of $\varepsilon_t$ of the potential investment objects;

$\sigma_p$: Standard deviation of $\varepsilon_t$ of portfolio and assume that $\sigma_p > 0$.

In Eq. (4), given constant $USL$ and $LSL$, $C_{BAQ}$ changes inversely to $\sigma_p$. The distribution of residuals of investment portfolio gets more dispersed, then $\sigma_p$ is

$^6$ According to the Central Limit Theorem, if the sample size is large, it is close to the normal distribution $\varepsilon \sim \text{N}(0,\sigma_e)$.

$^7$ We adopt the DD model in pooled Regression method and assume $\varepsilon_T = 0$ based on Eq. (1).
bigger and $C_{BAQ}$ is smaller and vice versa. Therefore, we establish a measurement benchmark of accrual quality for the DD model and further infer the investment risk $P_i$ from the perspective of investment portfolio. Thus, we extend the DD model and develop a basic capability index of accrual quality, which is based on investors’ perspective.

### 2.4 Investment Risk

After the establishment of accrual quality capability index ($C_{BAQ}$), we resolve the measurement of investment risk caused by lowered accrual quality. In this stage, Eq. (4) will be changed further and the mathematic relationship between $C_{BAQ}$ and $P_i$ will be constructed. Under the normal distribution assumption, $P_i$ is expressed as follows.\(^8\)

\[
P_i = 1 - P(Z < \frac{USL - \varepsilon_T}{\sigma_T}) + P(Z \leq \frac{LSL - \varepsilon_T}{\sigma_T})
\]  

(5)

$P_i$ stands for investment risk caused by lowered accrual quality in sampling investment portfolio $i$. Let $TSL = USL - LSL$ or $TSL = 2(USL - \varepsilon_T) = 2(LSL - \varepsilon_T)$.

The upper specification limit and lower specification limit distribute symmetrically given normal distribution assumption. Accordingly, $TSL = 2*USL = 2*LSL$ and Eq. (5) can be rewritten as follows.

\[
P_i = 1 - P(Z < \frac{USL}{\sigma_T}) + P(Z \leq \frac{LSL}{\sigma_T})
\]  

(6)

Apparently it can be seen from Eq. (6) that the higher the $C_{BAQ}$ is, the lower the $P_i$ is, thereby the lower investment risk is, and vice versa. As long as $C_{BAQ}$ of investment portfolio was estimated, the corresponding investment risk $P_i$ will be

\(^8\) Establishing $C_{BAQ}$ assuming $\sigma_T = \sigma_p$
obtained. (6) is revised as follows if we consider 90% probability of $\varepsilon_i$ within the acceptable range:

$$P_i = 1 - P(Z < \frac{2 \text{USL}}{2 \times 1.645 \times \sigma_T} \times 1.645) + P(Z \leq \frac{2 \text{LSL}}{2 \times 1.645 \times \sigma_T} \times 1.645)$$

$$= 1 - P(Z < 1.645C_{BAQ}) + P(Z \leq -1.645C_{BAQ})$$

$$= 1 - \left[1 - P(Z \leq -1.645C_{BAQ})\right] + P(Z \leq -1.645C_{BAQ})$$

$$= P(Z \leq -1.645C_{BAQ}) + P(Z \leq -1.645C_{BAQ})$$

$$= 2P(Z \leq -1.645C_{BAQ})$$  \hspace{1cm} (7)

In Eq. (7), it is obvious that $P_i$ is smaller when $C_{BAQ}$ is larger. That means the investment risk is lower. On the contrary, $P_i$ is larger when $C_{BAQ}$ is smaller, which means the investment risk is higher. Therefore, we should know investment risk given the $C_{BAQ}$. We develop a measure model of $P_i$ taking estimation errors of working capital as the foundation of our study. In other words, we establish the mathematical relationship between $\sigma_p$ and $P_i$. Our model not only complements the deficiencies left by the DD model but also provides investors with a useful decision-making model for estimating investment risk.

3 Information Sources and Discussion of Estimates

3.1 Information Sources, Sampling and Study Duration

We have selected financial data from the financial database of listed companies provided by the Taiwan Economic Journal (TEJ) as our samples. It takes a long time to estimate $\sigma_p$ and $\sigma_T$ so we adopted the TSE and OTC Listed Companies from year 1996 to 2007 as our samples, and excluded those companies from the insurance, security industry and without sufficient information. Furthermore, the sample companies had to be listed at or before the end of year 2005. Since we need at least 3 years of consistent data for the calculation of
variables, the sampling duration is divided into 10 periods based on this criterion. There is sample information from at least 6 periods for a single company. In the end, we obtain 8,346 firm-years from 932 companies.

### 3.2 Estimates and Examination of $C_{BAQ}$

In the calculation of $C_{BAQ}$, we estimate the population basing on the sample information. From Eq. (4), the estimation statistics of $\sigma_p^2$ is $S_p^2 = \frac{\sum (e_i - \bar{e})^2}{n-1}$

Hence, the predictor of $C_{BAQ}$ can be written as:

$$C_{BAQ} = \frac{USL - LSL}{3.29\sigma_p}$$

$$= \frac{USL - LSL}{3.29S_p} \times \sqrt{\frac{S_p^2}{\sigma_p^2} \times \frac{(n-1)}{(n-1)}}$$

$$= \hat{C}_{BAQ} \times \sqrt{\frac{\chi^2}{n-1}}$$  \hspace{1cm} (8)

From Eq. (8), the relationship between $C_{BAQ}$ and $\hat{C}_{BAQ}$ is:

$$\frac{C_{BAQ}}{\hat{C}_{BAQ}} = \sqrt{\frac{\chi^2}{v}} (v = n-1)$$  \hspace{1cm} (9)

Investors may face a situation where they have to determine whether the accrual quality of a particular portfolio is good since investment decisions are diversified. On the other hand, investors have to select an investment portfolio of minimum risk from potential investment objects. Therefore, we divide the $C_{BAQ}$ examination into 3 stages: first of all, we test $C_{BAQ}$ of portfolio to see if it is larger than 1. Secondly, we conduct Hartley’s Homogeneity test for the $C_{BAQ}$ of

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9 The 10 periods are divided as: 1st period is year 95-97; 2nd period is year 96-98; 3rd period is year 97-99 and so on.

10 So the estimate of $\sigma_p^2$ is $S_p^2$. 

investment set. Finally, basing on the result of investment set\textsuperscript{11} test, we make two-pair $C_{BAQ}$ test between two portfolios. Consequently, we provide a reference for investors when they are making investment decisions.

### 3.2.1 Capability Test of $C_{BAQ}$

When establishing $C_{BAQ}$, we assume the degree of variance between potential investment objects and investment portfolio is equal. Hence, investment risk is $P_r=10\%$ when $C_{BAQ}$ equals 1. We examine if $C_{BAQ}$ is larger than 1 by means of one-tailed test, and the null hypothesis should be:

$$H_1: \text{Portfolio } C_{BAQ} \leq 1$$

By now, we obtain a confidence interval of the left-tailed test as below (Appendix A):

$$\hat{C}_{BAQ} \times \sqrt{\frac{\chi^2_{(n-1,a)}}{n-1}} \leq C_{BAQ}$$

### 3.2.2 Homogeneity Test of investment set

Assuming that we select three investment portfolios from the potential investment objects, we use the $F_{max}$ method, which is introduced by Hartley [6] to test if there are significant differences among 3 sets of $C_{BAQ}$. The chosen Portfolios $C_{BAQi}$ ($i=1,2,3$) are classified as $C_{BAQ1}$, $C_{BAQ2}$ and $C_{BAQ3}$, and examined by the Hartley’s homogeneity test. Null hypothesis should be:

$$H_2: C_{BAQ1} = C_{BAQ2} = C_{BAQ3}$$

We obtain a test statistic of homogeneity test for three portfolios as follows:

$$F_{max} = \frac{\text{Min}\{\hat{C}_{BAQ1}, \hat{C}_{BAQ2}, \hat{C}_{BAQ3}\}}{\text{Max}\{\hat{C}_{BAQ1}, \hat{C}_{BAQ2}, \hat{C}_{BAQ3}\}}$$

\textsuperscript{11} An investment set is formed by our choosing multiple investment portfolios from the potential investment objects.
It proves that $F_{\text{max}} \sim F_{\text{max}(3,7-1)}$ (Appendix B).

3.2.3 Two-pairs Comparison of $C_{BAQ}$

If the result of the above homogeneity test rejects the null hypothesis, that means “at least one $C_{BAQ_i}$ is unequal.” In this circumstances, we have to compare two $C_{BAQ_i}$ each other. We compare the value of $C_{BAQ_i}$ and $C_{BAQ_j}$, and the null hypothesis is:

$$H_3 : \frac{C_{BAQ_i}}{C_{BAQ_j}} = 1$$

By now, it is proved that the $1-\alpha$ maxima and minima confidence interval (Appendix C) of $(C_{BAQ_i} / C_{BAQ_j})$ are:

$$\sqrt{\text{UCI}} = \sqrt{\frac{C_{BAQ_i} / C_{BAQ_j}}{F_{1/2}(v_1, v_2)}}$$

$$\sqrt{\text{LCI}} = \sqrt{\frac{C_{BAQ_i} / C_{BAQ_j}}{F_{1-\alpha/2}(v_1, v_2)}}$$

When confidence interval of $(C_{BAQ_i} / C_{BAQ_j})$ are both larger than 1, $C_{BAQ_i} > C_{BAQ_j}$. When confidence interval of $(C_{BAQ_i} / C_{BAQ_j})$ are both smaller than 1, $C_{BAQ_i} < C_{BAQ_j}$. When confidence interval of $(C_{BAQ_i} / C_{BAQ_j})$ include 1, $C_{BAQ_i} = C_{BAQ_j}$ may happen.

4 Empirical Results

We measure and analyze the capability index of accrual quality and investment risk basing on the above concepts.
4.1 Regression Analysis

Regression analysis is conducted according to Eq. (1). The results are shown on Panel A of Table 1.

From Panel A of Table 1, we know that the regression results are exactly the same as those of Dechow and Dichev [1]. In addition, the explanatory power (Adj. R\(^2\)) of the regression model is 25.9%, which is very close to the explanatory power of Dechow and Dichev[1], 29%. From Panel B of Table 1, the mean of residual is 0, which complies with the basic assumption of the linear regressive model. The standard deviation of the residual is 0.079, which is \(S_\tau^2\) and is the numerator in calculating \(\hat{C}_{BAQ}\).

4.2 Statistical Tests of \(C_{BAQ}\)

4.2.1 Capability Test of \(C_{BAQ}\)

We start with discussing the formation of the portfolio of a particular company. We assume investors’ decision-making criterion Portfolio \(\sigma_P\) equals to potential investment objects \(\sigma_T\) at most. Thus, it is necessary to test whether or not \(C_{BAQ}\) is larger than 1. We select Portfolios A, B, C, D, E from 5 companies respectively and calculate their \(S_P, \hat{C}_{BAQ}, P_i\) and the lower confidence interval according to Eq. (10). The results are indicated in Table 2.

We know from Table 2 that \(\hat{C}_{BAQ}\) of the five portfolios are 3.039, 1.502, 1.000, 0.722 and 0.696 respectively. Then the \(P_i\) of five portfolios are 0%, 1.4%, 10%, 23.5% and 25.2% respectively. It is obvious that the result of Portfolio C (\(\hat{C}_{BAQ}=1, P_i=10\%\)) fits our previous assumption. Apparently, if \(S_P\) is higher in the chosen portfolio, \(\hat{C}_{BAQ}\) is smaller, whereas the \(P_i\) is higher. On the contrary, if
$S_p$ is lower in the chosen portfolio, $\hat{C}_{BAQ}$ is larger and the $P_i$ is lower. Thus, there is a regular pattern among $S_p$, $\hat{C}_{BAQ}$ and $P_i$.

Table 1: Regression Results and Residuals Analysis (N=8,346)

<table>
<thead>
<tr>
<th>Panel A: Regression Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>$CFO_{t-1}$</td>
</tr>
<tr>
<td>$CFO_t$</td>
</tr>
<tr>
<td>$CFO_{t+1}$</td>
</tr>
<tr>
<td>Sample</td>
</tr>
<tr>
<td>F Value</td>
</tr>
<tr>
<td>DW</td>
</tr>
<tr>
<td>Adj. R²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Descriptive statistic of residuals from Eq.(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td>$\epsilon_i$</td>
</tr>
</tbody>
</table>

Explanations:
1. *** stands for $P<0.01$; ** stands for $0.01<P<0.05$; * stands for $0.05<P<0.1$.
2. $\Delta WC_t = \alpha + \beta_1 CFO_{t-1} + \beta_2 CFO_t + \beta_3 CFO_{t+1} + \epsilon_i$
   $\Delta WC_t$: changes of working capital in period t;
   $CFO_{t-1}$: cash-flow of operating activities in period t-1;
   $CFO_t$: cash-flow of operating activities in period t;
   $CFO_{t+1}$: cash-flow of operating activities in period t+1.

To enhance statistical confidence, we further examine the result by means of a one-tailed confidence interval ($a = 0.1$). In Table 2, the left-tailed confidence interval of Portfolio A, B and C is larger than 1, so the null hypotheses H1 are
rejected. In other words, A, B and C have accrual quality capability. When the left-tailed confidence interval of Portfolio D and E is smaller than 1, the null hypotheses H1 cannot be rejected. In other words, D and E do not have accrual quality capability.

Table 2: Test of Capability of Portfolio $C_{BAQ}$ ($a = 0.1$)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$S_p$</th>
<th>$\hat{C}_{BAQ}$</th>
<th>Lower Limit of Interval Estimation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.024</td>
<td>3.039</td>
<td>4.166</td>
<td>Reject H1</td>
</tr>
<tr>
<td>B</td>
<td>0.064</td>
<td>1.140</td>
<td>1.563</td>
<td>Reject H1</td>
</tr>
<tr>
<td>C</td>
<td>0.073</td>
<td>1.000</td>
<td>1.371</td>
<td>Reject H1</td>
</tr>
<tr>
<td>D</td>
<td>0.101</td>
<td>0.722</td>
<td>0.990</td>
<td>Did not reject H1</td>
</tr>
<tr>
<td>E</td>
<td>0.104</td>
<td>0.696</td>
<td>0.955</td>
<td>Did not reject H1</td>
</tr>
</tbody>
</table>

The above results match up our prerequisite assumption: when $C_{BAQ}=1$ and $P_i=10\%$. This can be a test basis for portfolio comparison, which helps investors find out whether or not a particular portfolio possesses accrual quality capability.

4.2.2 Homogeneity Test of Investment Set

The homogeneity test we conduct is based on investment set formed by multiple portfolios. First of all, we use $\hat{C}_{BAQ1}$, $\hat{C}_{BAQ2}$ and $\hat{C}_{BAQ3}$ which are proved to possess accrual quality capability in Table 2 to conduct the homogeneity test. In the test, we also refer to the $F_{max}$ method in Eq. (11). Then we calculate the values of $S_{p_{max}}^2$, $S_{p_{mix}}^2$ and $F_{max}$, and the results are listed in Table 3.
Table 3: Homogeneity Test of Portfolio $C_{BAQ}$ ($a = 0.05$)

<table>
<thead>
<tr>
<th>$V$ (Average Degree of Freedom)</th>
<th>$S_{P_{max}}^2$</th>
<th>$S_{P_{mix}}^2$</th>
<th>$\hat{F}_{max}$</th>
<th>Threshold $F_{max}$ ($K=5$, $V=8$)</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.005329</td>
<td>0.000576</td>
<td>9.233</td>
<td>5.900</td>
<td>Reject H2</td>
</tr>
</tbody>
</table>

Explanations:
1. The $F_{max}$ threshold table provided by Hartley [6] only provided 0.05 and 0.001 and did not have 0.1 threshold. Hence, we select $a=0.05$ for the test.

In Table 3, result for the homogeneity test rejects the null hypothesis H2, so at least one out of the three chosen portfolios is unequal to $C_{BAQ}$. This proves there is significant difference in the accrual quality capability among the three chosen portfolios. A further test aiming at comparing $C_{BAQ}$s of portfolios is needed if we want to discover the difference among portfolios.

**4.2.3 Two-pairs Comparison of $C_{BAQ}$**

Since the previous test results of $C_{BAQ}$ are unequal, we should compare $C_{BAQ_i}$ and $C_{BAQ_j}$ individually. Three compared combinations are formed by pairing up three portfolios. We pair up Portfolio A with Portfolios B and C (hereafter “AB” and “AC”) and so on. Then we calculate $\hat{C}_{BAQ_i}/\hat{C}_{BAQ_j}$, $\sqrt{UCI}$ and $\sqrt{LCI}$ of each match using Eq. (12) and (13). The results are indicated in Table 4.

The results shown in Table 4 obviously point out that only $\hat{C}_{BAQ_i}/\hat{C}_{BAQ_j}$ interval estimation range for Match BC includes 1, so it cannot reject null hypothesis H3. Therefore, whether investors either choose Portfolio B or C, their accrual quality capability is no different in terms of statistical confidence. $\hat{C}_{BAQ_i}/\hat{C}_{BAQ_j}$ Interval estimation ranges are both larger than 1 for other matches.
(AB and AC), so they reject null hypotheses H3. In terms of statistical confidence, Portfolio A outperforms Portfolios B and C.

From our results above, investors can pick the best portfolio by conducting two-pair comparison against the chosen portfolio from investment set.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Portfolio</th>
<th>$\hat{C}<em>{BAQi} / \hat{C}</em>{BAQj}$</th>
<th>$\sqrt{UCI}$</th>
<th>$\sqrt{LCI}$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>2.666</td>
<td>5.137</td>
<td>1.382</td>
<td>Reject H3</td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>3.039</td>
<td>5.858</td>
<td>1.576</td>
<td>Reject H3</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>1.30</td>
<td>2.198</td>
<td>0.591</td>
<td>Did not reject H3</td>
</tr>
</tbody>
</table>

Remark: $\sqrt{UCI}$、$\sqrt{LCI}$，are calculated based on Eq. (12), Eq. (13) and the results are $F(\alpha/2, v_1, v_2) = 3.717$ and $F(1-\alpha/2, v_1, v_2) = 0.269$ respectively.

### 4.2.4 Statistical test for diversified Portfolio $C_{BAQ}$

To go a step further, we discuss the diversified Portfolios, X, Y and Z from 3 different companies and calculate the $S_p$、$\hat{C}_{BAQ}$ and $Pi$ and lower limit of confidence interval. The results are listed in Table 5. From Table 5; we know that Portfolios X, Y and Z have accrual quality capability.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$S_p$</th>
<th>$\hat{C}_{BAQ}$</th>
<th>Lower Limit of confidence interval</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.036</td>
<td>2.031</td>
<td>2.460</td>
<td>Reject H1</td>
</tr>
<tr>
<td>Y</td>
<td>0.043</td>
<td>1.689</td>
<td>2.046</td>
<td>Reject H1</td>
</tr>
<tr>
<td>Z</td>
<td>0.067</td>
<td>1.076</td>
<td>1.303</td>
<td>Reject H1</td>
</tr>
</tbody>
</table>
In addition, we conduct the homogeneity test on Portfolios X, Y and Z and calculate their $S_{P_{\text{max}}}^2$, $S_{P_{\text{mix}}}^2$ and $F_{\text{max}}$. Results are listed in Table 6.

Table 6: Hartley’s Homogeneity Test for Diversified Portfolio $C_{\text{RAQ}}$ ($a = 0.05$)

<table>
<thead>
<tr>
<th>Average Degree of Freedom</th>
<th>Statistic $F_{\text{ma}}$</th>
<th>Threshold $F_{\text{max}}$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0.005</td>
<td>0.001</td>
<td>3.565</td>
</tr>
</tbody>
</table>

Explanations:
1. $F_{\text{ma}}$ conducted by Hartley [6] did not obtain threshold $K=3$, $V=28$. So we replace it with the mean of $F_{\text{max}}$ threshold from: $K=3$, $V=20$, $F_{\text{max}}$ threshold=2.92 and $K=3$, $V=30$, $F_{\text{max}}$ threshold.

From Table 6, we know that there are significant differences in the capability of accrual quality among Portfolios X, Y and Z. Hence, we take two-pair test for diversified Portfolios X, Y and Z, and the matches are XY, XZ and YZ and so on. $\hat{\hat{C}}_{\text{RAQ}_i} / \hat{\hat{C}}_{\text{RAQ}_j}$, $\sqrt{UCI}$ and $\sqrt{LCI}$ are calculated by using Eq. (12) and (13). The results are listed in Table 7.

Table 7: Joint Confidence Intervals Test on Diversified Portfolio $C_{\text{RAQ}}$

<table>
<thead>
<tr>
<th>Portfolio i</th>
<th>Portfolio j</th>
<th>$\hat{\hat{C}}_{\text{RAQ}<em>i} / \hat{\hat{C}}</em>{\text{RAQ}_j}$</th>
<th>$\sqrt{UCI}$</th>
<th>$\sqrt{LCI}$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
<td>1.446</td>
<td>1.732</td>
<td>0.835</td>
<td>Did not reject H3</td>
</tr>
<tr>
<td>X</td>
<td>Z</td>
<td>3.565</td>
<td>2.719</td>
<td>1.311</td>
<td>Reject H3</td>
</tr>
<tr>
<td>Y</td>
<td>Z</td>
<td>2.466</td>
<td>2.261</td>
<td>1.090</td>
<td>Reject H3</td>
</tr>
</tbody>
</table>

It is clear that Portfolios X and Y have the same accrual quality capability in terms of statistic confidence. The accrual quality capabilities of Portfolios X and Y also outperform that of Portfolio Z.
4.3 Investment Decision Process

A flow chart of the investment decision process is illustrated in Figure 2 for investors who take the firm’s accrual quality into consideration in investment decision-making. In Figure 2, the original decision-making point from the investment risk is calculated to be within the risk tolerance range of investors. Even so, the decision-making point can be moved forward another step - the capability index analysis $C_{BAQ}$ of accrual quality, which provides for investors use of the test procedures developed in our study. Information cost can be saved during the investment decision-making.

Figure 2: Flow Chat of Investment Decision Process
5 Conclusions

When investors are making investment decisions, accrual quality decreases due to estimation errors of working capital, and the investment risk increases as a consequence. To reduce investment risk in terms of accrual quality, we refer to the concept of quality capability. We also adopt the regression model of accrual quality that introduced by Dechow and Dichev [1]. The capability index of basic accrual quality was established in order to distinguish the degree of accrual quality. To go a step further, we develop the mathematical relationship between the capability index of basic accrual quality and investment risk from the viewpoint of investors. Our empirical results show that the higher the capability index of basic accrual quality for portfolio, the lower the investment risk, and vice versa. Tendency range and dispersion range can be considered in future studies about capability index of accrual quality to develop a modifying capability index of accrual quality.

References


Appendix A

To test whether the capability of $C_{BAQ}$ is larger than 1, we extend the basic concept of confidence interval of variance to write Eq. (A1):

\[
\sqrt{\frac{(n-1)S_p^2}{\chi^2_{(n-1, -\alpha/2)}}} \leq \sigma_p \leq \sqrt{\frac{(n-1)S_p^2}{\chi^2_{(n-1, \alpha/2)}}}
\]  

(A1)

To further work out Eq. (A2):

\[
\sqrt{\frac{\chi^2_{(n-1, -\alpha/2)}}{(n-1)S_p^2}} \leq \frac{1}{\sigma_p} \leq \sqrt{\frac{\chi^2_{(n-1, -\alpha/2)}}{(n-1)S_p^2}} = \frac{USL - LSL \times \chi^2_{(n-1, -\alpha/2)}}{3.29S_p} \leq \frac{USL - LSL \times \chi^2_{(n-1, -\alpha/2)}}{3.29S_p} \leq \frac{\chi^2_{(n-1, -\alpha/2)}}{(n-1)}
\]

\[
= \hat{C}_{BAQ} \times \sqrt{\frac{\chi^2_{(n-1, -\alpha)}}{n-1}} \leq C_{BAQ} \leq \hat{C}_{BAQ} \times \sqrt{\frac{\chi^2_{(n-1, -\alpha/2)}}{n-1}}
\]  

(A2)

Because we adopt the left-tailed test, the confidence interval is

\[
\hat{C}_{BAQ} \times \sqrt{\frac{\chi^2_{(n-1, -\alpha)}}{n-1}} \leq C_{BAQ} \leq \infty
\]  

(A3)

Appendix B

If null hypothesis H1: $C_{BAQ1} = C_{BAQ2} = C_{BAQ3}$ is true, and then Eq. (11) can be written as:

\[
F_{max} = \frac{\text{Min} \left\{ \left( \hat{C}_{BAQ1} \right)^2, \left( \hat{C}_{BAQ2} \right)^2, \left( \hat{C}_{BAQ3} \right)^2 \right\}}{\text{Max} \left\{ \left( \hat{C}_{BAQ1} \right)^2, \left( \hat{C}_{BAQ2} \right)^2, \left( \hat{C}_{BAQ3} \right)^2 \right\}}
\]
Construction of Investment Risk Measure ...

\[ \text{Max} \left\{ \left( \frac{C_{BAQ1}}{\hat{C}_{BAQ1}} \right)^2, \left( \frac{C_{BAQ2}}{\hat{C}_{BAQ2}} \right)^2, \left( \frac{C_{BAQ3}}{\hat{C}_{BAQ3}} \right)^2 \right\} \]

\[ \text{Min} \left\{ \left( \frac{C_{BAQ1}}{\hat{C}_{BAQ1}} \right)^2, \left( \frac{C_{BAQ2}}{\hat{C}_{BAQ2}} \right)^2, \left( \frac{C_{BAQ3}}{\hat{C}_{BAQ3}} \right)^2 \right\} \]

(B1)

If \( d = \text{USL} - \text{LSL} \), then:

\[ C_{BAQ} = \frac{1.645 \sigma_p}{\hat{C}_{BAQ}} = \frac{d}{1.645 \sigma_p} = \frac{S_p}{\sigma_p} = \sqrt{\frac{S_p^2}{\sigma_p^2} (n-1)} = \sqrt{\frac{(n-1)S_p^2}{\sigma_p^2} * \frac{1}{n-1}} = \sqrt{\frac{\chi^2}{v}} \quad [v = n-1] \]  

(B2)

To substitute \( \left( \frac{C_{BEQI}}{\hat{C}_{BEQI}} \right)^2 \sim \chi^2_{v_i}/v_i \) into Eq. (B1), it is written as:

\[ F_{\text{max}} = \frac{\text{Max}\left\{ \chi^2_{v_1}/v_1, \chi^2_{v_2}/v_2, \chi^2_{v_3}/v_3 \right\}}{\text{Min}\left\{ \chi^2_{v_1}/v_1, \chi^2_{v_2}/v_2, \chi^2_{v_3}/v_3 \right\}} \]

(B3)

Furthermore, if we conduct a variance test toward the five portfolios by \( F_{\text{max}} \) method in Hartley [6], its null hypothesis is \( H_2b: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \) and its test statistic is:

\[ F_{\text{max}} = \frac{s_{\text{max}}^2}{s_{\text{min}}^2} = \frac{\text{Max}\{s_1^2, s_2^2, s_3^2\}}{\text{Min}\{s_1^2, s_2^2, s_3^2\}} \]  

(B4)

If \( H_2b \) is true, and then Eq. (B4) can be:

\[ F_{\text{max}(5,\bar{v}-1)} = \frac{\text{Max}\left\{ \frac{v_1s_1^2}{\sigma_1^2}, \frac{v_2s_2^2}{\sigma_2^2}, \frac{v_3s_3^2}{\sigma_3^2} \right\}/v_1, \frac{v_2s_2^2}{\sigma_2^2}, \frac{v_3s_3^2}{\sigma_3^2} \right\}/v_2, \frac{v_3s_3^2}{\sigma_3^2} \right\}/v_3 \right\}}{\text{Min}\left\{ \frac{\chi^2_{v_1}}{v_1}, \frac{\chi^2_{v_2}}{v_2}, \frac{\chi^2_{v_3}}{v_3} \right\}} \]

(B5)

Since (B3)=(B5), so \( F_{\text{max}} \sim F_{\text{max}(5,\bar{v}-1)} \), in which \( \bar{v} = \sum v_i / 3 \).
Appendix C

To test whether the confidence interval of $\frac{C_{BAQi}}{C_{BAQj}}$ includes 1, we come up with equality according to Eq. (B2):

$$
\left( \frac{\hat{C}_{BAQi}}{\hat{C}_{BAQj}} \right)^2 = \left( \frac{C_{BAQi}}{C_{BAQj}} \right)^2 = \frac{\chi^2_{v_i} / v_i}{\chi^2_{v_j} / v_j} \sim F(v_1, v_2)
$$

(C1)

Therefore, we obtain the Upper Confidence Interval (hereafter UCI) and the Lower Confidence Interval (hereafter LCI) of $1-\alpha$ for $\left( \frac{C_{BAQi}}{C_{BAQj}} \right)^2$ and provide the following equality:

$$\text{UCI} = \left( \frac{\hat{C}_{BAQi}}{\hat{C}_{BAQj}} \right)^2 \times F_{\frac{\alpha}{2}}^\prime (v_1, v_2)$$

(C2)

$$\text{LCI} = \left( \frac{\hat{C}_{BAQi}}{\hat{C}_{BAQj}} \right)^2 \times F_{1-\frac{\alpha}{2}}^\prime (v_1, v_2)$$

(C3)

Then the UCL and LCI of $1-\alpha$ for $\left( \frac{C_{BAQi}}{C_{BAQj}} \right)$ is as below respectively:

$$\sqrt{\text{UCI}} = \left( \frac{\hat{C}_{BAQi}}{\hat{C}_{BAQj}} \right) \times \sqrt{F_{\frac{\alpha}{2}}^\prime (v_1, v_2)}$$

(C4)

$$\sqrt{\text{LCI}} = \left( \frac{\hat{C}_{BAQi}}{\hat{C}_{BAQj}} \right) \times \sqrt{F_{1-\frac{\alpha}{2}}^\prime (v_1, v_2)}$$

(C5)